System Design: The Absentee in System Theory

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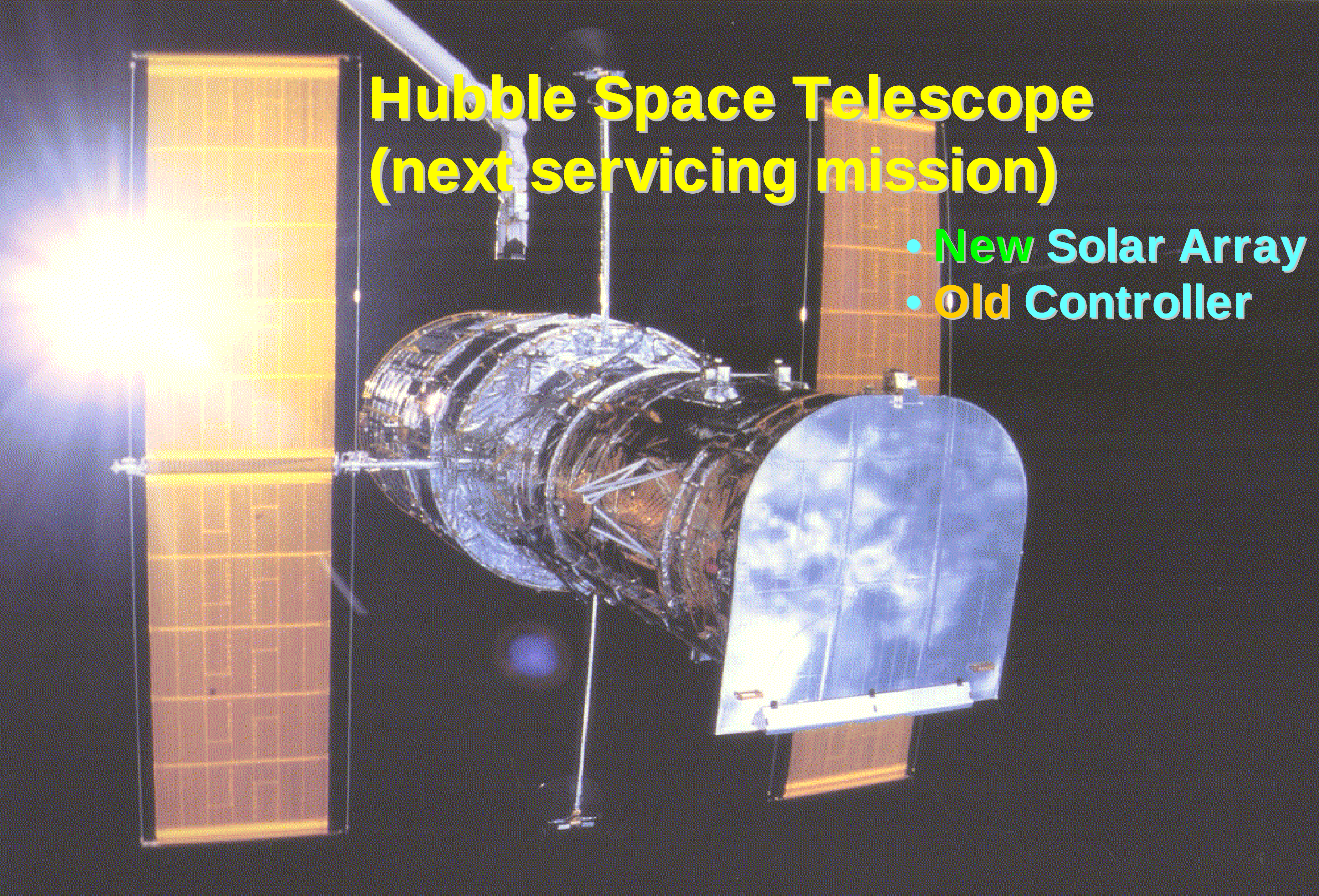
Dedicated to:

Osita Nwokah,

Friend, Scholar

With a Little Help From My Friends

Adhikari
Aldrich
Callafon
Grigoriadis
Helton
Kiichiro
Lu
Mingori
Murakami
Pinaud
Roman
Sato
Sultan
Williamson
Yamashita



Hubble Space Telescope (next servicing mission)

- New Solar Array
- Old Controller

Technology: What Paved the Way?

- 1st Half of 20th Century: **PHYSICS**
- 2nd Half of 20th Century: **ENGINEERING**

  (Component Technology)

  (Control is a component)
- Next? **SYSTEMS**

  (Interdisciplinary Technology)

Outline

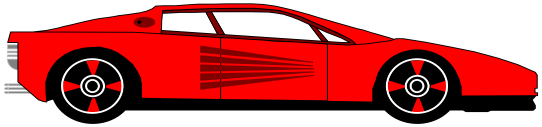
- Designing Systems
- Designing Models for Systems
- Inspirations From

ART

NATURE

- Designing Controlled Structures

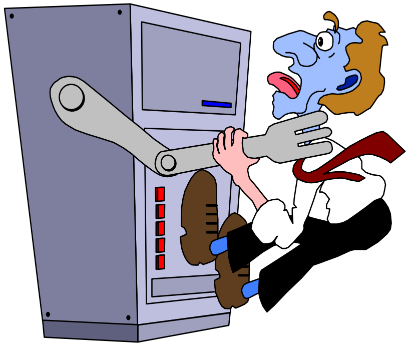
Pin the Tail on the Performance Limiting Technology

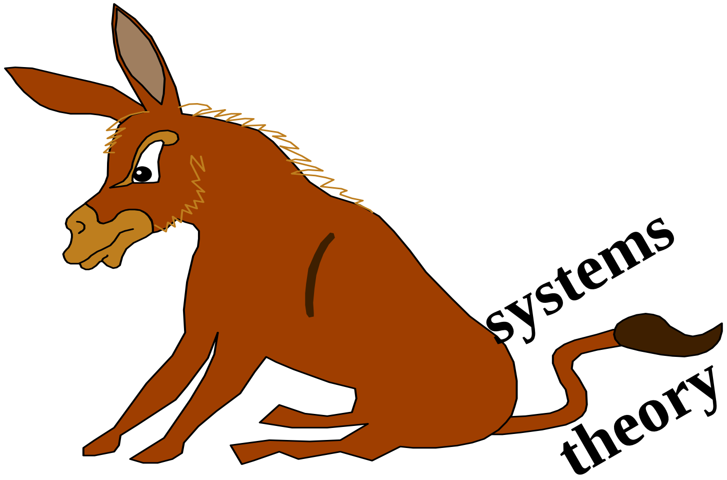


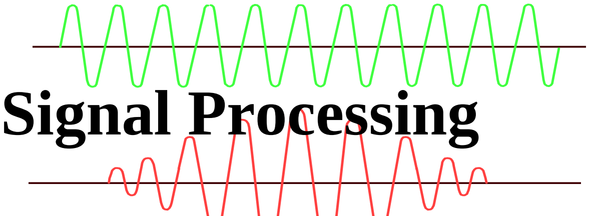
Manufacturing

Modeling

Computing

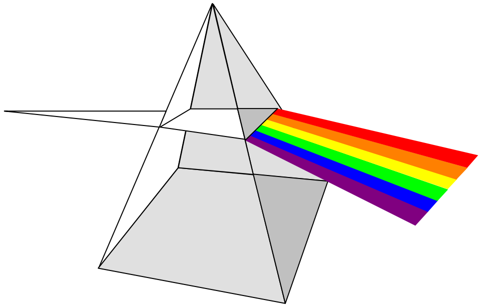






Signal Processing

Control



Physics

Sensing

Systems Design Today

- **Universities Teach Component Technology**
  - **materials, dynamics, signal processing, control**
- **Leading to: uncoordinated multidisciplinary tasks**
  - **Manufacture components, then**
  - **Model components (physics), then**
  - **Connect, Measure, Actuate components, then**
  - **Control the interconnected components**
- **Problem: dealing with sufficient rather than necessary**
- **Systems approach needed**
- ***Michael Faraday*: "Begin with the whole, then construct the parts"**

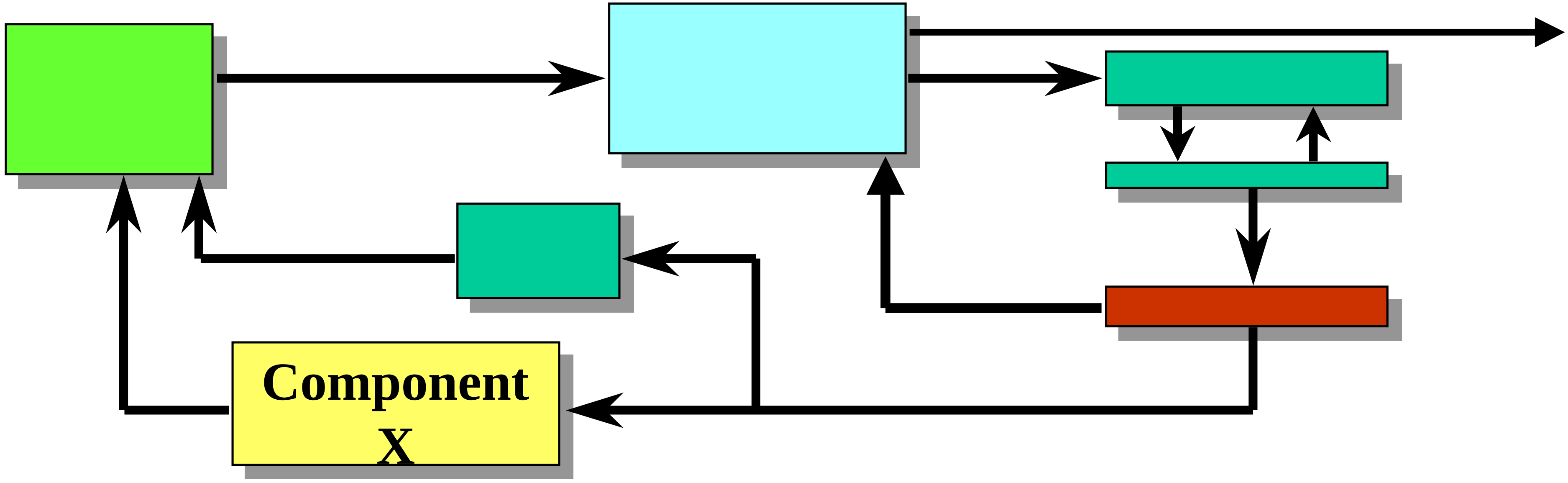
When is the Whole LESS Than the Sum of the Parts?

- The answer: usually
- Today, components are overdesigned to compensate for the lack of coordination in their design
- A Misconception: "The best system is made from the best components"
- Often, more gain in integrating two disciplines, than gained by new technology in either discipline
- Examples …………

Given a System Requirement

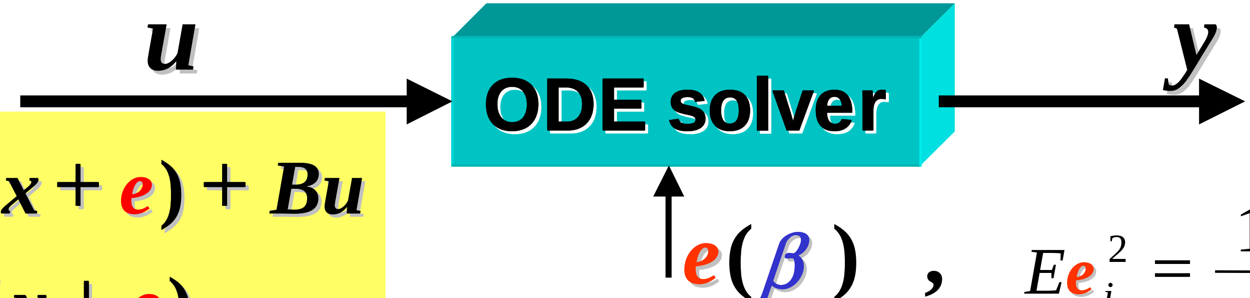
Where Should We Invest?

- How accurately to **Model** component X?
- How accurately to **Manufacture** component X?
- Is Component X even **Necessary**?
- How should the components be **Connected**?



- **Control** is a **Component** technology

Finite Precision Computing



$$\delta x = A(x+e) + Bu$$

$$y = C(x+e)$$

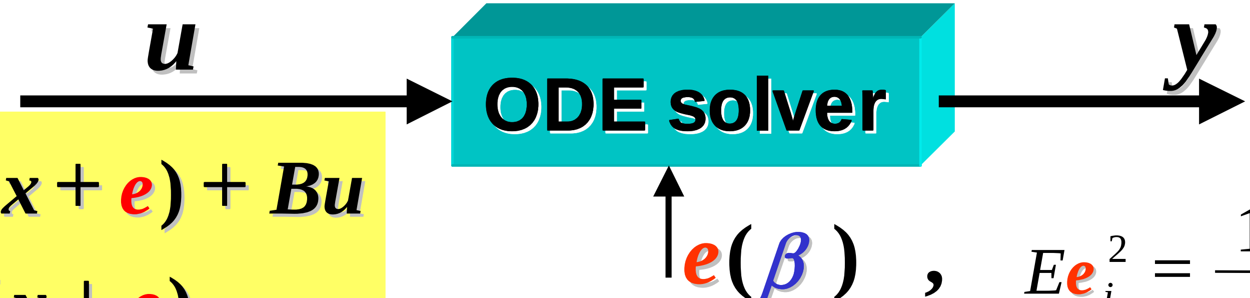
$e(\beta)$, $Ee_i^2 = \frac{1}{12}2^{-2\beta}$

$x = Tv$: **No Clue about Basis T From Physics**

$$y = G(z)u + G_e(z,T)e$$

- **Useless to Model Better Than Error *e (unbounded over T)***
- **How We.... MODEL One Component Affects The Dynamics of Another**

