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UNCERTAINTY MODELING FOR LINEAR TAPE-OPEN DRIVES*

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Introduction

An essential step in designing a robust embedded high performance servo controller is the characterization of product variability of the servo actuator. This paper shows how product variations can be modeled using an unstructured uncertainty model that uses knowledge of an initial and possibly low bandwidth embedded servo controller to parametrize the uncertainty. Using the knowledge of an initial controller to formulate the uncertainty model is based on a so-called dual-Youla parametrization [1, 2]. Instead of modeling uncertainty in an additive or multiplicative way, it is shown that our approach models product variations as perturbations on closed-loop transfer functions, making it (a) less conservative and (b) closely related to the actual control design methodology needed to achieve a high performance servo system for track following.

Experimental Setup and Frequency Domain Data

In the System Identification and Control Laboratory (SICL) at UCSD we have mechanically identical LTO3 drives depicted in Figure 1 used in the experimental work of this study. The two systems are representative of the extreme situations of a well and a bad performing drive produced during manufacturing. Both LTO3 drives are controlled by the same and known (initial) servo control algorithm C_i and are equipped with I/O hardware to perform experiments to measure the dynamic response of the servo actuators.

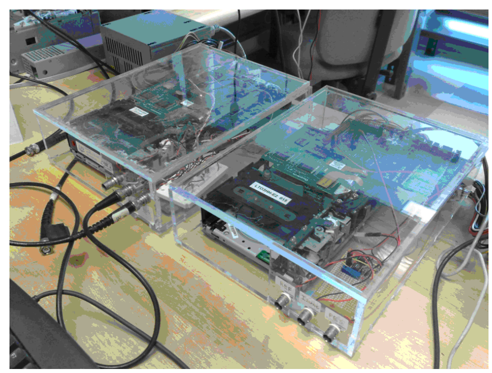

Figure 1. LTO3 drives with IO/hardware for experimentation purposes.

Using closed-loop experiments with an external excitation signal r on the servo controller output u, we measured the position error signal (PES) y. With the signals u and y related to r via

$$y(t) = G_0 S_i r(t) + S_i v(t),\ u(t) = S_i r(t) - C_i S_i v(t)$$

where G_0 denotes the (to be modelled) servo actuator dynamics, C_i is the dynamics of the (known initial) servo controller, $S_i = (1 + G_0 C_i)^{-1}$ is the sensitivity function and v represent the external disturbances such as lateral tape motion (LTM) present during track servoing. As our reference signal r is statistically uncorrelated with v, closed spectral analysis techniques can be used to obtain an estimate $\hat{G}(\omega)$ of the frequency response of the servo dynamics via

$$\hat{G}(\omega) = \frac{\hat{\Phi}yr(\omega)}{\hat{\Phi}ur(\omega)}$$

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where $\hat{\Phi}xr(\omega)$ denotes an estimate of the cross-spectral density function based on windowed and averaged cross-correlation functions [3] between a signal x and the reference signal r. The resulting frequency responses $\hat{G}_1(\omega)$ and $\hat{G}_2(\omega)$ of the LT03 drives used in our study has been depicted in Figure 2.

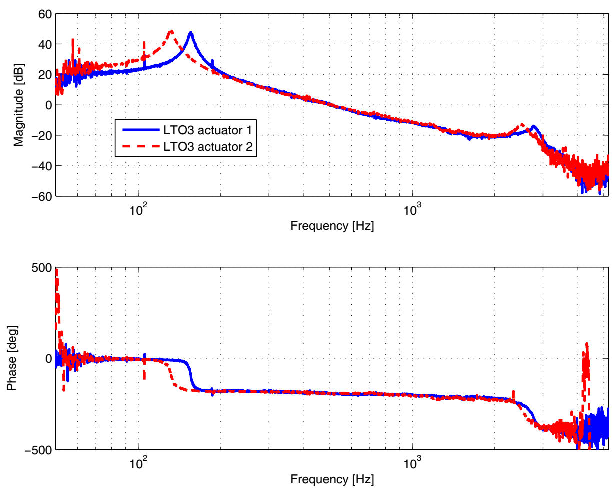

Figure 2. Estimated frequency responses of the two LTO3 drives.

