Material instability in complex fluids - from conjecture to reality

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Summary

- 1996 workshop had an important impact on subsequent activity, particularly in the areas related to material (constitutive) instability: Instability of homogeneous deformations, irrespective of boundary conditions.
- Review of main points of my Institute Lecture* in the light of subsequent experimental and theoretical developments.
- Some open theoretical questions.

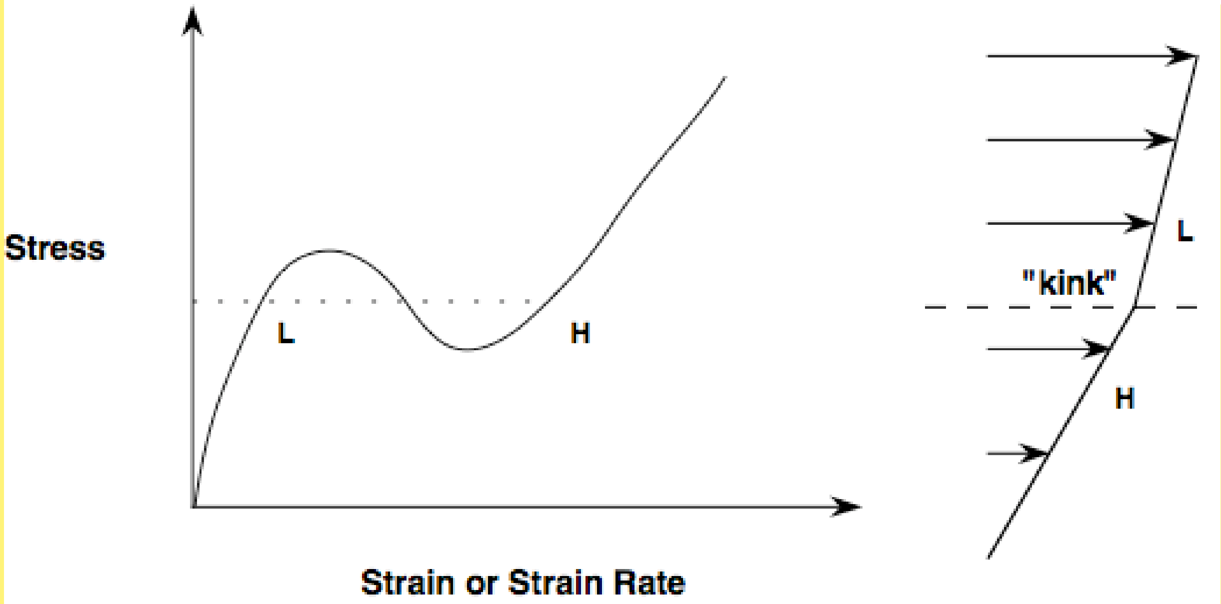
* Transparencies available in pdf format at:

http://maeresearch.ucsd.edu/goddard/archives/GoddardLectureINI96.pdf

or at http://ucsd.edu on Mechanical and Aerospace Engineering Faculty site.

Material Instability

- Material (or constitutive) instability concerns the instability of spatially homogeneous states arising from strain softening.
- Mathematically it involves "change of type" and singularities in field equations, viz.
    - loss of dynamic hyperbolicity coupled with short wave instability
    - loss of static ellipticity accompanied by steady-state multiplicity.