Unified Signal Processing/Control



$$\delta x = A(x+e) + Bu$$

$$y = C(x+e)$$

$$e(\beta)\ , \quad Ee_i^2 = \frac{1}{12}2^{-2\beta}$$

$$x = T\ v$$

No Clue about Basis T From Physics

$$y = G(z)u + G_e(z,T)e$$

$$\min_T \left\| G_e(T)\ e(\beta) \right\|$$

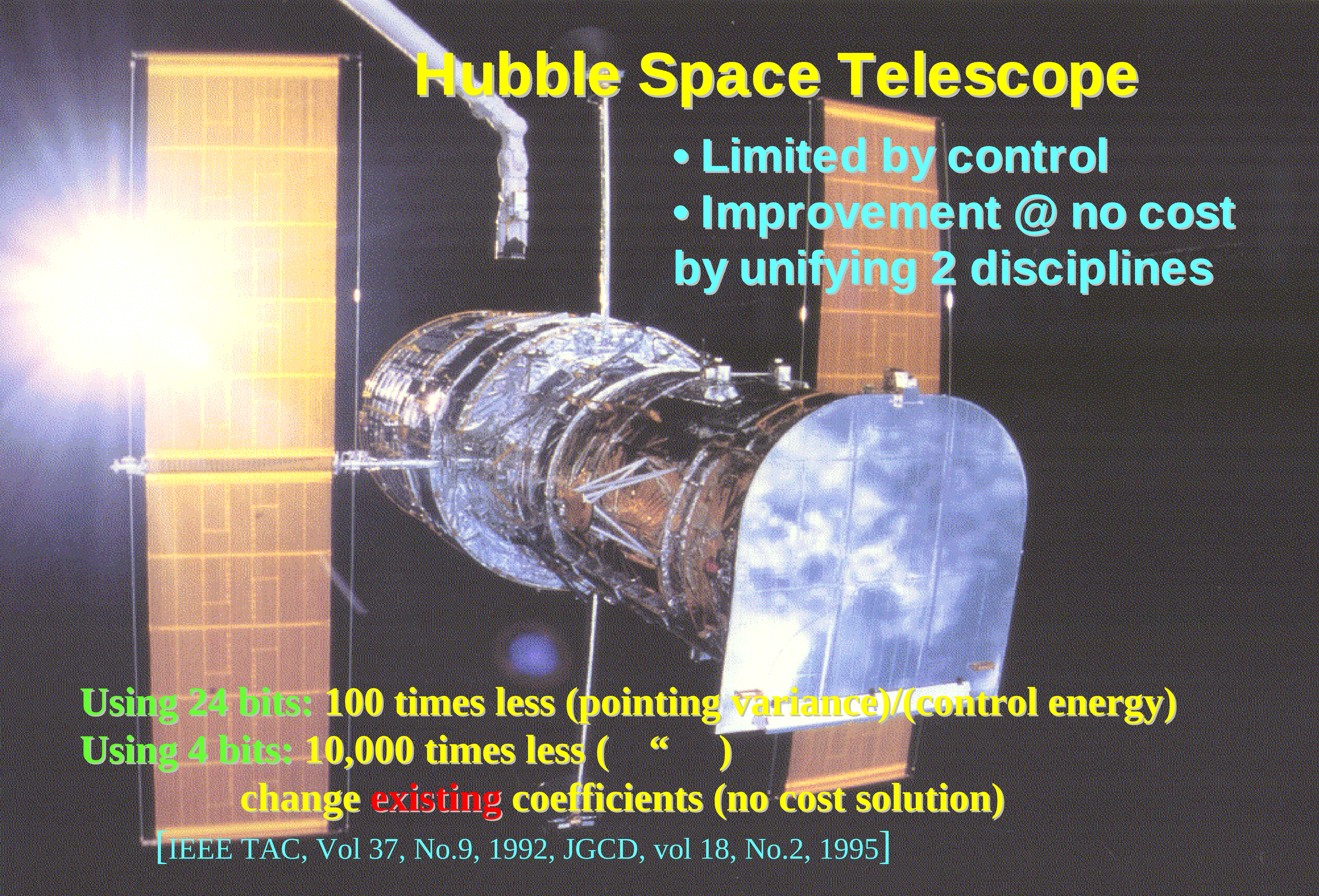
$$\ni (T^{-1}(Exx^T)T^{-T})_{ii} = 1$$

[Mullis/Roberts 76]
[Williamson 86]
[liu/Grigoriadis/Skelton 88]
[Gevers 92, Bamieh 94]

- **Component technology: Design (A,B,C), then T**
- **Systems technology: Design (A,B,C,T) jointly**

Control: Coupled ARE(β)

Hubble Space Telescope



- Limited by control
- Improvement @ no cost by unifying 2 disciplines

Using 24 bits: 100 times less (pointing variance)/(control energy)
Using 4 bits: 10,000 times less (“)
change existing coefficients (no cost solution)

[IEEE TAC, Vol 37, No.9, 1992, JGCD, vol 18, No.2, 1995]

Optimal Mix of Plant/Control Design

[Grigoriadis, Zhu, Skelton, 1992]

$$\min_{K} E\, u^T u$$

$$E\, y\, y^T \le Y$$

Guarantee Performance Y

Convex, given Y

$$\underline{p}_i \le p_i \le \bar{p}_i$$

$$\dot{x} = A(p)x + B(p)u + Dw\,,$$
$$y = Cx$$

$$[A \;\; B] = M + \sum p_i M_i$$

Optimal Mix of Plant/Control Design

[Grigoriadis, Zhu, Skelton, 1992]

$$\min_{p,\,K} \; E\, u^T u$$

$$E\, y\, y^T \leq Y$$

Update

Plant p,

Control K

Not Convex

$$\underline{p}_i \leq p_i \leq \bar{p}_i$$

$$\min_{K} \; E\, u^T u$$

$$E\, y\, y^T \leq Y$$

Guarantee

Performance Y

Convex, given Y

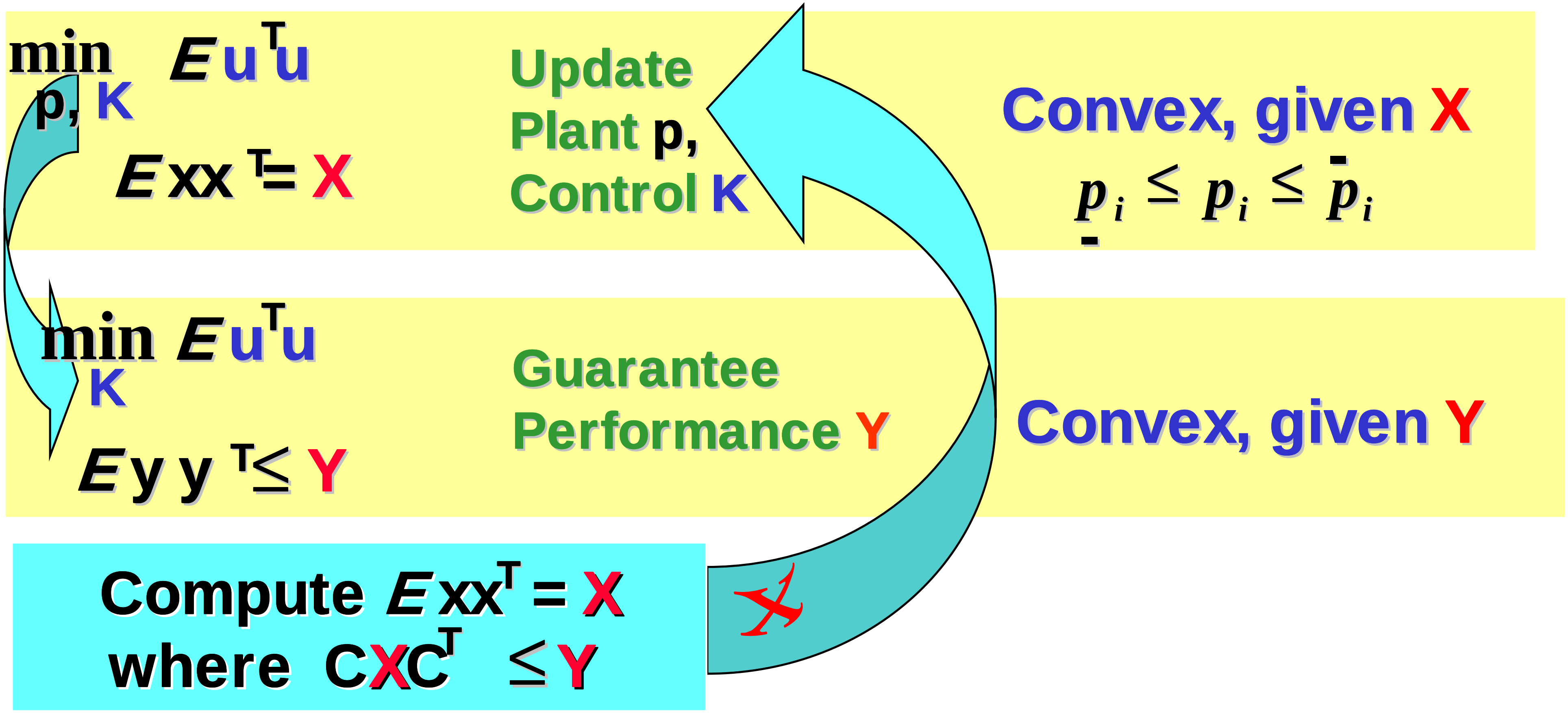
$$\dot{x} = A(p)x + B(p)u + Dw\,,$$

$$y = Cx$$

$$[A \;\; B] = M + \sum p_i M_i$$

Optimal Mix of Plant/Control Design

[Grigoriadis, Zhu, Skelton, 1992]



$$\min_{p,\,K}\ E\, u^T u$$

$$E\, x x^T = X$$

Update Plant p, Control K

Convex, given X

$$\underline{p}_i \le p_i \le \bar{p}_i$$

$$\min_{K}\ E\, u^T u$$

$$E\, y y^T \le Y$$

Guarantee Performance Y

Convex, given Y

Compute $E\, x x^T = X$

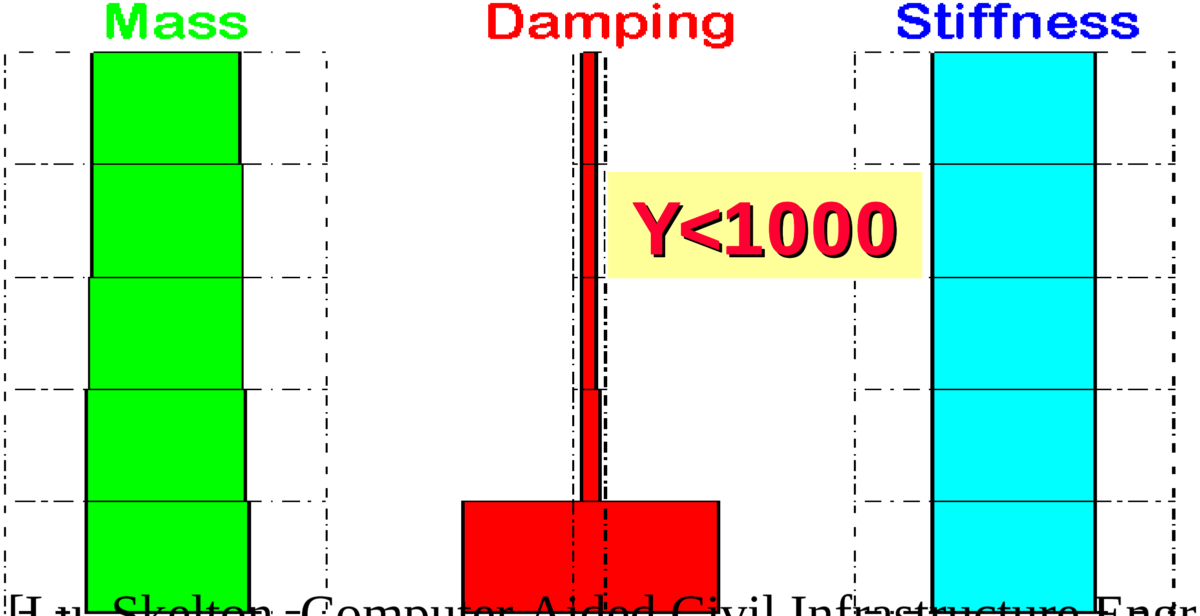
where $C X C^T \le Y$

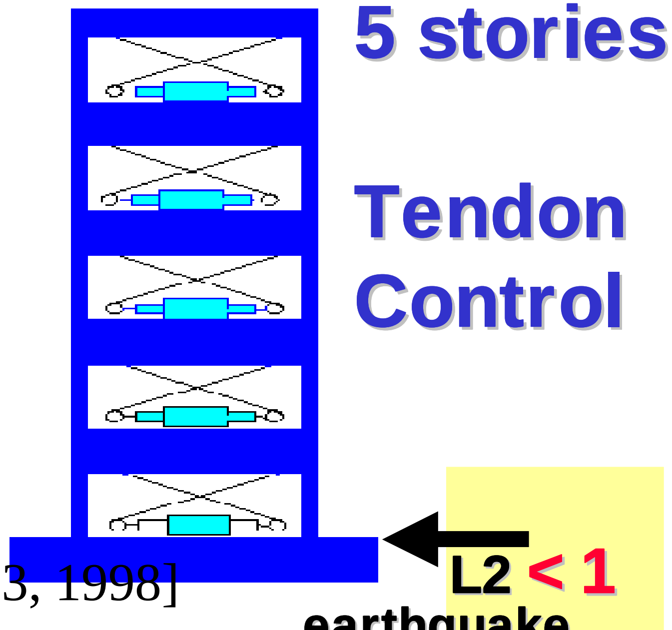
$$\dot{x} = A(p)x + B(p)u + Dw\,,$$

$$y = Cx$$

$$[A \;\; B] = M + \sum p_i M_i$$

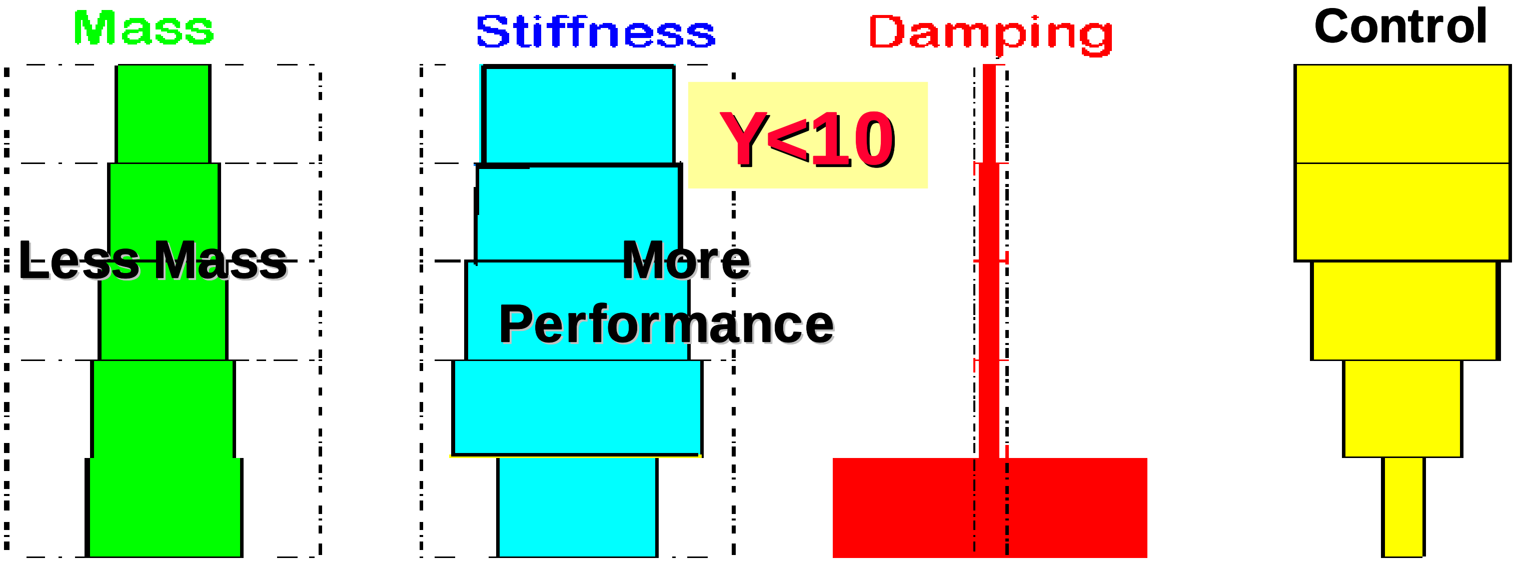
Optimal Mass/Damping/Stiffness/Control





earthquake

[Lu, Skelton, Computer Aided Civil Infrastructure Engr, vol 13, 1998]