From Figure 2 we can observe the obvious variations in the main resonance modes around 150Hz and 2.5kHz. In addition, but not clearly visible, there are variations in small resonance modes in the range from 500 - 2kHz. These variations around the anticipated servo bandwidth play an important role in the stability and performance robustness of a servo system.

Uncertainty Characterization

Based on the measured frequency responses, a nominal model $\hat{P}$ along with an bounded model perturbation or uncertainty Δ can be formulated to construct an uncertainty model P of our LTO3 drives. To be consistent with standard robust control analysis and design tools [4], the uncertainty model is of the form

$$\mathcal{P} = \{P \mid P = \mathcal{F}_u(Q,\Delta),\ |\Delta(\omega)| \leq \delta(\omega)\}$$

where $\mathcal{F}_u(Q,\Delta) = Q_{22} + Q_{21}\Delta(1+\Delta Q_{11})^{-1}Q_{12}$ denotes the upper fractional transformation of the model perturbation Δ and a 2×2 transfer function Q. The transfer function Q is a function of the nominal model $Q_{22} = \hat{P}$ and the way in which the uncertainty Δ is characterized on the nominal model $\hat{P}$.

A standard (unstructured) additive uncertainty model of the form $\mathcal{P} = \{P \mid P = \hat{P} + \Delta\}$ can be realized via $Q_{11} = 0$, $Q_{21} = Q_{12} = 1$ but is known to create an overly conservative representation of the allowable model perturbation [4].

Instead, we opt to choose an uncertainty model $\mathcal{P}$ that is a special case of the so-called dual-Youla uncertainty model [5] in which we use the knowledge of the stable nominal model $\hat{P}$ and the stable initial servo controller C_i. The resulting dual-Youla uncertainty model is given by

$$\mathcal{P} = \{P \mid P = (\hat{P}+\Delta)(1-C_i\Delta)^{-1},\ |\Delta(\omega)| < \delta(\omega)\} \quad (1)$$

which reduces to a standard additive uncertainty description if and only if the initial servo controller is chosen as $C_i = 0$.

Computation of the dual-Youla uncertainty Δ in (1) is straightforward. Given a stable nominal model $\hat{P}$ and a stable initial controller C_i that from a stable feedback,

$$\begin{aligned} \delta(\omega) &= \max_k |\Delta_k(\omega)|, \text{ where} \\ \Delta_k(\omega) &= (1+\hat{G}_k(\omega)C_i(\omega))^{-1}(\hat{G}_k(\omega) - \hat{P}(\omega)) \end{aligned} \quad (2)$$

in which $\hat{G}_k(\omega)$ denotes the frequency response of different servo actuators G_k, $k = 1,2,\ldots$ stabilized by the same initial servo controller C_i. Again, it can be seen that (2) reduces to an additive uncertainty bound in case $C_i = 0$.

Stability Robustness

The knowledge of the initial controller C_i is used in (2) to shape the model perturbation $\Delta_k(\omega)$ with the sensitivity function $(1+G_kC_i)^{-1}$ for each of the servo actuators G_k. With $|(1+\hat{G}_k(\omega)C_i(\omega))^{-1}| < 1$ for frequencies below the cross-over frequency of the servo system, any (low frequent) additive perturbations not relevant for stability or performance robustness are automatically reduced in the uncertainty bound of (2). This makes the uncertainty model (1) less conservative and better suited for robustness analysis and robust controller design.

In this short paper, only the results for stability robustness are included. Given the uncertainty model $\mathcal{P}$ in (1), the robust stability of a newly designed and enhanced servo controller C_{new} applied to all possible models $P \in \mathcal{P}$ can be guaranteed if

$$|(1+C_{new}(\omega)\hat{P}(\omega))^{-1}(C_{new}(\omega) - C_i(\omega)|\delta(\omega) < 1\ \forall\omega \quad (3)$$

where $\hat{P}$ is the nominal model, C_i the initial controller and $\delta(\omega)$ is the uncertainty bound computed via (2). The robust stability test (3) is derived from the small gain theorem [4] and has to be performed over a densely chosen frequency grid ω to be valid.

The robust stability test (3) also indicates another favorable property of the proposed model uncertainty set P in (1). If the newly designed controller C_{new} is equal or close to the initial controller C_i, stability robustness is trivially satisfied.