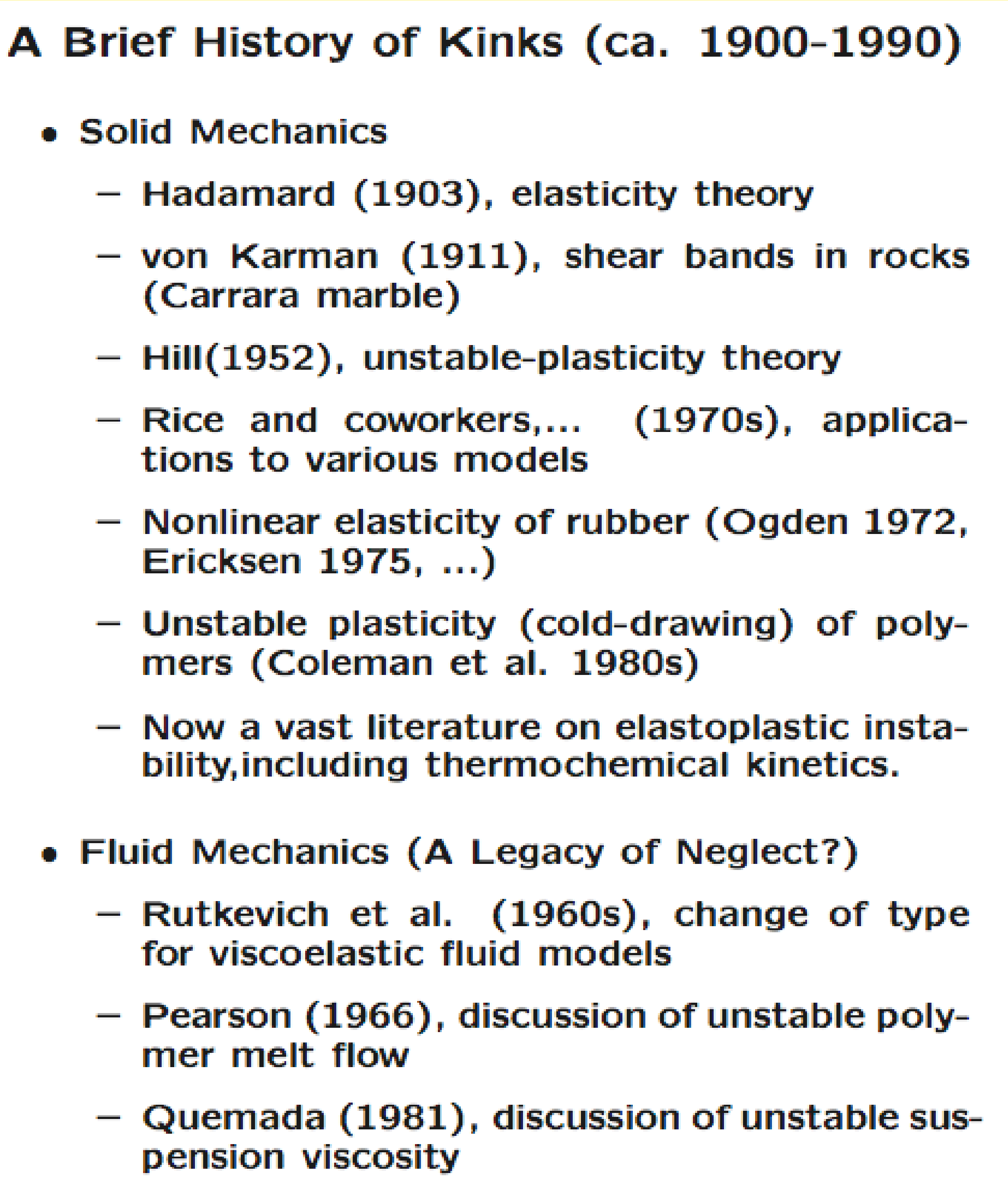


- Rest-state (in)stability is tantamount to thermostatic (in)stability against phase transition.

(Kink pair gives shear band. Length scale?)

Plan of Lecture

- Phenomenological Aspects - examples of material instability in solids and, possibly, in fluids.
- Analogies: thermostatic and elastic
- Unstable mechanical models and a proposed general test
  - elastic
  - viscous fluids
  - viscoelastic fluids
  - (elastoplastic solids)
  - possible relevance to "melt fracture" and unsteady flow phenomena
- Diffusion and other effects



A Brief History of Kinks (ca. 1900-1990)

- Solid Mechanics
  - Hadamard (1903), elasticity theory
  - von Karman (1911), shear bands in rocks (Carrara marble)
  - Hill(1952), unstable-plasticity theory
  - Rice and coworkers,... (1970s), applications to various models
  - Nonlinear elasticity of rubber (Ogden 1972, Ericksen 1975, ...)
  - Unstable plasticity (cold-drawing) of polymers (Coleman et al. 1980s)
  - Now a vast literature on elastoplastic instability,including thermochemical kinetics.
- Fluid Mechanics (A Legacy of Neglect?)
  - Rutkevich et al. (1960s), change of type for viscoelastic fluid models
  - Pearson (1966), discussion of unstable polymer melt flow
  - Quemada (1981), discussion of unstable suspension viscosity

(Oldroyd, 1958)

Conclusions

- One cannot rule out material instability (as defined above) in complex fluids, or in constitutive models, on thermodynamic or other grounds. They may describe real phenomena.

- As in other branches of mechanics and physics, one should strive to assess the (in)stability of models and the stable flow domain. It may be possible to lay down a general criterion, but easier to perform direct stability analysis on homogeneous flows.