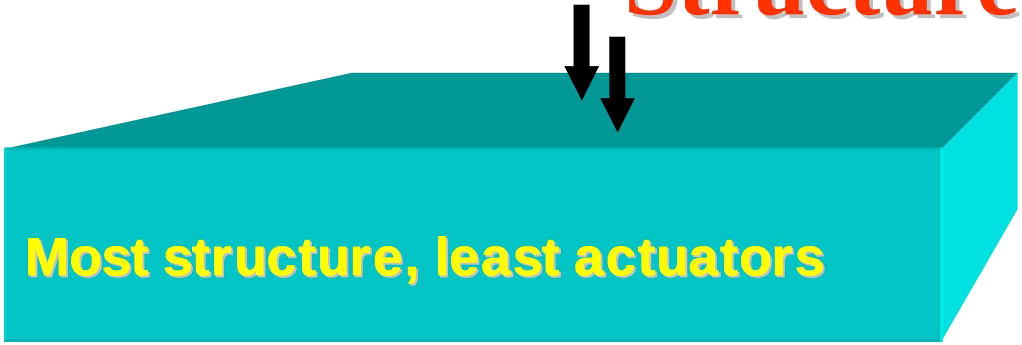


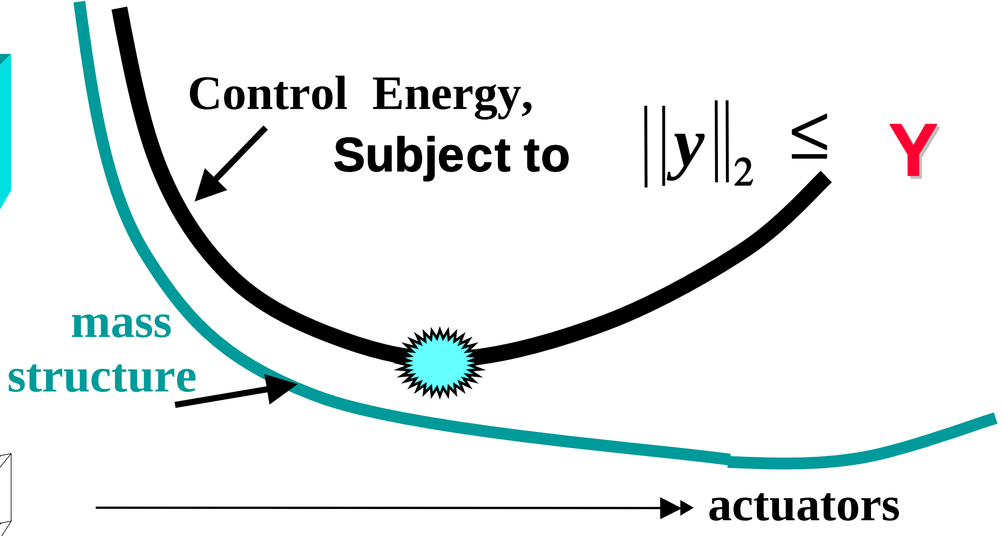
Less Mass

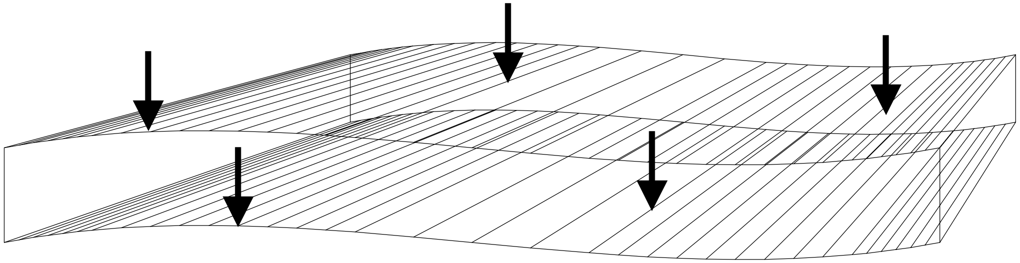
More Performance

More Actuators

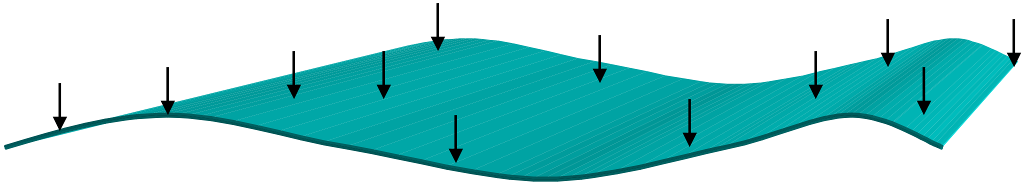
Optimal Mix of Physics/Information, Structures/Control







less structure, more actuators



less structure, more actuators

- **Optimal Number Actuators**
- **Less energy, more robust**
- **More complex controller**
- **Avoiding integer program……**

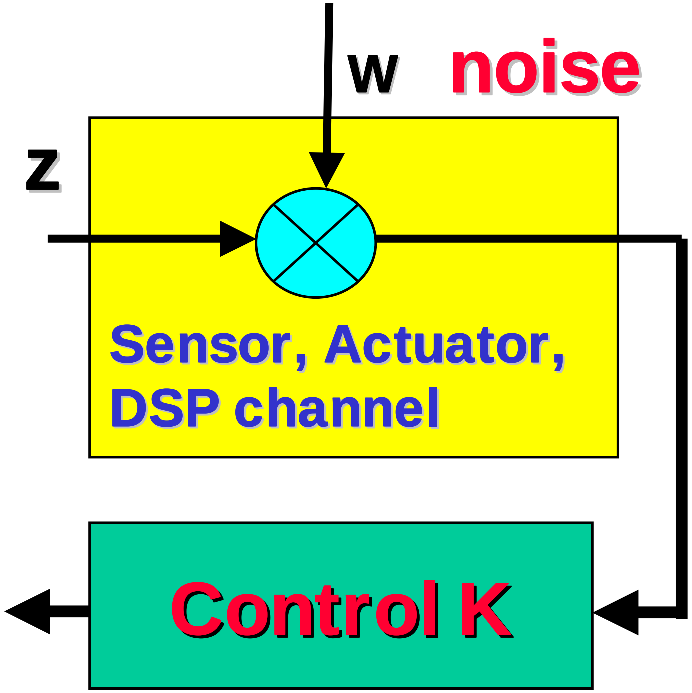
The Economic Design Problem

$$W = \sigma Z$$

Finite Signal-to-Noise ratio

$$E w_j w_k = W \delta_{jk} \quad , \quad E z z^T = Z$$

$$\$ = \sum_i p_i \sigma_i^{-1}$$



Min $\$$, subject to $E yy^T < Y$
$\sigma,$ K

Convergent algorithm, but no global

If $\sigma_1^{-1} >> \sigma_2^{-1}$, then delete sensor 2, or off-the-shelf

[Lu, Skelton, Automatica, to appear]

Outline

- **Designing Systems**
- **Designing Models for Systems**
- **Inspirations From**

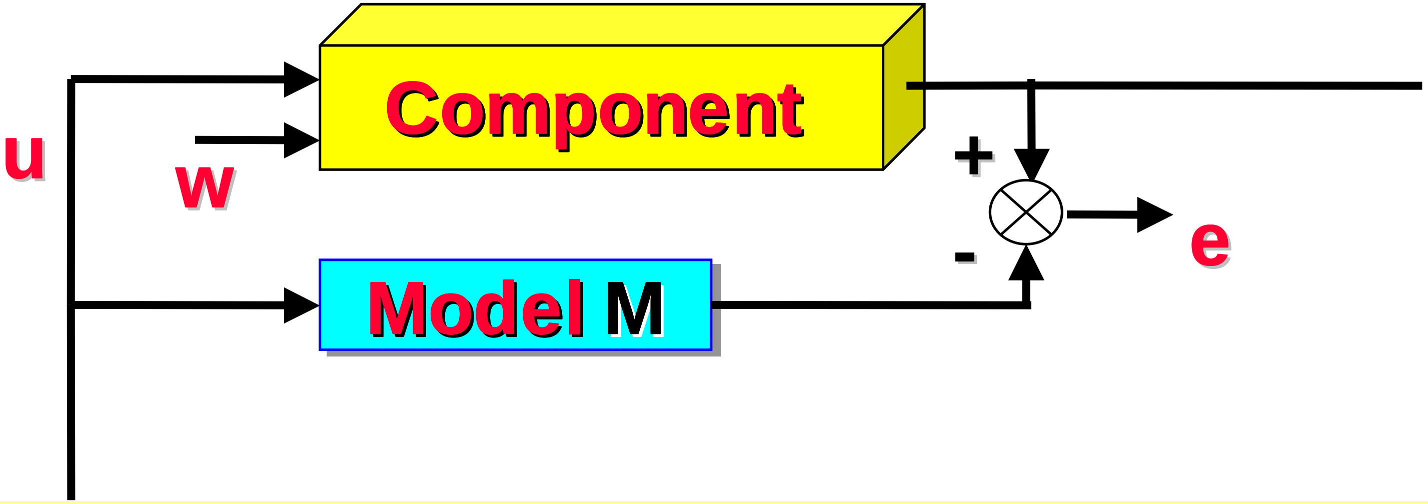
ART

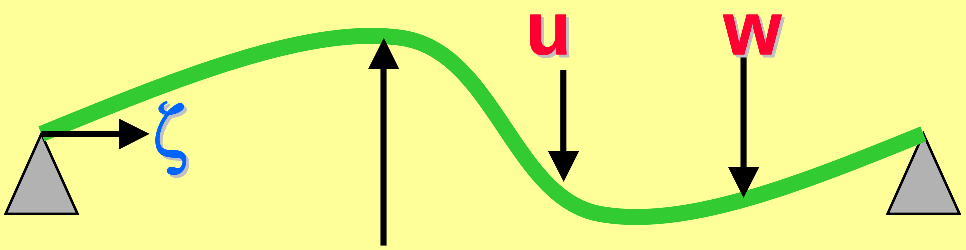
NATURE

- **Designing Controlled Structures**

Component Modeling

[Hu/Skelton, Computers and Structures,1985]





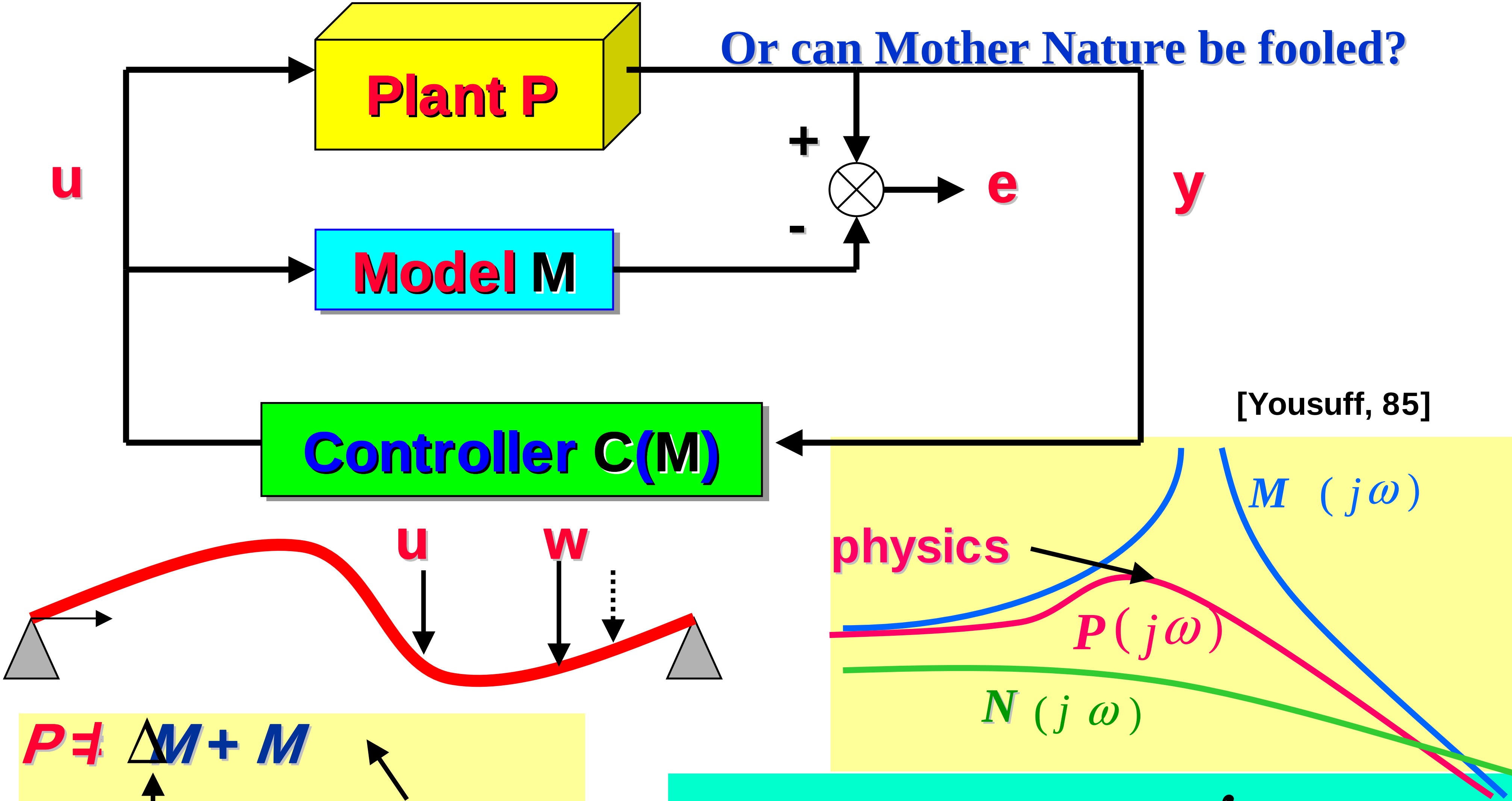
$$\mu(\zeta, t) = \Phi(\zeta)\ v(t)$$

- From physics,
  Choose Φ for small e
- From systems criteria,
  Choose Φ to depend on u,w

recall $x = T\,v$

Choose T to depend on w(b)

Is System Modeling Just Physics, Physics?



Or can Mother Nature be fooled?

[Yousuff, 85]

$P = \Delta M + M$

Desired focus

Robust Control Theory

$$C(P) = C(N) = C(M) \Longleftarrow \int (8y^2 + u^2)\,dt$$

Control Models : How Much Info is Really Necessary?

$$\textit{Minimize} \quad \sum_{k=1}^{N} (y_k^T Q y_k + u_k^T R u_k)$$

$$\textit{Subject to} \quad x_{k+1} = A x_k + B u_k \quad , \quad y_k = C x_k$$

[Shi, Skelton, DATA-BASED CONTROL, '94], [Furuta, '93], [Ikeda, '99]

Theorem

Optimal Controller **Requires Only**

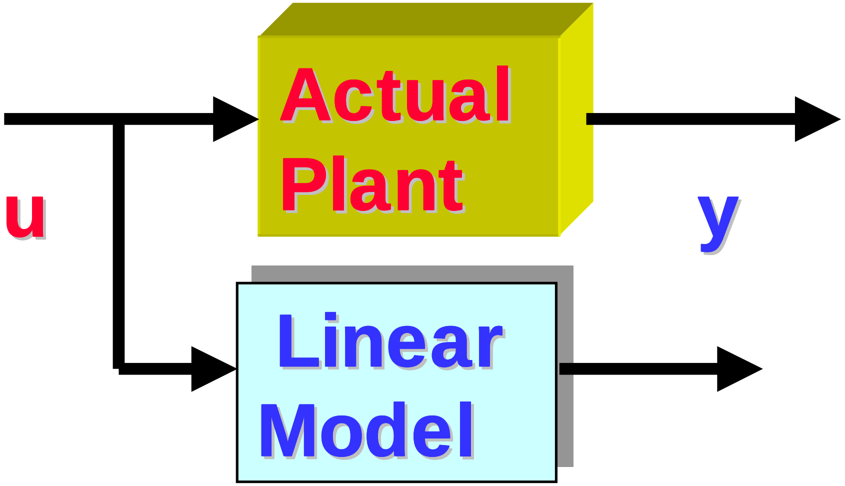
$CA^i B \;, \; i = 0,1,2,\ldots\ldots, N-1$

- **Only Errors in $CA^i B$ Affect Control Performance**
- **Any QMC from data yields the optimal control**
- **Why compute Markov Parameters, Use Data Directly**

Data Equivalent Models

[Skelton, Zhu, Qmarkov COVER, 1991]

Does there exist any linear model to Match the input/output data?



