Mixing efficiency in decaying stably stratified turbulence

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1. Introduction

Mixing efficiency in stratified flows is a measure of the proportion of turbulent kinetic energy that goes into increasing the potential energy of the fluid by irreversible mixing. This is an important issue in the parameterization of mixing in geophysical flows. The laboratory experiments of Britter (1985), Rottman & Britter (1986) and Rehmann & Koseff (2000) have attempted to measure the mixing efficiency of grid turbulence in a uniformly stratified fluid. These experiments have been done mostly by towing a grid horizontally through water that has been stratified using either salt or heat. The mixing efficiency in these experiments is measured as a function of the Richardson number $R_i = (N M / U)^2$, in which $N$ is the buoyancy frequency of the fluid, $M$ is the grid mesh length, and $U$ is the towing speed. The mixing efficiency itself is measured as the flux Richardson number $R_f$, defined as the ratio of the change in potential energy of the fluid to the amount of work done towing the grid through the tank. The Reynolds numbers $Re$ for these experiments, based on the tow speed and grid mesh length, range from 1,000 to 10,000. The Schmidt number $Sc$ is about 700 for the salt-stratified experiments and the Prandtl number $Pr$ is about 7 for the heat-stratified experiments.

The results of these experiments are fairly consistent; there appears to be no strong dependence on the molecular diffusivity (although there are some uncertainties about this conclusion). For $R_i < 1$ the mixing efficiency increases approximately as $R_i^{1/2}$. For larger $R_i$ the mixing efficiency appears to approach a constant value of about 6 although since none of the laboratory experiments can achieve values of $R_i$ much greater than about 10 there is some uncertainty about whether the mixing efficiency will remain constant or not for larger $R_i$. Some different types of experiments have suggested that $R_f$ should decrease for sufficiently large $R_i$. There have been many attempts to develop scaling arguments to describe this mixing efficiency behavior. The $R_i^{1/2}$ behavior can be explained using simple energy arguments. The behavior at large $R_i$ is not well understood.

In the research described here, direct numerical simulations (DNS) of transient turbulent mixing events are carried out in order to study the detailed physics of the mixing efficiency. In particular, DNS results of decaying, homogeneous, stably-stratified turbulence are reviewed and used to determine the mixing efficiency as a function of the initial turbulence Richardson number.

2. Numerical Methods

Most of the numerical experiments described here were carried out with the pseudo-spectral DNS code used by Riley, Metcalfe & Weissman (1981) and is described in detail by those authors. This code simulates a flow field that is periodic in all three spatial co-ordinates, with uniform background density gradient.

Some additional results have been obtained using the DNS code described in Gerz, Schumann & Elgobashi (1989) and Diamessis & Nomura (1999). For the same flow conditions, this code produced results in agreement with those obtained with the previously described code.
2.1 Initialization

For all our numerical experiments, the turbulent flow field was initialized as a Gaussian, isotropic and solenoidal velocity field in the usual way using random Fourier modes with a specified energy spectrum. The turbulence was then allowed to decay until approximately 99% of the initial turbulence energy had dissipated. Decay times of about ten times the initial time scale of the turbulence $L_0/u_0$ were typically required, where $L_0$ and $u_0$ are the initial length and velocity scales as determined by the specified initial spectrum. The stable stratification for each transient experiment was specified by the initial Richardson number, which was varied in the range $0 < Ri < 40$. The Richardson number for these simulations is defined as $Ri = (NL_0/u_0)^2$.

The energy spectrum of the initial turbulence was chosen to have the exponential form (see e.g., Townsend, 1976):

$$E(k) = Cu_0^2 L_0^5 k^4 \exp\left[-\frac{1}{2} k^2 L_0^2 \right]$$  \hspace{1cm} (1)

where $C$ is a constant scaling factor.

For most of the simulations, the initial energy was exclusively kinetic in form i.e., the initial density fluctuations (and hence $P E_0$) were set to zero. Since one of our main objectives is to compare the simulation results with experimental measurements, the appropriate modelling of the initial conditions is necessary. Experimental measurements downstream of grid generated turbulence in stratified fluids suggest that the $P E$ close to the grid (say at $x/M = 10$) is typically only a small fraction of the $K E$, approximately 10% (possibly increasing with increasing stratification). On this basis it seems that the above initialization scheme is reasonable, however Hunt, Stretch & Britter (1988) have indicated that the flow evolution can be sensitive to the $P E$ initial conditions. As a preliminary test, two sets of simulations were done with non-zero initial density fluctuations with $P E_0$ equal to 24% and 95% of the initial turbulent kinetic energy.