- Understanding of instability could be useful for describing real phenomena as wells as for avoiding artificial instabilities in models, e.g. in engineering flow calculations.

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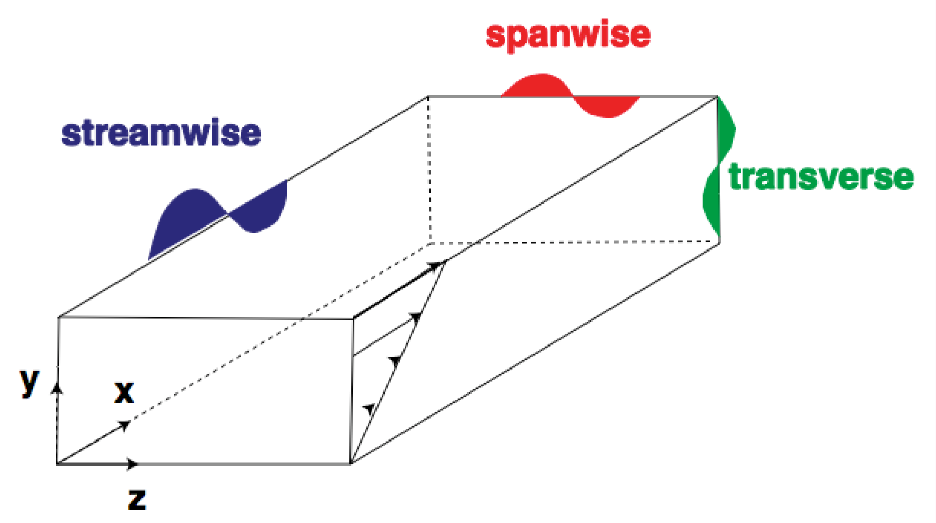
Update

Cursory count of research papers involving experiment on “shear banding”*

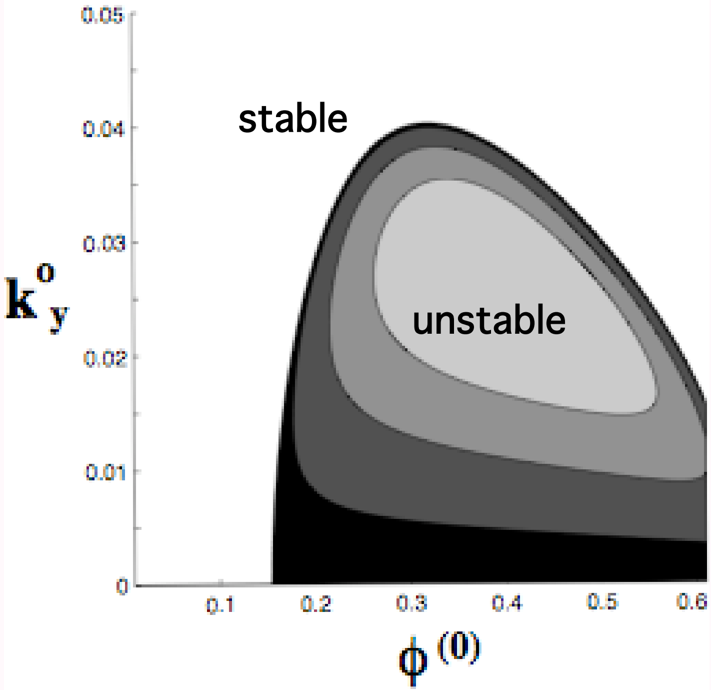
- Micellar solutions, 37 references (1 before 1996)
- Polymers, 21 (4 before 1996)
- Colloids, 4
- Suspensions (noncolloidal), 1 (unpublished)
- Foams (?)

* A more complete bibliography will be posted on my website in bibtex format.

Clustering instabilities in standard granular gas model*



Instability modes: Streamwise (“slugging”), Spanwise (“streaking”) and transverse (“layering”). All involve competition between “cold” dense and “warm” rarefied regions.