IFF $$R - HH^T \geq 0$$

$$R_i = E y_{k+i} y_k^T \;, \qquad H_i = E y_{k+i} u_k^T$$

$$R = \begin{bmatrix} R_0 & R_1^T & R_2^T & R_3^T \\ R_1 & R_0 & R_1^T & R_2^T \\ R_2 & R_1 & R_0 & R_1^T \\ R_3 & R_2 & R_1 & R_0 \end{bmatrix}, \qquad H = \begin{bmatrix} H_0 & 0 & 0 & 0 \\ H_1 & H_0 & 0 & 0 \\ H_2 & H_1 & H_0 & 0 \\ H_3 & H_2 & H_1 & H_0 \end{bmatrix}$$

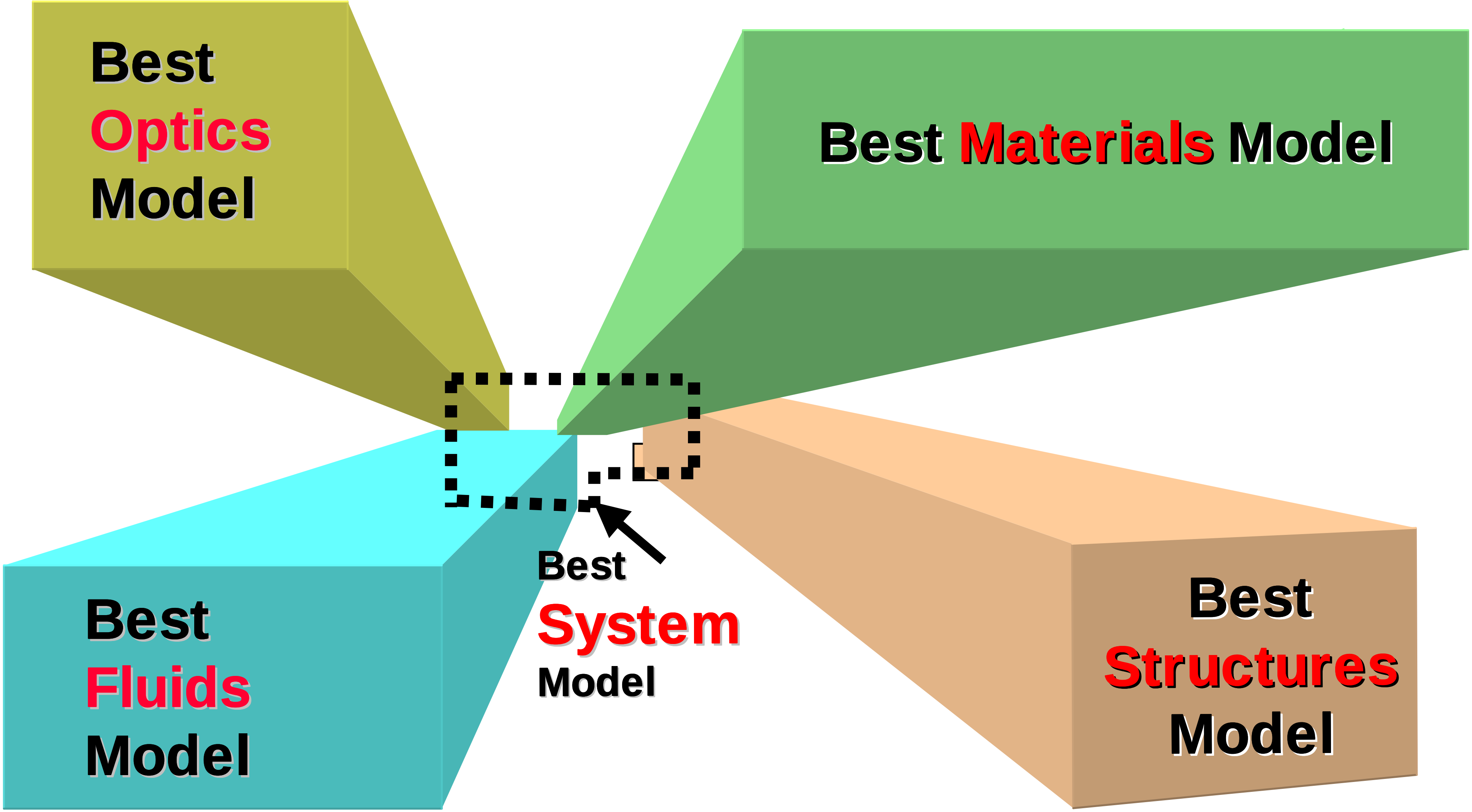
Many Models Equivalent to the One From Physics

Absentee: A System Modeling Theory

- **The size of the error is Not Continuous from Component (OL) to System (CL)**
  - **Unbounded OL error, but zero CL error**
  - **Smaller OL errors $\not\Rightarrow$ smaller CL errors. Hence….**
- **Good Component Models $\not\Rightarrow$ Good System Models**
  - **There might exist a Better model for System Design than the Actual model from physics.**
  - **Bad News: Investments in Component Modeling may Not Help System Modeling**
  - **Good News: Good System Models can be Simpler Than Component Models (Yes, There Exists an Optimal Size)**
- **Should control design occur Before, After, or During plant modeling?**

The Best System Model

- May have little to do with the best **Component Model**



Best
Optics
Model

Best Materials Model

Best
Fluids
Model

Best
System
Model

Best
Structures
Model

Outline

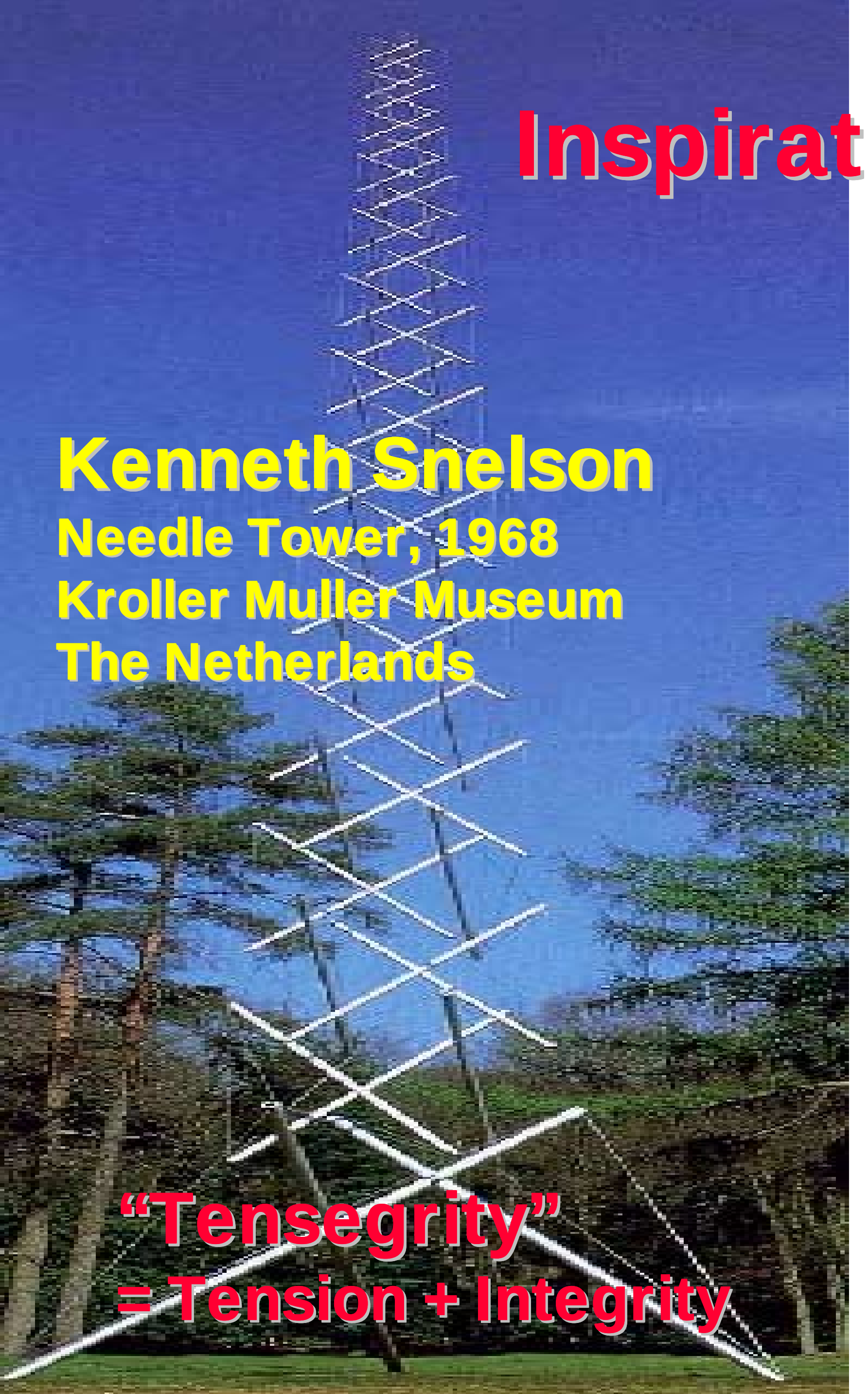
- **Designing Systems**
- **Designing Models for Systems**
- **Inspirations From** ART

  BIOLOGY
- **Designing Controlled Structures**

Integrating Material, Structure, Control

- The Theory of Continua is inadequate to account for the strength of materials.
- Challenge for Man-made Systems: Architecture
  - **Information Architecture**: selection of sensors, actuators, and feedback paths.
  - **Material Architecture:** selection of the material geometry
- Look to Biological and Natural systems, where
  - Mechanical, Chemical, and Electrical forces are involved in complex patterns of information flow, sensing, and feedback

Inspiration From Art



Kenneth Snelson

Needle Tower, 1968

Kroller Muller Museum

The Netherlands

“Tensegrity”

= Tension + Integrity

After 30 Years of

- **Forcing Continua**
- **Adding Actuators to**

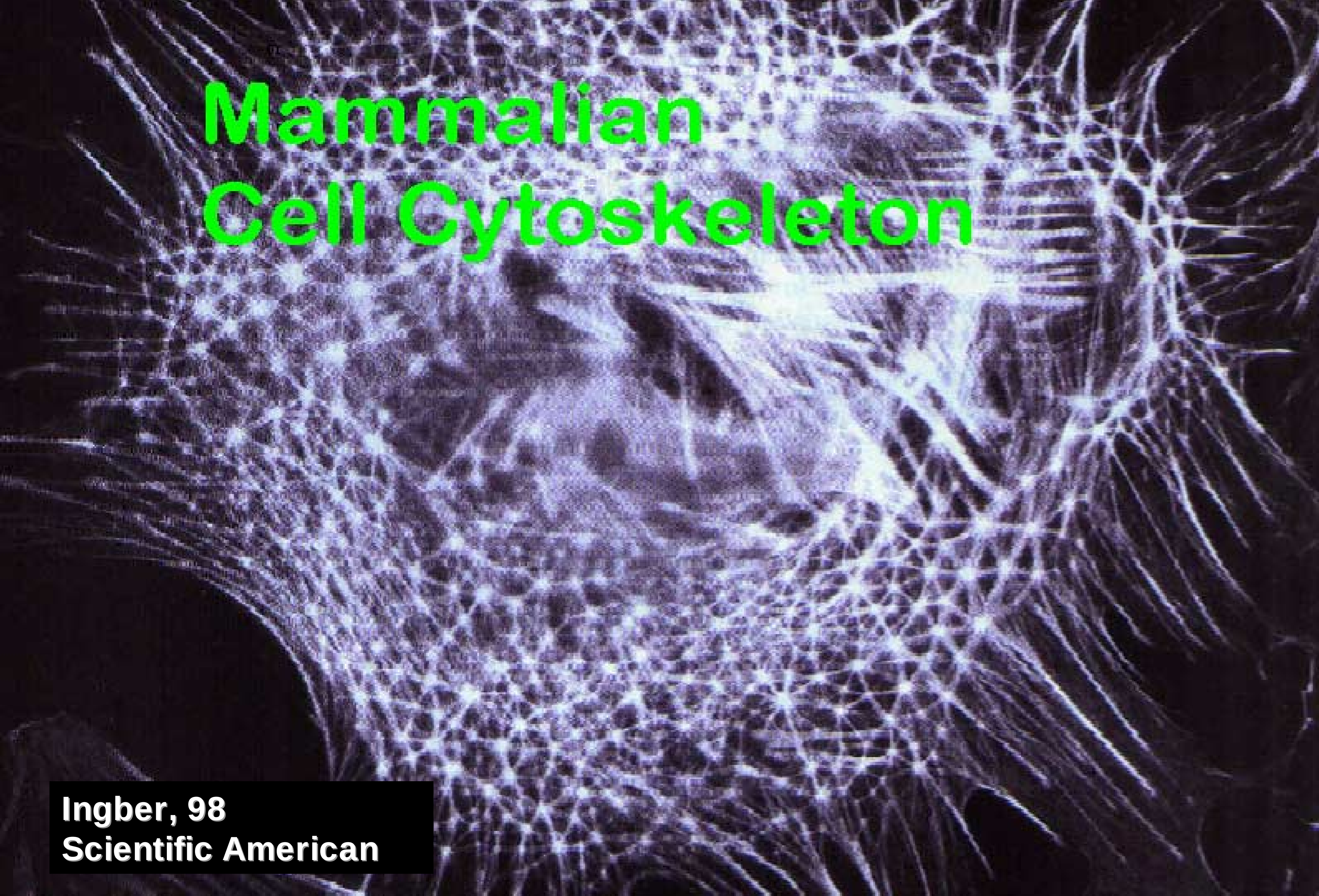
Old Paradigms:

Beams, Plates, Shells

Eureka !!!!

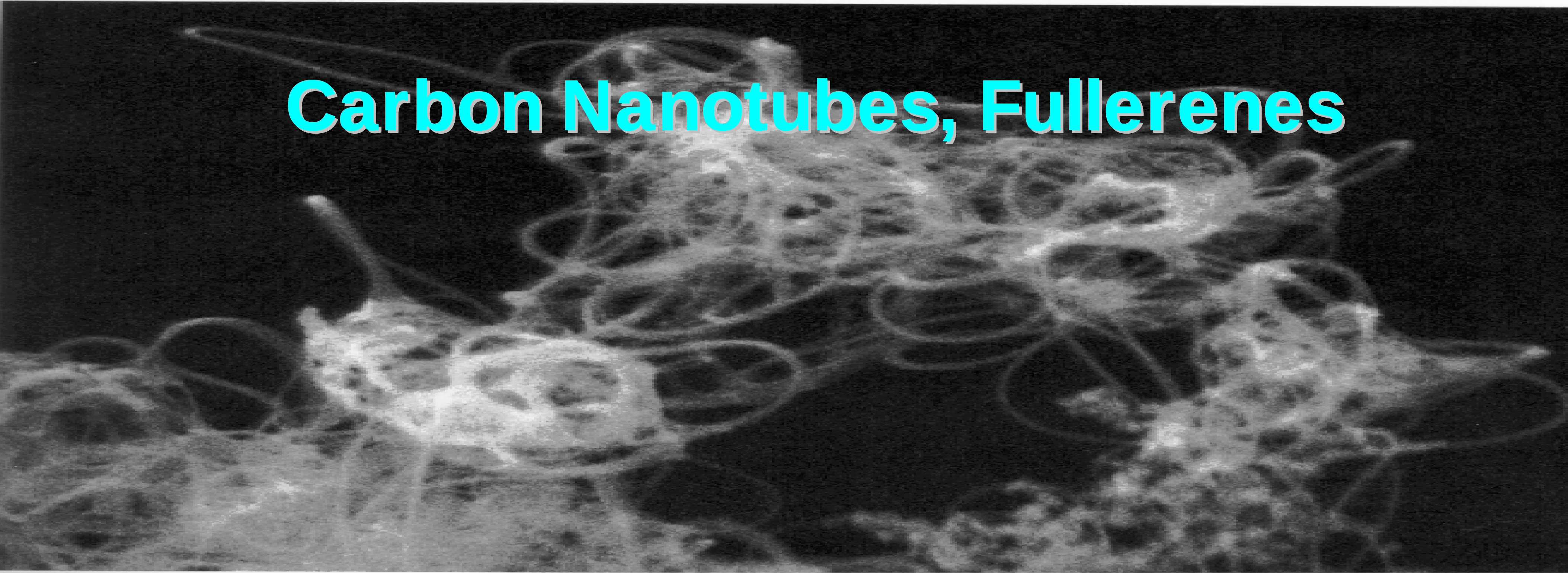
- **No Joints**
- **No Load Reversals**
- **No Friction**
- **No Member Bending**
- **Easy to Change Equilibrium**

