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ROBUST ESTIMATION AND ADAPTIVE CONTROLLER TUNING FOR VARIANCE MINIMIZATION IN SERVO SYSTEMS

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ABSTRACT

Improving storage density by means of an enhanced servo system typically requires the reduction of variance of the Position Error Signal (PES). The variance of the PES is determined by both period and non-repeatable disturbances for which the characteristics are often not known *a-priori* during the servo algorithm design. Moreover, the servo control algorithm is often limited to a standard Proportional, Integral and Derivative (PID) controller as more complicated algorithms are viewed to be less robust. In this document it is shown how a standard (PID) servo control algorithm can be augmented with an additional feedback loop that can be tuned automatically by estimating the actual disturbance spectra seen in the PES. Adaptation to the disturbance spectra is done in lieu of possible model uncertainties in the servo actuator, guaranteeing stability robustness. As such, the control algorithm provides a Robust Estimation and Adaptive Controller Tuning (REACT) to disturbance spectra to minimize PES variance in high performance servo systems in data storage applications.

1. INTRODUCTION

Optimizing servo controllers for high performance data storage applications require an intricate tuning performance in terms of Position Error Signal (PES) variance minimization, while maintaining robustness. PES variance minimization is required to improve storage density, while performance robustness is the mathematical concept of guaranteeing that a single and well designed algorithm is guaranteed to yield the same performance on a large class of servo actuator with similar dynamical properties. Obviously, performance and robustness are often conflicting requirements [1], unleashing the need to compromise performance to ensure, at least, stability robustness. In addition, for optimal tuning of the servo algorithm in terms of PES variance minimization, a detailed model is needed of all possible disturbances that will be present in the servo loop and contribute to PES variance. Such a model can be formulated reasonably well for repeatable disturbances, but the knowledge on random or non-repeatable disturbances is often lacking and only becomes available after the servo algorithm has been implemented.

To facilitate automatic tuning of the servo control algorithm to the disturbances seen in the servo loop this document proposes a methodology for Robust Estimation and Adaptive Controller Tuning (REACT) [2] to minimize PES variance in high performance servo systems for data storage applications. The algorithm formulates the tuning as a perturbation on an existing standard Proportional, Integral and Derivative (PID) controller and formulates conditions for stability robustness by considering a special so-called Youla [3] and dual-Youla [4] parametrization and possible uncertainty in the actuator dynamics.

This document gives an overview of the main idea behind the REACT algorithm to tune servo controllers for data storage systems. Applications of REACT in the field of Active Noise Control can be found in [5], while an application to a Linear Tape Open (LTO) drive system is presented at the MIPE'09 conference [6]. The outline of this document is as follows. Section 2 first gives an overview of the Youla parametrization used to formulate the perturbation of the servo controller. This is followed in Section 3 by presenting the results on how model uncertainty in actuator dynamics can be incorporated to ensure stability robustness during servo controller tuning. Section 4 and 5 show how the parametrization can be used to tune the servo control algorithm directly on the basis of data measured directly in an operational servo loop and an example is given in Section 6.

2. CONTROLLER PARAMETRIZATION

A well-know result in controller design and optimization is the Youla parametrization [3] that allows the parametrization of the class of all stabilizing feedback controllers C for a given dynamical system G by a single stable dynamical system Q. The parametrization is given in terms of coprime factorization of the dynamical system G and the feedback controller C that are defined as follows for Single Input, Single Output systems.

Definition 1 (right coprime factors): Consider the pair of stable transfer functions (N, D). The pair (N, D) is a right coprime factorization (*rcf*) of G if the following three items hold:

a. there exists a pair of stable transfer function (X, Y) such that $XN + YD = 1$,
b. $\det\{D\} \neq 0$ and
c. $G = ND^{-1}$ ◊

According to Definition 1, coprime factors (N, D) are always stable transfer functions and item a. guarantees they do not have common unstable zeros, whereas item b. and c. guarantee that G can be written as the product of N and the inverse of D. A similar definition can also be given for the coprime factors (N_c, D_c) of the feedback controller $C = N_c D_c^{-1}$. With the definition of coprime factors, the Youla parametrization [3] applied to a servo system reads as follows.

Definition 2 (Youla paramerization): Let (N, D) be a *rcf* of an actuator transfer function G and let (N_c, D_c) be a *rcf* of a an *initial* feedback controller C that stabilizes G. Then all controllers C_Q of the form

$$\begin{aligned} &C_Q = N_Q D_Q^{-1} \text{ with} \\ &N_Q = N_c + DQ \text{ and} \\ &D_Q = D_c - NQ \end{aligned} \tag{1}$$

where Q is *any* stable transfer function, stabilize the same actuator transfer function G. ◊

The Youla parametrization is a powerful result, as it allows us to parametrize all stable controllers for a specific servo actuator dynamics G via (right) coprime factors and a stable perturbation Q. Hence, as long as we keep the transfer function Q stable, the newly tuned or perturbed controller $C_Q = N_Q D_Q^{-1}$ in Definition 2 will still stabilize the actuator dynamics G.