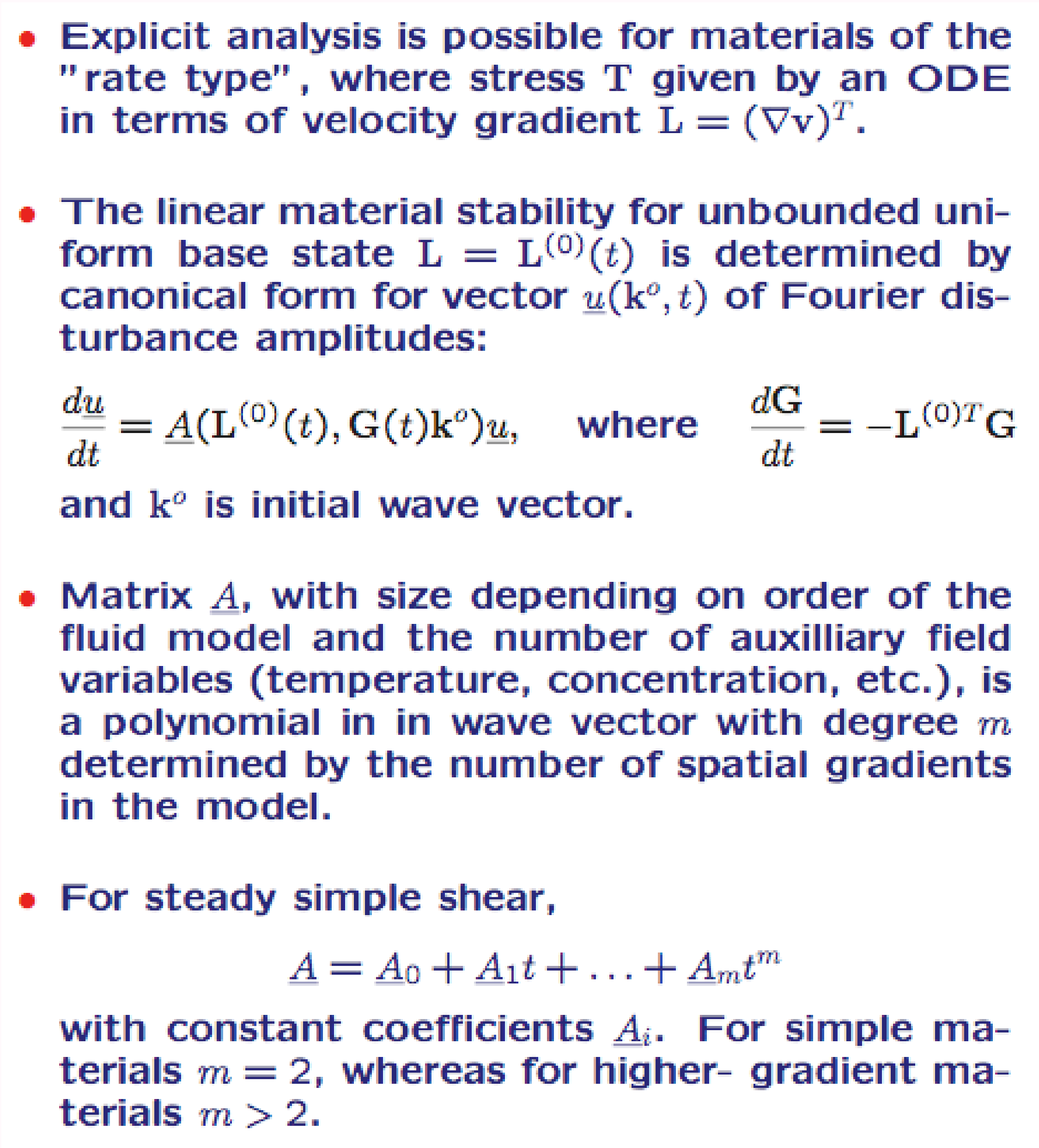


Contours of growth rates vs. initial wave number and initial particle fraction for layering modes in zero g. (Collisional restitution =0.99)

- Dominant linear mechanism is (Kelvin) “wave-vector stretching’ by base flow.
- Analysis suggests that theoretical model “self destructs”.

*M. Alam et al., *Part. Sci. Tech.* 17,69,1999; *JFM* 523, 277, 2005 (effect of gravity).

Linear stability analysis*



- Explicit analysis is possible for materials of the "rate type", where stress T given by an ODE in terms of velocity gradient $\mathrm{L} = (\nabla \mathrm{v})^T$.

- The linear material stability for unbounded uniform base state $\mathrm{L} = \mathrm{L}^{(0)}(t)$ is determined by canonical form for vector $\underline{u}(\mathrm{k}^o, t)$ of Fourier disturbance amplitudes:

  $$\frac{d\underline{u}}{dt} = \underline{A}(\mathrm{L}^{(0)}(t), \mathrm{G}(t)\mathrm{k}^o)\underline{u}, \quad \text{where} \quad \frac{d\mathrm{G}}{dt} = -\mathrm{L}^{(0)T}\mathrm{G}$$

  and k^o is initial wave vector.