Mammalian Cell Cytoskeleton

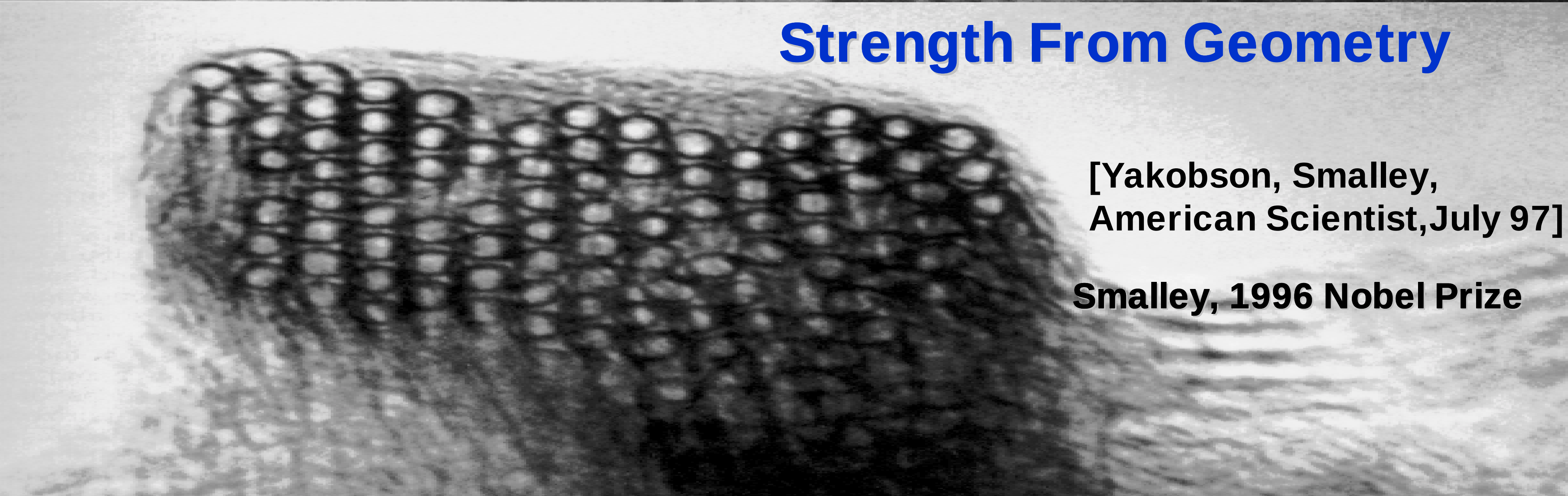


Ingber, 98
Scientific American

Carbon Nanotubes, Fullerenes

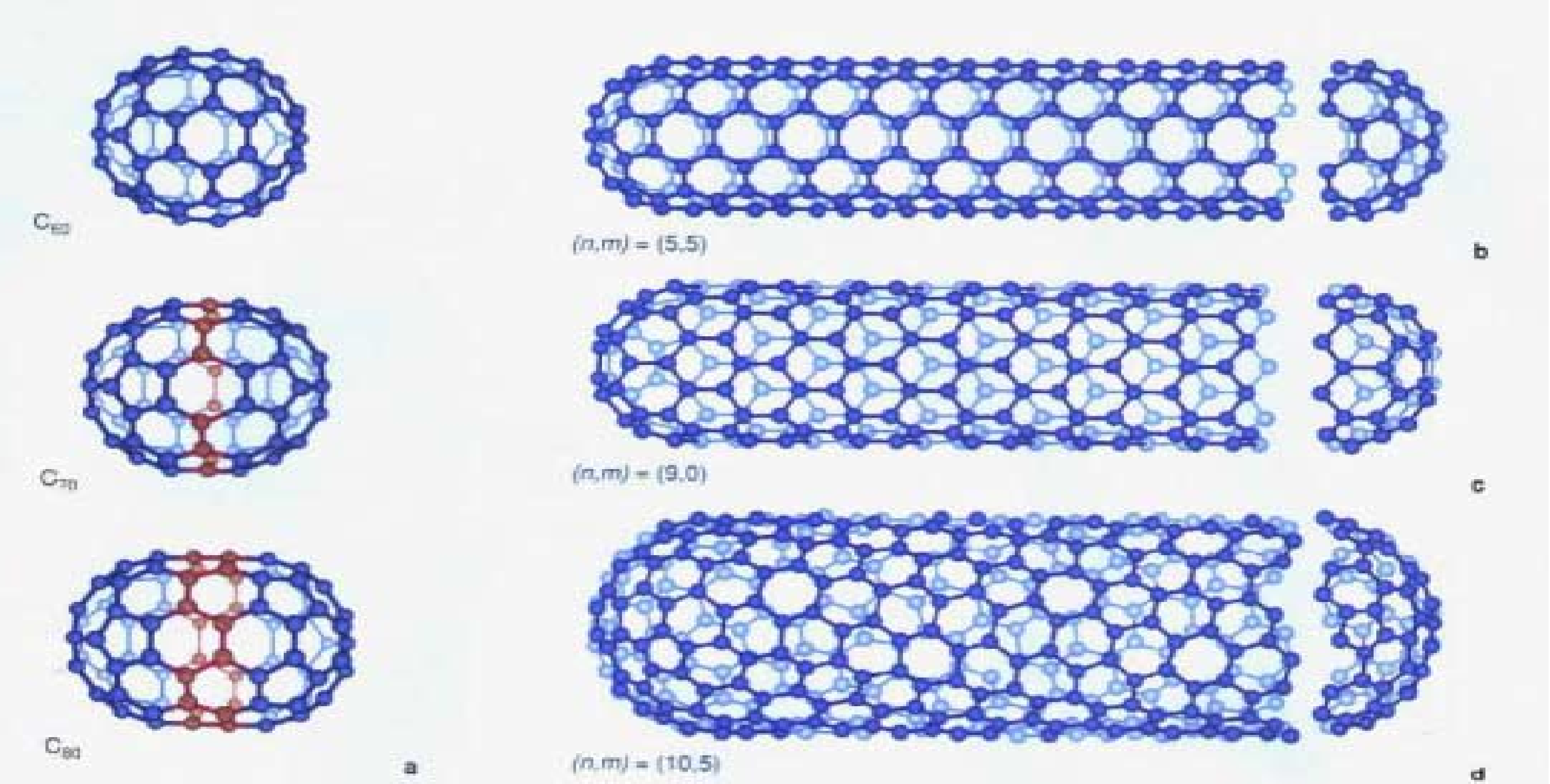


Strength From Geometry

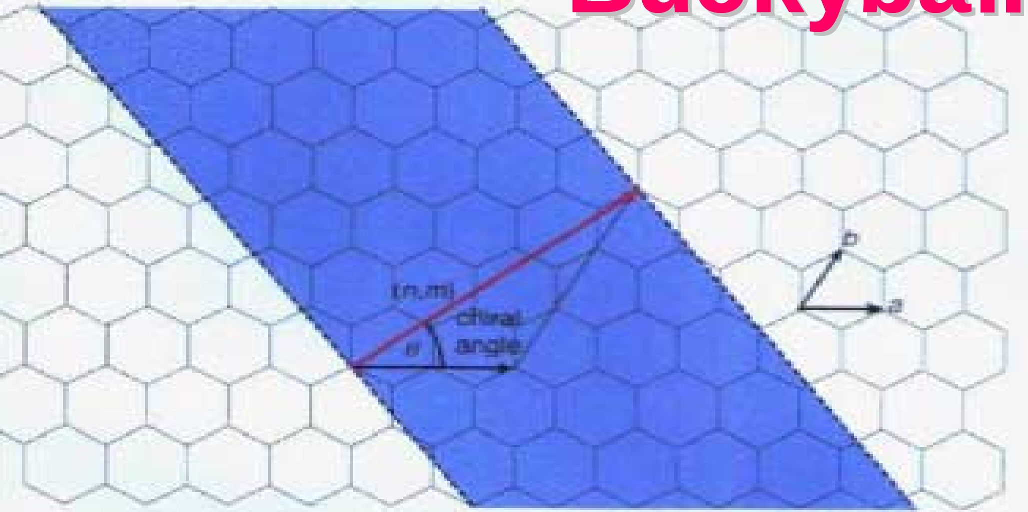


[Yakobson, Smalley,
American Scientist,July 97]

Smalley, 1996 Nobel Prize



Buckyballs and Fullerenes



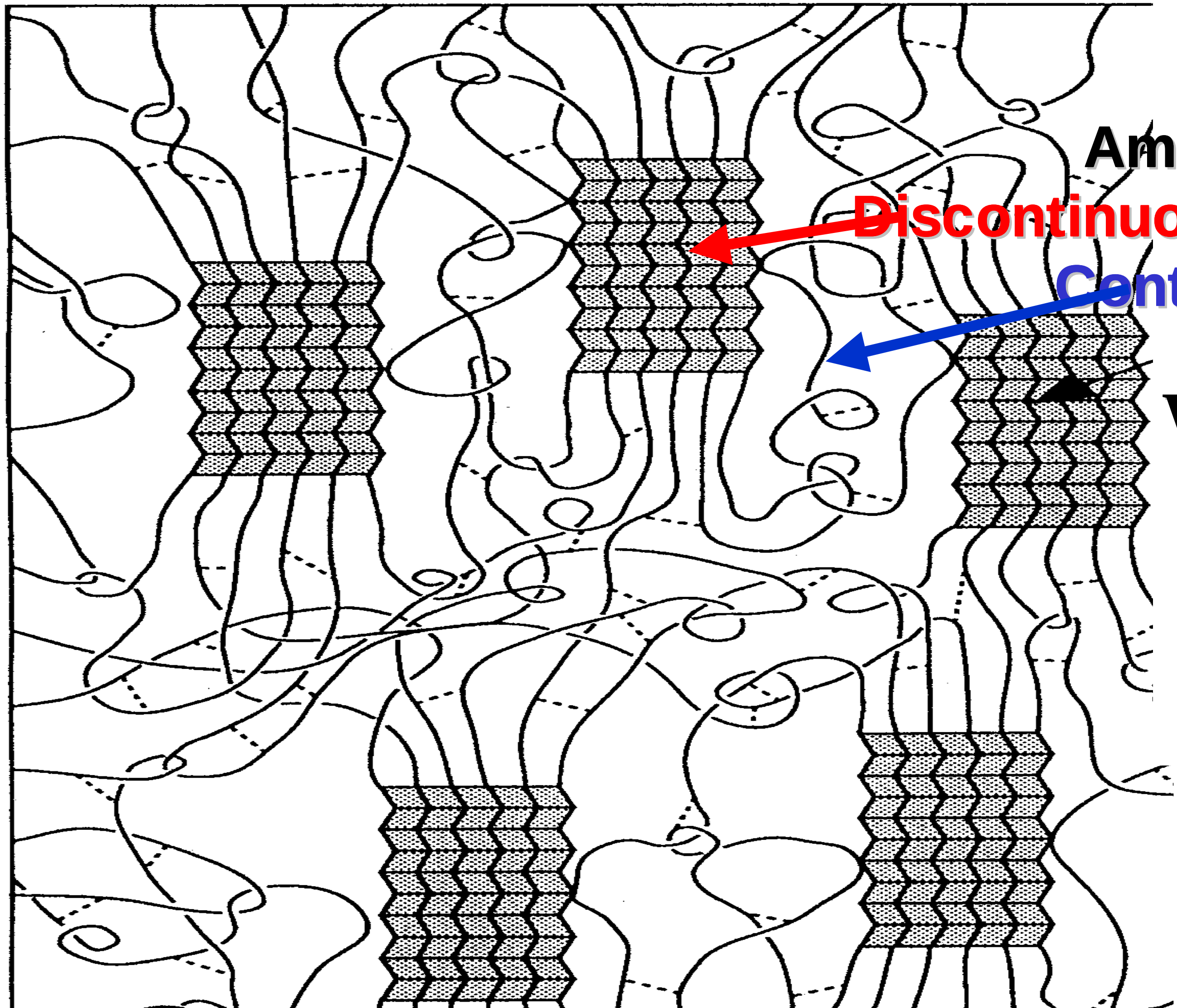
Geometry →

mechanical properties, electrical properties

[Yakobson, Smalley, American Scientist,July 97]

A Tensegrity Found in Nature,

Spider Fiber: **Nature’s Strongest**



Amino Acids:

Discontinuous compressive

Continuous tension

What’s in Common?

Fullerenes,

Spider fiber

Cell Cytoskeleton

Snelson’s Artform

Tensegrity topology

Outline

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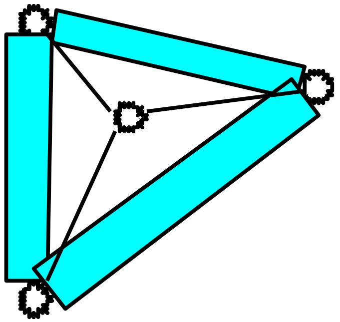
Definition: Tensegrity Systems

2 N Points form a Tensegrity Geometry if the set of Points are stabilizable with pretensioned axially loaded members connecting the points, with no more than two compressive members attached to a Point

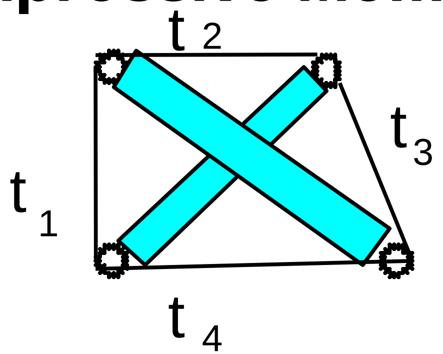
Class D: Discontinuous Compressive Members