2.2 Evaluating Mixing Efficiency

The main diagnostics describing the energetics of the decaying flow fields were archived from each run for subsequent analysis: these included the horizontal and vertical kinetic energy ($HKE$, $VKE$), potential energy ($PE$) and dissipation rates. In addition the time-integrated contributions of each of these aspects of the energy decay were also archived for subsequent analysis.

For the numerical simulations, the turbulence remains homogeneous (but not isotropic) for all times. This includes the buoyancy fluxes and dissipation rates. This implies that the mean fields are decoupled from the turbulence in this idealized problem, so that there is no change in the background potential energy. This model problem should be applicable to the homogeneous region of the flow fields investigated in the experiments. However, the turbulence in the experimental flows must be non-homogeneous near the upper and lower boundaries. It is in these regions that the gradients in the density flux produces changes to the background mean density profile, and hence in the $PE$ of the fluid column. To compare the simulations with the experiments, we use the time-integrated buoyancy flux from the homogeneous region to represent the changes that are reflected in the non-homogeneous regions in the experiments. That is, in the numerical simulations the integral flux Richardson number $R_f$ (or mixing efficiency) is defined as the time integral of the buoyancy flux divided by the change in turbulent kinetic energy over the same time period. We note that local irreversible mixing processes within the homogeneous region, represented by the local dissipation of scalar fluctuations (by molecular diffusion), do not contribute directly to changes in the background mean density field. They do however contribute indirectly by influencing the development of the buoyancy flux.
3. Results

The resolution of the simulations varied from $32^3$ for the initial series of numerical experiments, and was later increased to $64^3$. This allowed for about a factor of two increase in the Reynolds number $Re = u_0 L_0/\nu = 100$ and 200, respectively, in which $\nu$ is the kinematic viscosity. Most of the simulations reported here were carried out for Prandtl numbers $Pr = 0.5$. Limited investigation of Prandtl number effects were carried out by varying the Prandtl number in the range $0.1 < Pr < 2$. Higher Prandtl numbers could not be achieved without problems in resolving the scalar field: Reynolds numbers were reduced to $Re = 50$ for the $Pr = 2$ cases. Dissipation spectra were used to evaluate resolution issues. These values of $Pr$ are substantially smaller than those for the laboratory experiments.

There is some arbitrariness in relating the different definitions of $Ri$ used in the DNS and laboratory experiments. This relationship could be established using existing grid turbulence data at, say, $10M$ downstream from the grid. However, since our interest initially is in the trends of $Rf$ and its maximum magnitude, we will assume the two definitions of $Ri$ are approximately equal.

Figure 1 is a plot of $Rf$ versus $Ri$ for a selection of the DNS and laboratory results. To keep the figure simple, and since all the laboratory data is fairly consistent, we have plotted only the results from the laboratory experiments of Rottman & Britter (1986) and Rehmann & Koseff (2000). The results from three sets of numerical simulations are shown in the plot. These simulations correspond to the initial conditions: (1) $Re = 100$, $Pr = 0.5$ and $PE_0/VK E_0 = 0$; (2) $Re = 100$, $Pr = 0.5$ and $PE_0/VK E_0 = 24\%$; and (3) $Re = 100$, $Pr = 0.5$ and $PE_0/VK E_0 = 94\%$. The DNS results for other values of $Pr$ used did not differ greatly from those shown. The DNS results for all three initial conditions compare well qualitatively with the experimental data. Quantitatively, the best comparison with the experimental results is for the DNS case for which $PE_0/VK E_0 = 94\%$. The other two initial conditions appear to produce mixing efficiencies that are much higher than was measured in the laboratory.

It appears, then, that the initial conditions are the controlling feature of the mixing efficiency in this kind of flow. However, our numerical simulations do not span a large enough range of $Pr$ to definitively rule out any significant $Pr$ effects. Looking at the simulations in more detail we note that most of the interesting dynamics, and in particular the contributions to integrated buoyancy flux, are completed by about 2-3 initial time scales of the turbulence. We suspect that these mixing efficiency results can be reproduced by inviscid, non-diffusive Rapid Distortion calculations. If this is true, it would rule out completely the differences in $Pr$ or $Sc$ as an explanation for the observed differences in mixing efficiency between the DNS and laboratory experiments.

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4. References