- Matrix $\underline{A}$, with size depending on order of the fluid model and the number of auxilliary field variables (temperature, concentration, etc.), is a polynomial in in wave vector with degree m determined by the number of spatial gradients in the model.

- For steady simple shear,

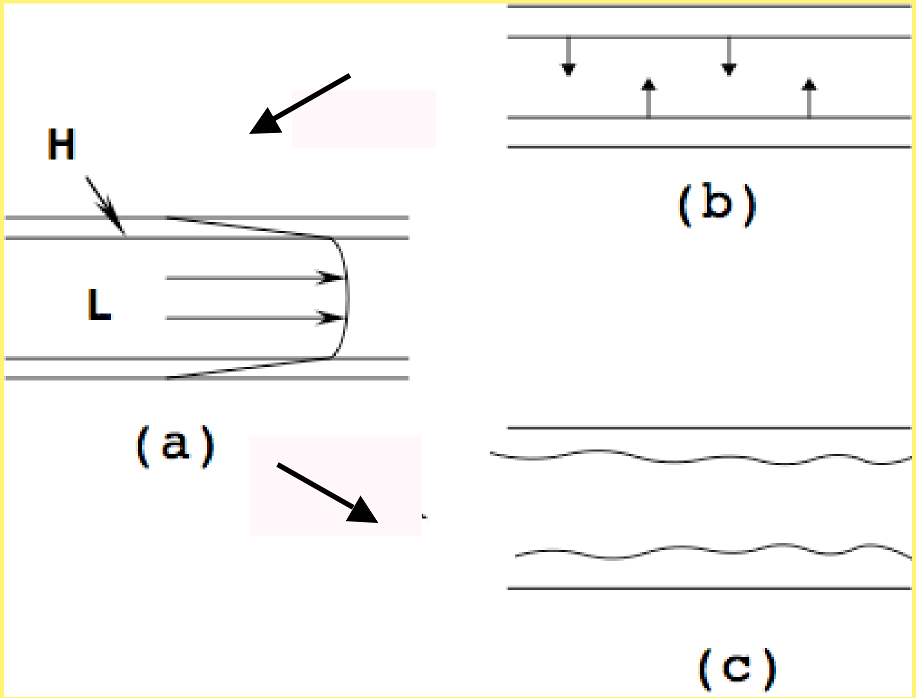
  $$\underline{A} = \underline{A}_0 + \underline{A}_1 t + \ldots + \underline{A}_m t^m$$

  with constant coefficients $\underline{A}_i$. For simple materials $m = 2$, whereas for higher- gradient materials $m > 2$.

*Ann. Rev. Fluid Mech., 35,113-33, 2003

Still conjectural

Migrational Instabilities in Suspensions ??*



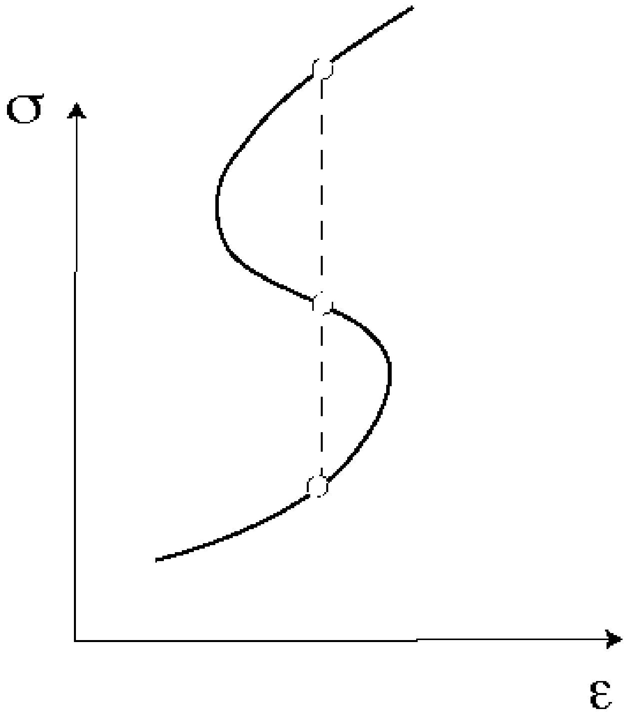
- (a)Two-phase structure arising from (b) particle migration from high to low shear regions [possibly followed by (Renardy) interfacial instability (c)].