Class C: Continuous Compressive Members



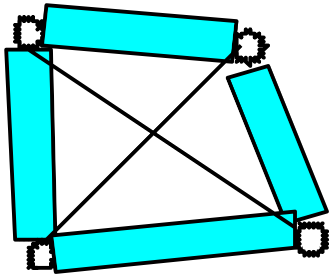
Tensegrity System

C3T3, Class C



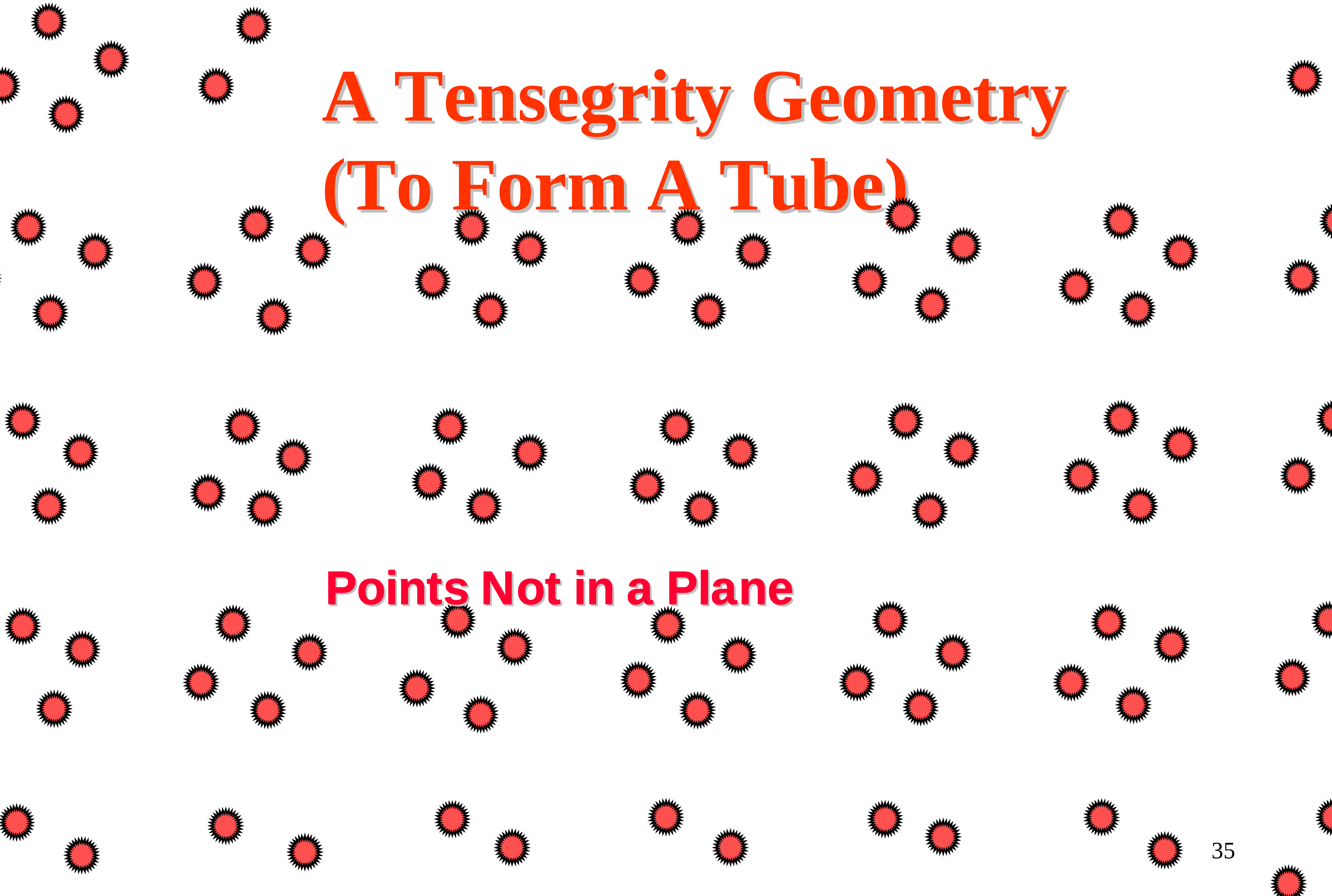
Tensegrity System

C2T4, Class D



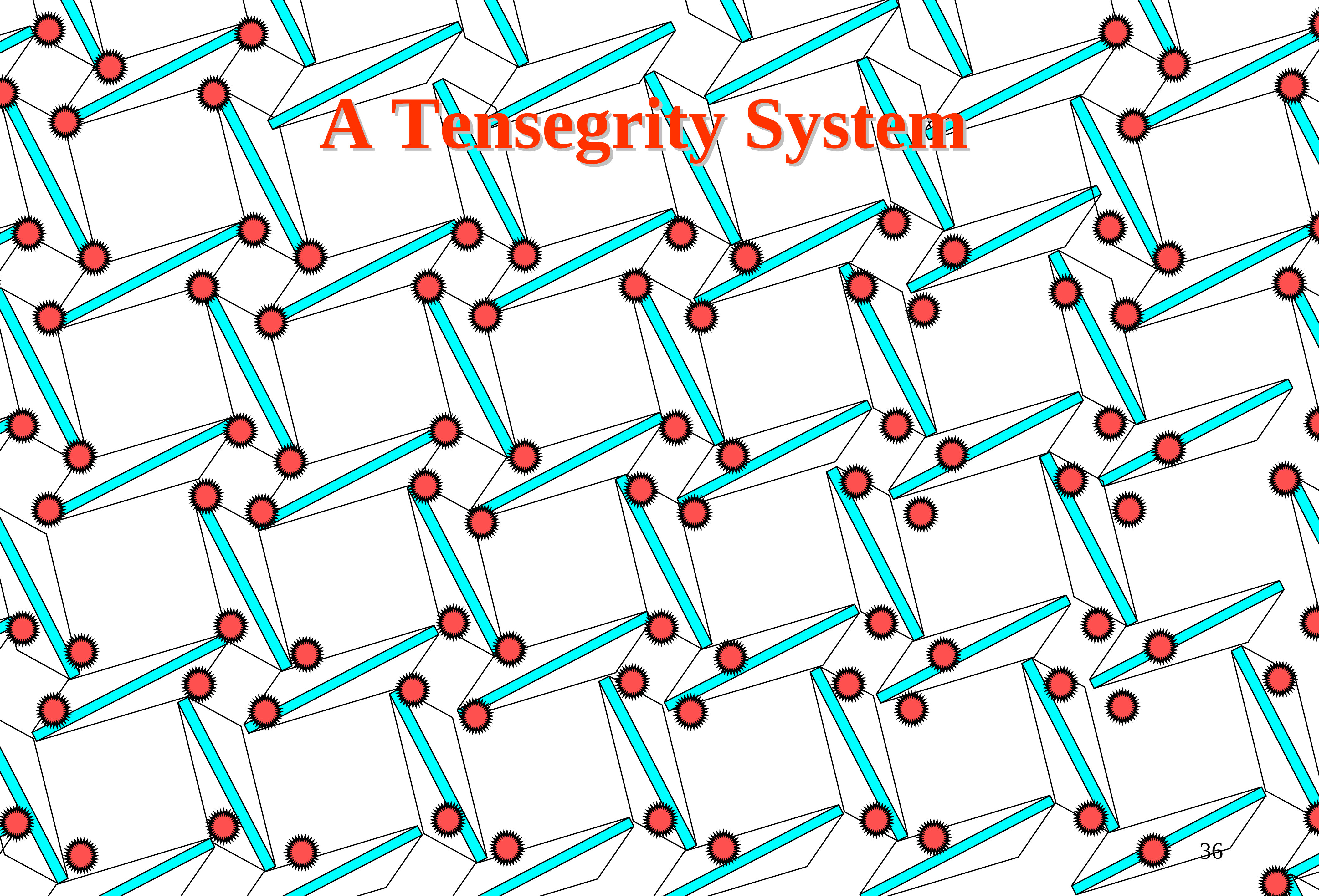
3D: No Tensegrity

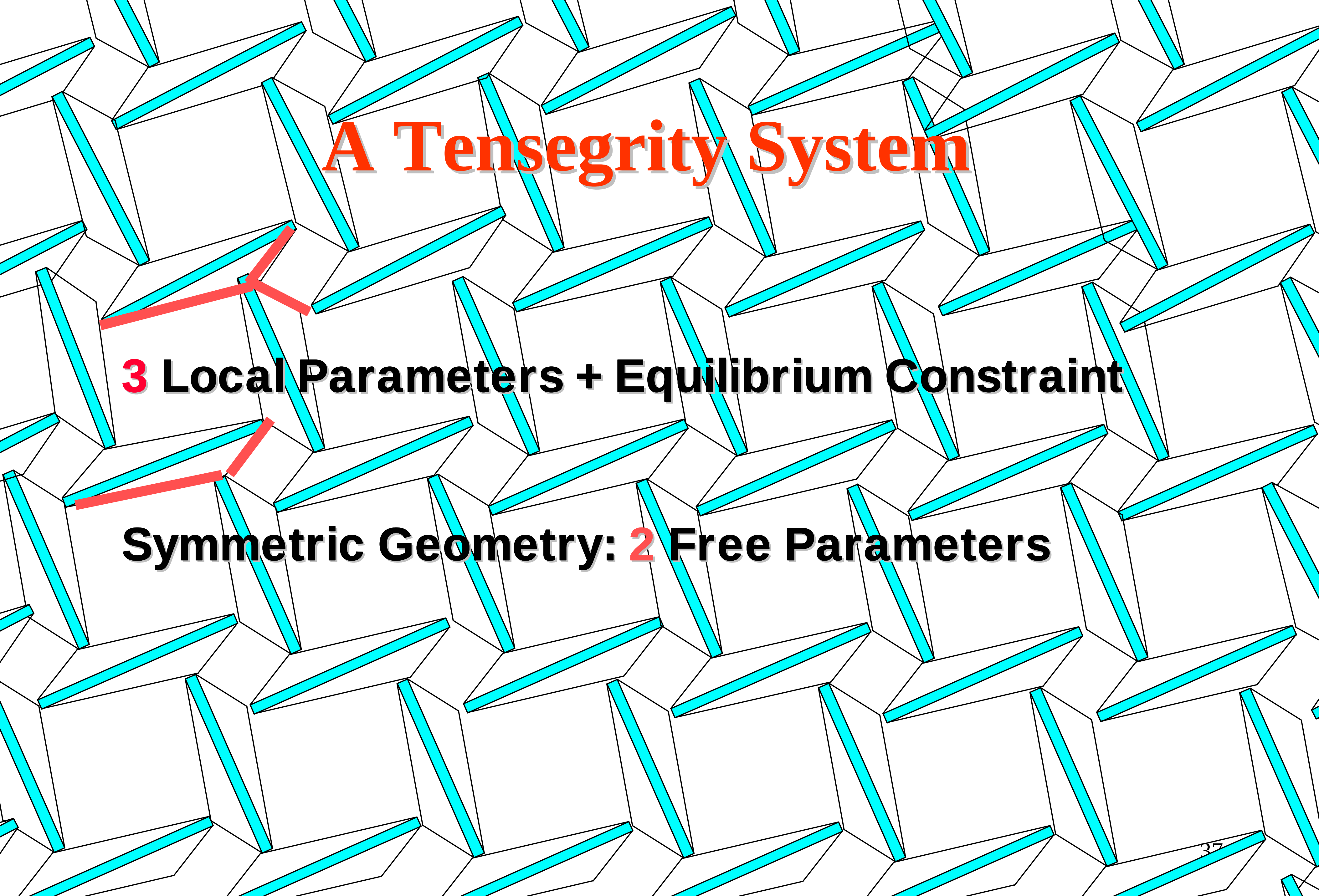
A Tensegrity Geometry (To Form A Tube)



Points Not in a Plane

A Tensegrity System



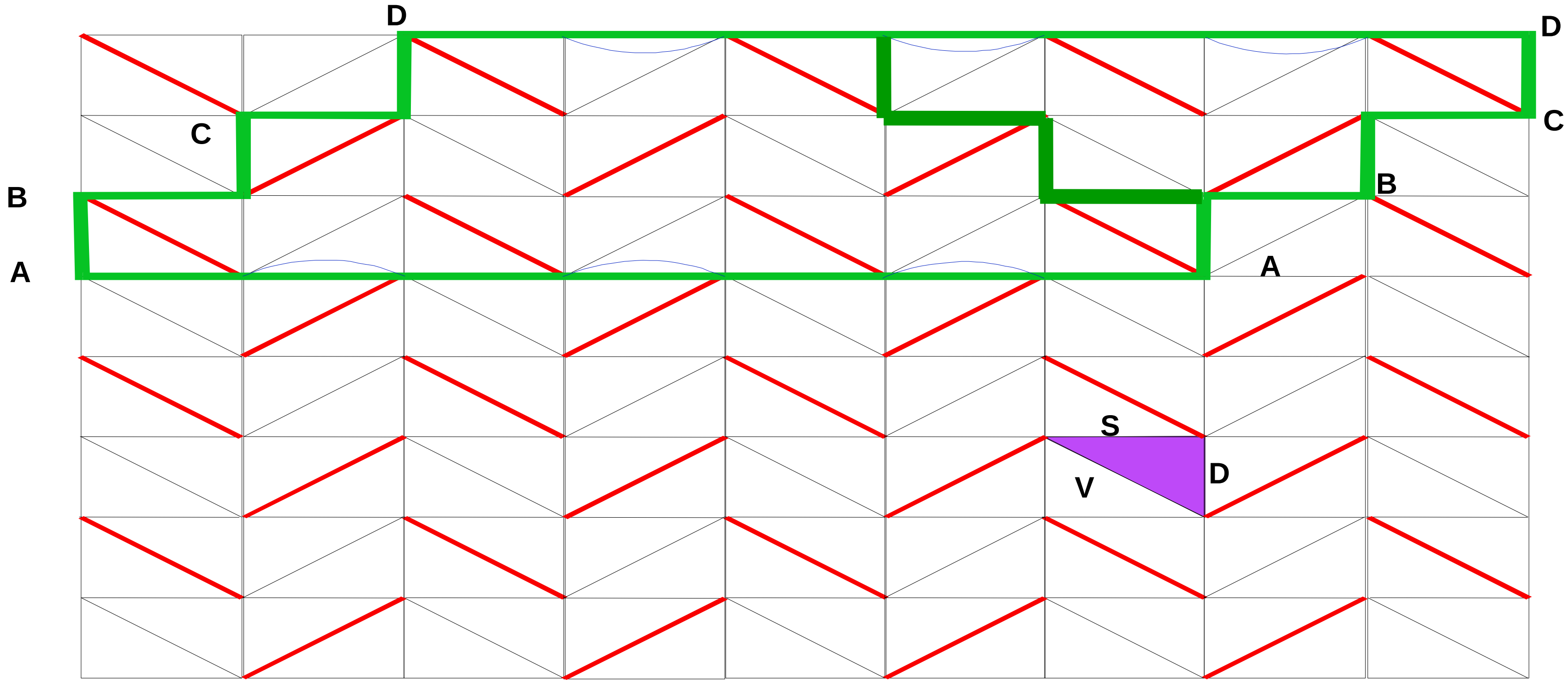


A Tensegrity System

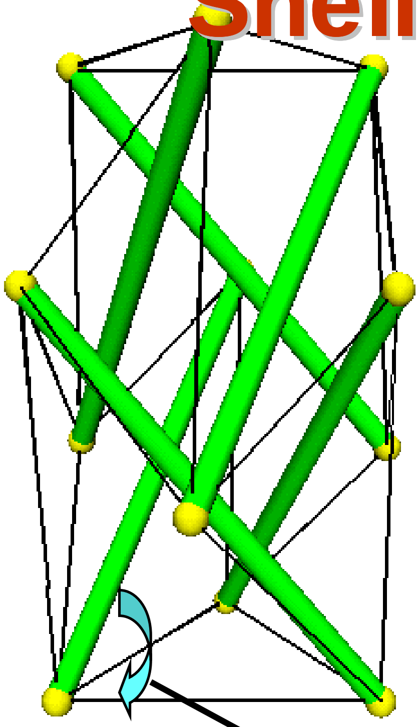
3 Local Parameters + Equilibrium Constraint