This makes sense, as our initial (low bandwith) servo controller C_i was know to stabilize all servo actuators G_k for which me measured the frequency responses $\hat{G}_k$. However, such a trivial result is not obtained with a more conservative additive uncertainty set with $C_i = 0$, as the condition $|(1+C_{new}(\omega)\hat{P}(\omega))^{-1}C_{new}(\omega)|\delta(\omega) < 1 \; \forall\omega$ would have to be verified.

Application to LTO3 drives

For the two measured frequency responses depicted in Figure 2, the damping and location of the two main resonance modes of the servo actuators is obtained via standard curve fitting routines [6]. Subsequently, a nominal model $\hat{P}$ is obtained by choosing the average of the damping and location of the resonance modes as depicted in Figure 3.

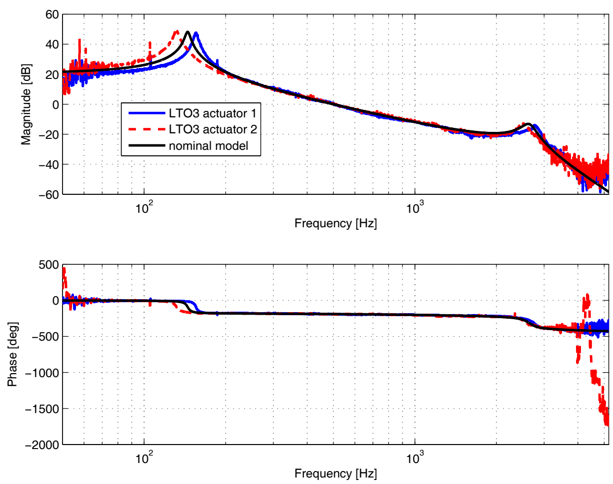

Figure 3. Nominal model $\hat{P}$ compared to the estimoated frequency responses of the two LTO3 drives.

Based on the nominal model $\hat{P}$ and the initial controller C_i used for the closed-loop experiments, the uncertainty bound $\delta(\omega)$ in (2) can be computed for $k = 1,2$ and the results are depicted in Figure 4. A comparison is made with the more conservative additive uncertainty bound $\delta(\omega)$ when the information on the initial controller is not used.

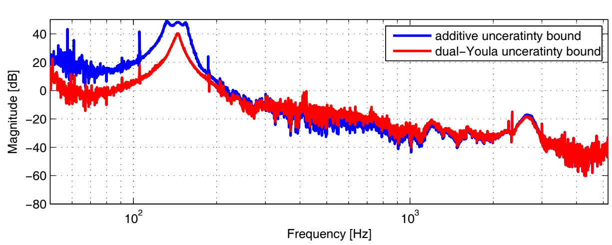

Figure 4. Uncertainty bound $\delta(\omega)$ in (1) for dual-Youla uncertainty model and additive uncertainty model $(C_i = 0)$.

It can be seen from Figure 4 that the dual-Youla uncertainty bound is indeed smaller at the lower frequencies in which perturbations on one of the main resonance modes of the servo actuator occurs. In addition, the most crucial advantage of the proposed dual-Youla uncertainty description to capture the variations in the LTO3 drives is depicted Figure 5 that evaluates the stability robustness test for $C_{new} = C_i$. For the dual-Youla uncertainty model in (1), the robust stability test (3) for $C_{new} = C_i$ is trivially satisfied, guaranteeing stability of the controller C_i. However, for the additive uncertainty model there exists a frequency ω for which $|(1+C_i(\omega)\hat{P}(\omega))^{-1}C_i(\omega)|\delta(\omega) > 1$, so that stability robustness cannot be guarantee for all models P in an additive uncertainty model. This clearly indicates that dual-Youla uncertainty model provides a better way for characterizing the unstructured variations in the dynamic response of the LTO3 servo actuators studied in this paper.

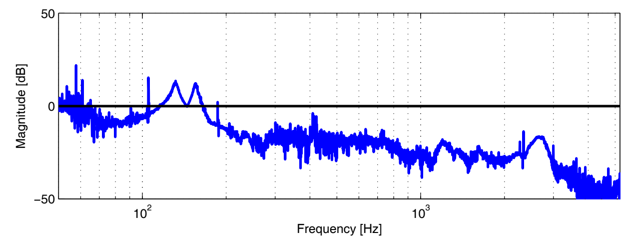

Figure 5. Robust stability test (3) for $C_{new} = C_i$.

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