- Can similar phenomenon occur in homogeneous simple shear?

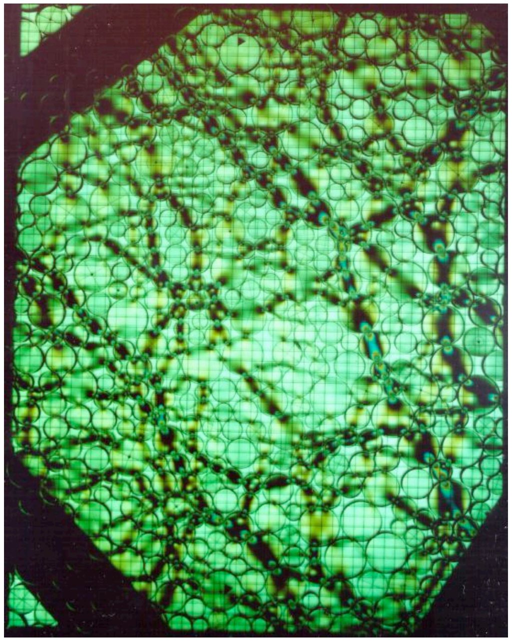
Italic note:

**INI Lecture 1996; Dynamics of Complex Fluids*, M. Adams *et al.* (eds.), Imperial College Press, 1998; *Proc. IUTAM Symposium: Lubricated Transport of Viscous Materials*, H. Ramkisoon (ed.), Kluwer, 1998; As yet unpublished work of Aksel et al., Bayreuth, 2006.

*Stress Localization??**



Coexistence of different stresses for given strain (rate)

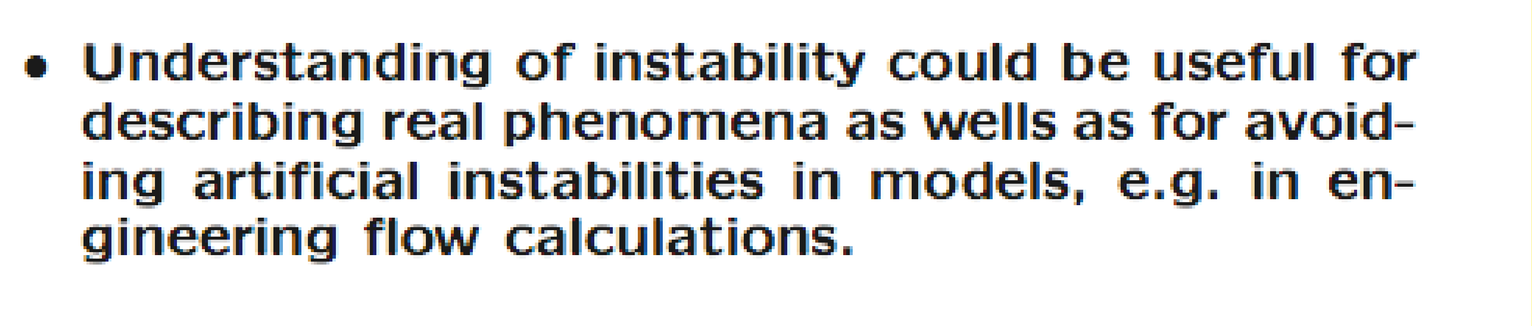


Granular "force chains" in photoelastic disks. (Drescher & De Josselin De Jong, 1972)

- Can non-convexity (e.g. buckling) lead to mesoscopic "two-phase" stresses?
- In fluids this could arise from viscosity jumps (e.g. from "coil-stretch" transitions, most plausibly in elongational flows ⇒ "stringiness").

******JNNFM*** 102, 251-261,2002; *Ann. Rev. Fluid Mech.***, 35,113-33, 2003; ***Proc. Int. Workshop Bifurc. Instab. Geomech*****. 2005, in the press, 2006 (granular- entropy model)**

Conclusions



- Understanding of instability could be useful for describing real phenomena as wells as for avoiding artificial instabilities in models, e.g. in engineering flow calculations.

- Some open theoretical issues:

  1. Non-linear instability (amplitude effects) ("Rheochaos"?).
  2. Boundary effects (nucleation, suppression).
  3. Gradient (non-local) effects and integral models.