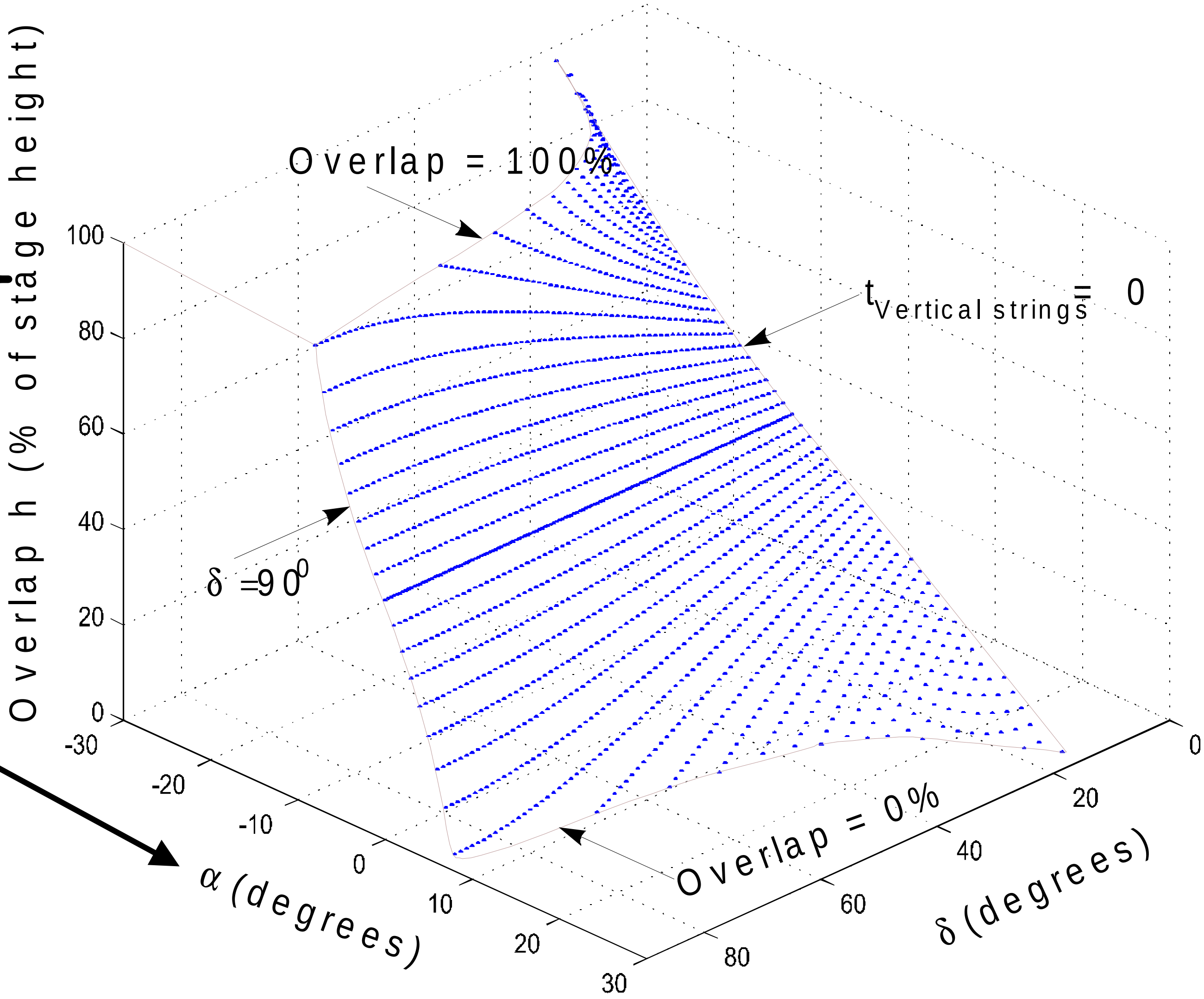
Symmetric Geometry: 2 Free Parameters

Shell Class of Tensegrity Systems



Shell Class Tensegrity Structures





Tensegrity Geometry

$$\left.\begin{array}{l} (h, \alpha, \delta) \;=\; \text{Stable equilibrium} \\ F(h, \alpha, \delta)t = 0, \quad , \quad t > 0 \\ \left| F^T F \right| = 0 \end{array}\right\} \text{Tensegrity Geometry}$$

Tensegrity Geometry

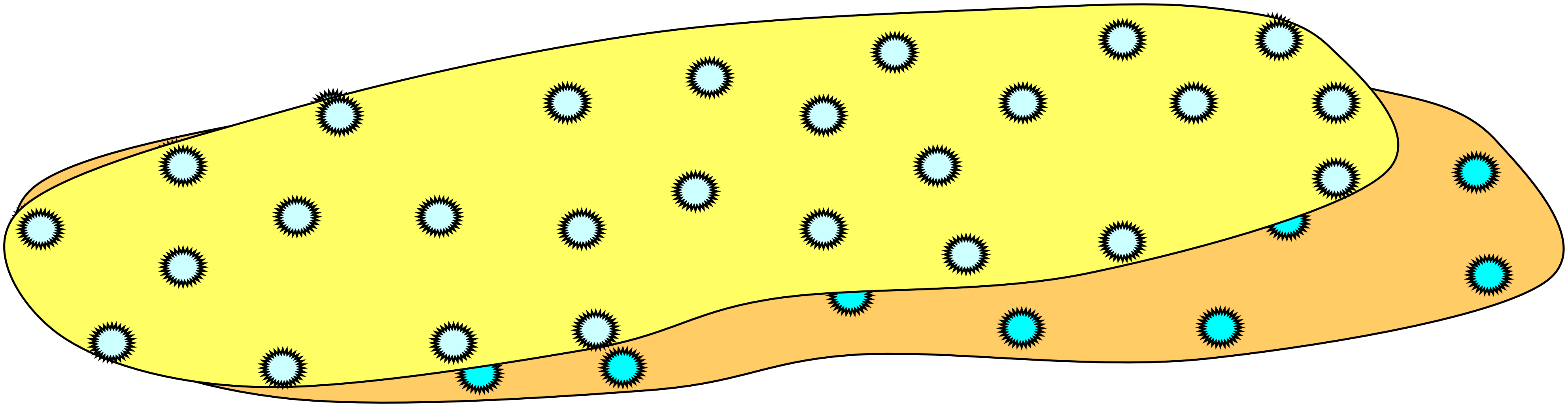
[Skelton, Helton, Adhikari, 1998],
[Sultan/Skelton, 1998]

$$h = \frac{1}{2 \tan \delta \cos\left(\alpha + \frac{\pi}{6}\right)} \left(-\frac{L_t}{\sqrt{3}} + L \sin \delta \cos\left(\alpha + \frac{\pi}{6}\right) + \sqrt{\frac{L_t^2}{3} - 3 L^2 \sin^2 \delta \cos^2\left(\alpha + \frac{\pi}{6}\right)} \right)$$

- Pugh, 1976
- Pelligrino, Calladine, 1986
- Motro, 1986
- Furuya, 1992
- Coughlin, Stamenovic, 1997

- Skelton, 1993 - 1999
- Sultan, 1996,1997, 1998, 1999
- Oppenheim, 1998
- Williamson, Skelton, 1999

Tensegrity Paradigm for Structural Control



- **Changing the Shape With Less Control Energy**
  - **Construct a Tensegrity Geometry with a specified shape**
  - **Actuate the tendons (rest lengths) to avoid straining the structure, moving from one equilibrium to another**

Shape Control with Tensegrity

Theorem [99]

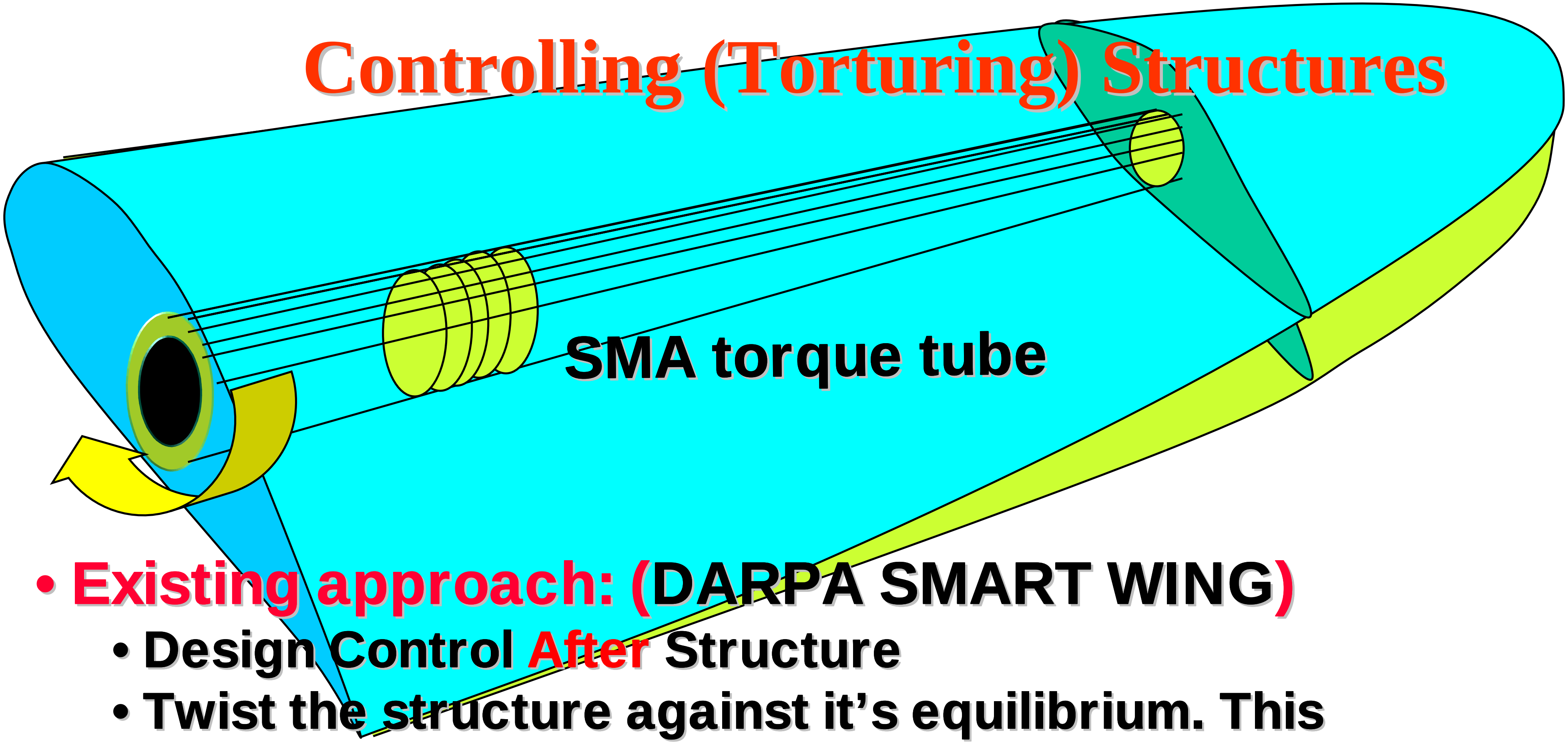
Let q1, q2 be two tensegrity geometries, associated with the same nullspace of F(q). Then,

- There exists a continuum of tensegrity geometries between q1 and q2.
- There exist tendon controls to change the geometry from q1 to q2 without changing potential energy.

$$\ddot{q} + (K_r(\dot{q}) + K_p(q))q = B(q)(u + v) + Dw$$

$$\dot{u} = K^{-1}(Rq - u)\frac{(Rq - u)^T KR\dot{q}}{(Rq - u)^T(Rq - u)} \quad , \quad Rq > u$$

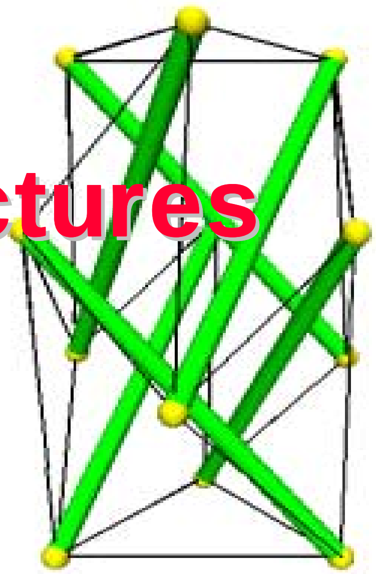
Controlling (Torturing) Structures



- **Existing approach: (DARPA SMART WING)**
  - **Design Control After Structure**
  - **Twist the structure against it’s equilibrium. This requires work (7 deg limit, 20 deg desired)**

- **New Paradigm: Unify at more fundamental level**
  - **Change shape by changing the equilibrium**

Minimal Mass Tensegrity Structures

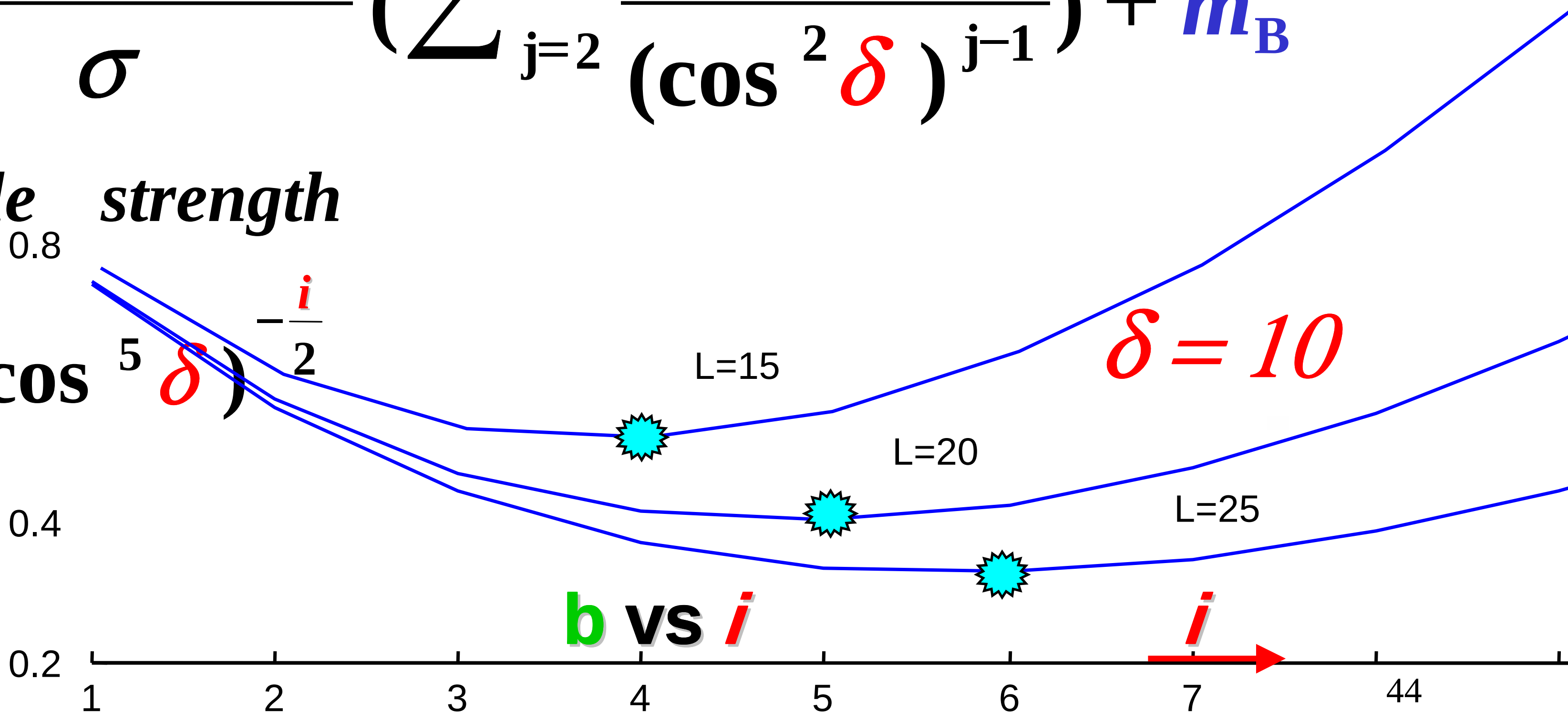


$$m_i = \mathbf{b}\, m_0$$

$$\mathbf{b} = \frac{\pi E_0 (\tan^2 \delta)}{\sigma}\left(\sum_{j=2}^{i} \frac{1}{(\cos^2 \delta)^{j-1}}\right) + m_B$$

$\sigma = Tensile\ \ strength$

$$m_B = (2\cos^5 \delta)^{-\frac{i}{2}}$$



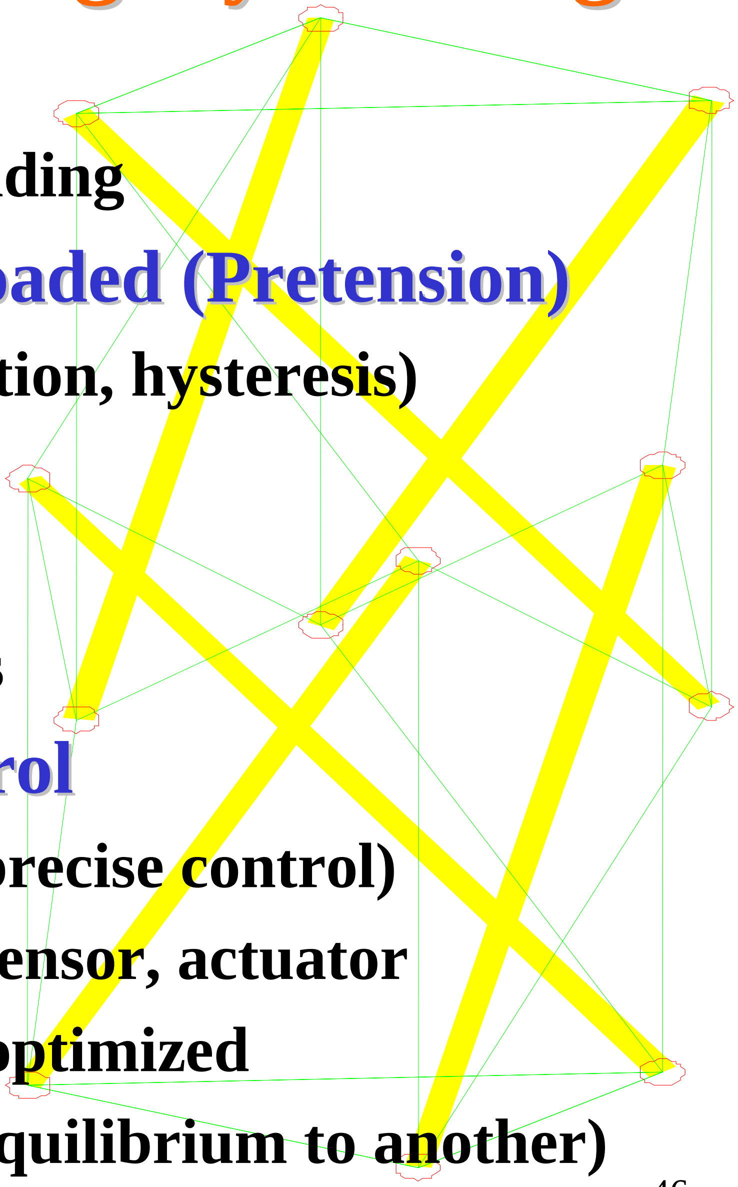
Stiffness-to-Mass Ratio of C4T1^i

Theorem

The compressive stiffness of the C4T1^i Structure is equal to the tensile stiffness of the tendons of the shortest tendons

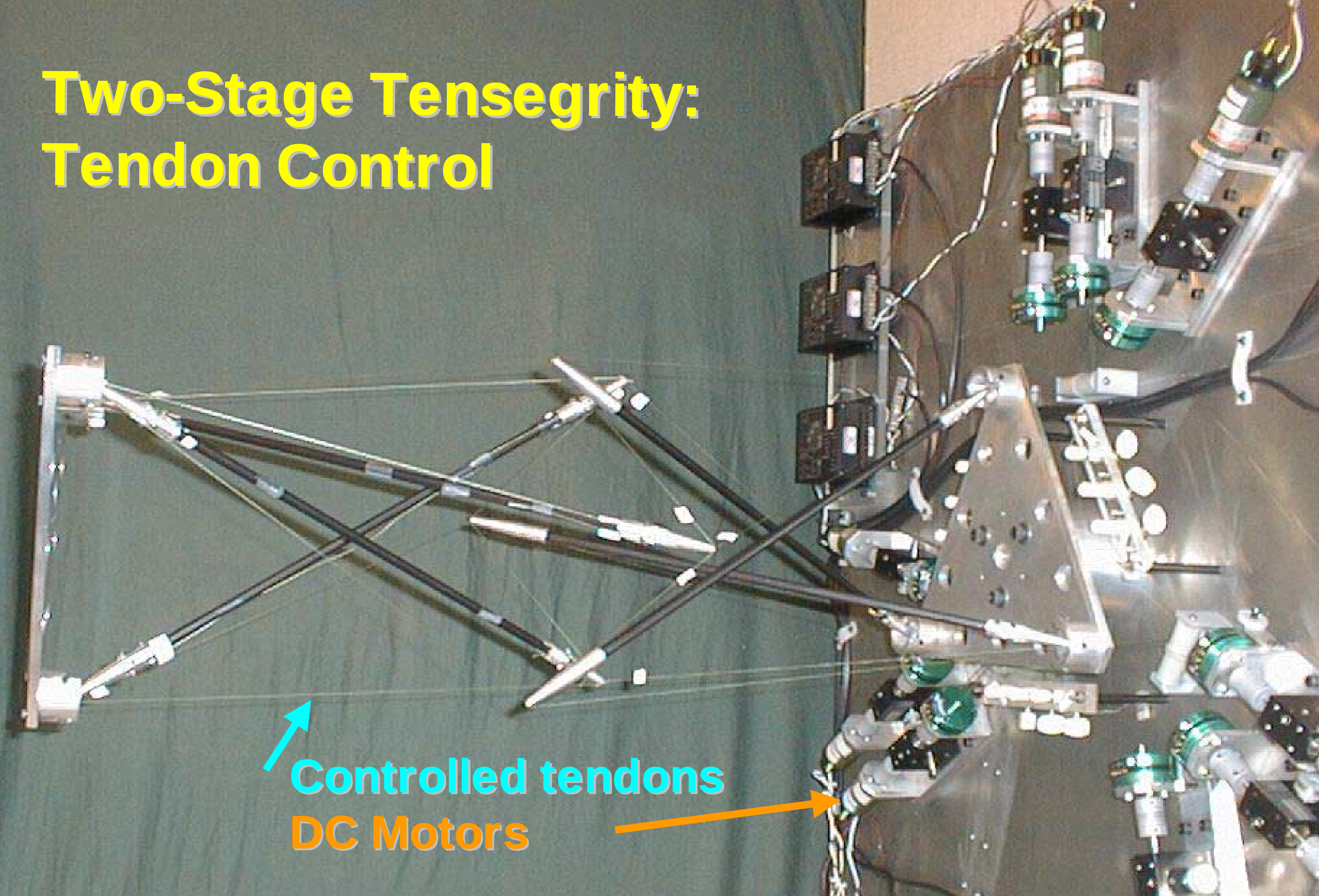
- **Controlling the tension of the shortest tendon controls the compressive stiffness of the entire structure**
- **All compressive members carry the same load**
- **Infinite buckling strength for finite i**
- **minimal mass occurs at smaller i than infinite strength**

Advantages of The Tensegrity Paradigm



- **All Members Axially Loaded**
  - Global bending without member bending
- **All Members Uni-Directionally Loaded (Pretension)**
  - No reversal of load direction (no friction, hysteresis)
- **Structural Efficiency**
  - Strength to mass very high
  - Inspired by Art and Biological forms
- **Easy to Integrate Structure/Control**
  - More accurate models (hence more precise control)
  - A structural member also serves as sensor, actuator
  - Actuator/Sensor architecture easily optimized
  - Change shape with little work (one equilibrium to another)

Two-Stage Tensegrity: Tendon Control



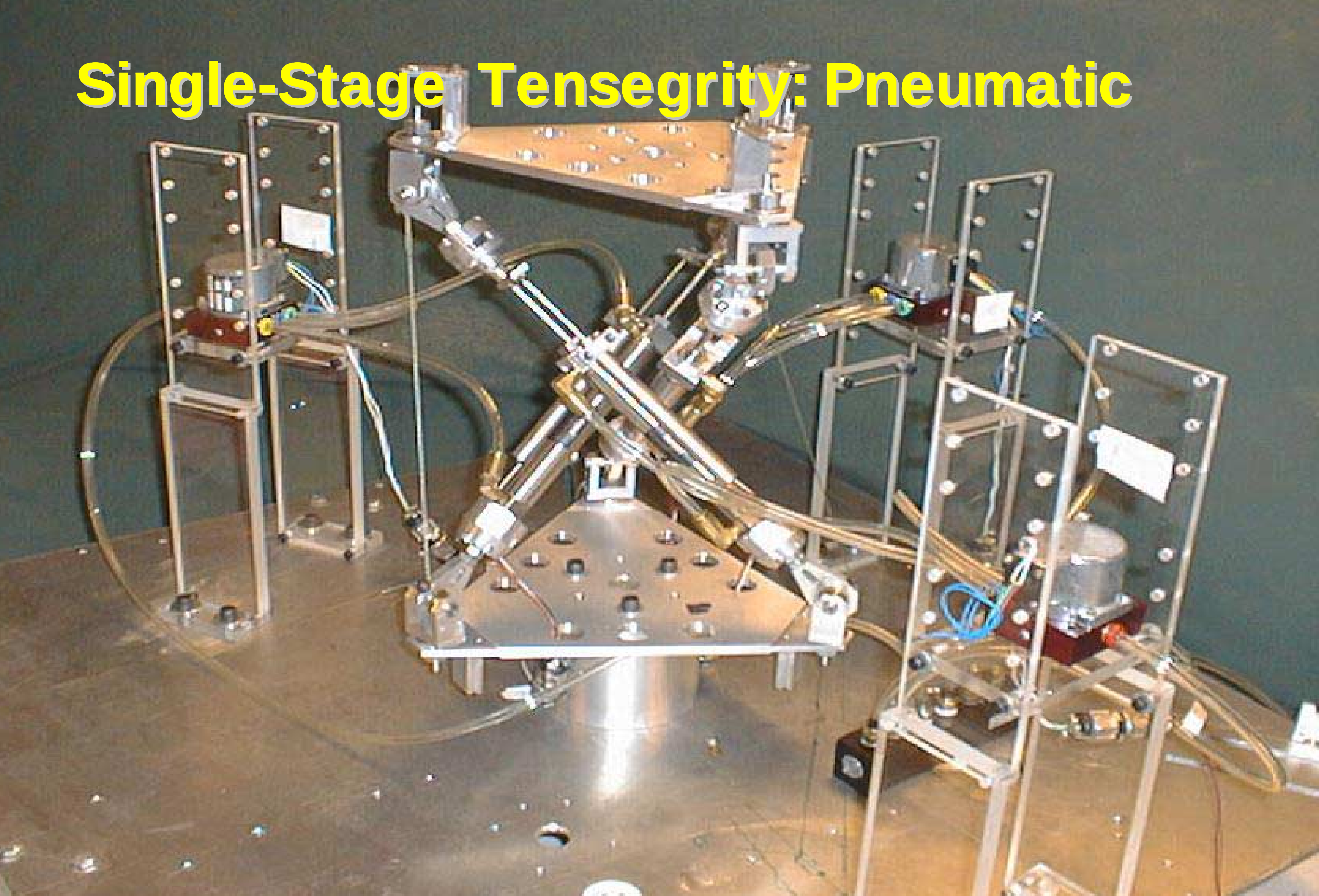
Controlled tendons

DC Motors

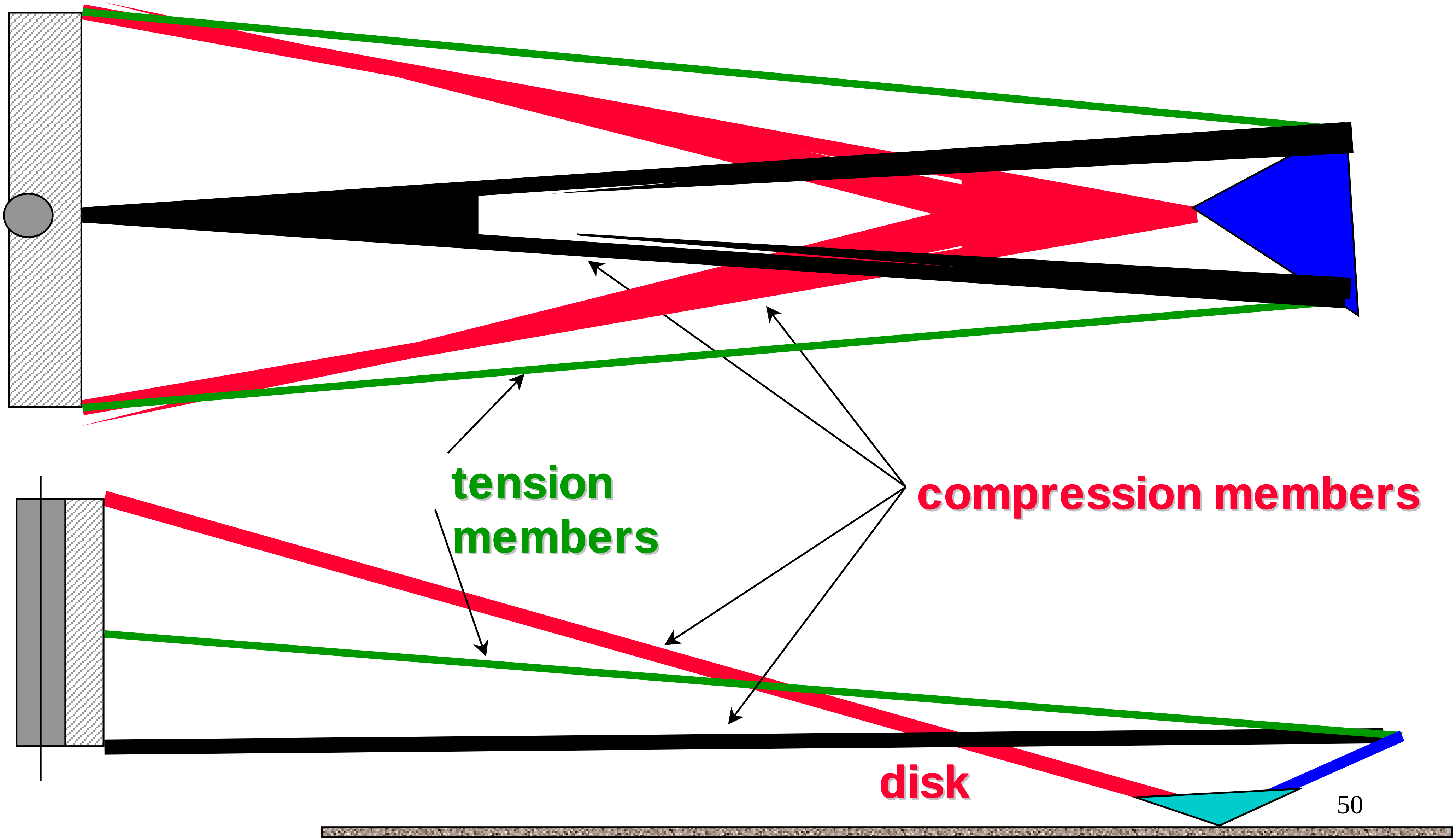
Deployable Tensegrity Heat Shield

Structural Systems and Control Laboratory
School of Engineering, UCSD

Single-Stage Tensegrity: Pneumatic



Tensegrity Suspension for Disk Drives



Conclusions

To Universities and funding agencies:

- **Give the Soul of Control a Body: System Design**
- **Give Modeling a Purpose: Systems Modeling**
- **Function following Form**
  - **Snelson's Tensegrity Artform Inspires a New Paradigm to Integrate Mechanics, Structures, and Control**

Conclusions

- After Component Technologies mature, the next quantum leap in technology must come from a scientific method to do Interdisciplinary System Design
- Why wait til component maturity to invest in a scientific method for Systems Design
- The biggest challenge: System Modeling
- System Design (and modeling) requires more than communicating what each discipline already knows
- We can exploit biological material architecture to
- Suggest a system design paradigm for designing to specific materials, thermal, electrical, mechanical properties:

What Your Data Never Told You

- **Data is not Information**
- **Information is not Knowledge**
- **Knowledge is not Understanding**
- **Understanding is not Wisdom**

[Howard Garner, Harvard]