The use of coprime factors is required for unstable actuator or controller dynamics to avoid the cancellation of unstable poles and zeros in our feedback loop during controller perturbations. However, the Youla parametrization can be simplified in case both the actuator dynamics G and the initial feedback controller C are known to be stable. With a stable actuator G, we obtain the trivial choice $(N, D) = (G, 1)$ for the *rcf* of G, while for a stable initial controller C we may choose $(N_c, D_c) = (C, 1)$ for the *rcf* of C. In that case, the newly tuned or perturbed controller $C_Q = N_Q D_Q^{-1}$ in Definition 2 will simplify to

$$C_Q = (C + Q)/(1 - GQ) \tag{2}$$

and writing the newly tuned or perturbed controller C_Q as a feedback connection of the stable transfer function Q and the actuator model G. Again, as long as Q is stable the controller C_Q stabilizes G.

As a final note, the assumption of a stable actuator G also allows the initial controller C to be chosen as $C = 0$ as a stable actuator G does not require a controller for stabilization. In that case C_Q can be simplified further to

$$C_Q = Q/(1 - GQ) \tag{3}$$

However, any information on an initially designed controller C used in the stabilization or control of the actuator dynamics G can be used in the parametrization of the newly tuned or perturbed controller C_Q.

3. ACTUATOR UNCERTAINTY

The Youla parametrization as given in Definition 2 assumes no modeling error or uncertainty on the (stable) actuator dynamics G to ensure stability of the newly tuned or perturbed controller C_Q in (1) or (2). As a result, the only requirement for stability robustness is the stability of the perturbation Q in (1). In case of actuator uncertainty, an additional constraint on the actual "size" of Q will have to be imposed to guarantee stability robustness.

The bound on the size of Q for stability robustness depends on how the uncertainty on the actuator dynamics G is described, e.g. additive or multiplicative uncertainty. However, describing the uncertainty on the actuator dynamics G also in a coprime framework allows a clear computation of the upper bound on the size of the perturbation Q. Following the ideas of a double-Youla parametrization [7], the following main result is given here.

Corollary 1 (stability robustness for dual-Youla): Let (N, D) be a *rcf* of an *nominal* actuator transfer function G and let (N_c, D_c) be a *rcf* of a an *initial* feedback controller C that stabilizes G. Now consider a stable uncertainty Δ perturbing $G = ND^{-1}$ to G_Δ given by

$$\begin{aligned} &G_\Delta = N_\Delta D_\Delta^{-1} \text{ with} \\ &N_\Delta = N + D_c \Delta \text{ and} \\ &D_\Delta = D - N_c \Delta \end{aligned} \tag{4}$$

where Δ is unknown but bounded by an H_∞-norm $\|\Delta\|_\infty < \gamma^{-1}$. Then the uncertain actuator dynamics G_Δ with the controller perturbation C_Q given in (1) yields a stable feedback system for all $\|\Delta\|_\infty < \gamma^{-1}$ if and only if Q is stable and $\|Q\|_\infty \leq \gamma$. ◊

The result in Corollary 1 indicates that next to the stability requirement of Q mentioned in Definition 2 we now also have a size constraint on Q measured by an H_∞-norm. If we have no uncertainty, $\gamma = \infty$ and only the requirement on the stability of Q remains.

The uncertainty Δ on the nominal actuator dynamics G is structured according to a dual-Youla parametrization, as indicated in Figure 1. Although unfamiliar at first, it has been shown to have many favorable properties [7] compared to standard additive or multiplicative uncertainties. Similar to the argumentation used for the Youla parametrization, it can be seen that a stable nominal model G with a *rcf* $(N, D) = (G, 1)$ with an

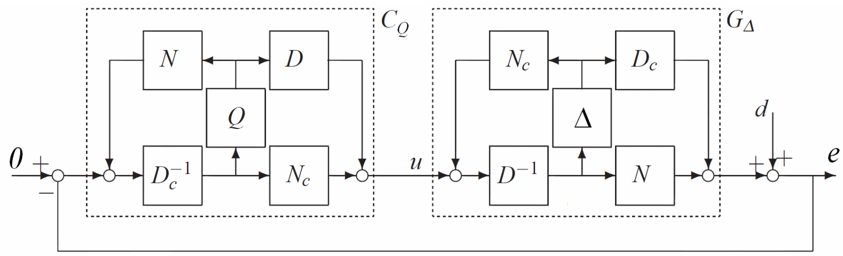

Fig. 1: Block diagram of combined controller perturbation according to (1) and actuator uncertainty given in (4).

initial controller $C=0$ reduces the uncertainty description in (3) back to a simple additive uncertainty $G_\Delta = G+\Delta$. However, any information on an initially designed controller C used in the stabilization or control of the actuator dynamics G can be used in the uncertainty description of the uncertain actuator dynamics G_Δ.

4. AFFINE OPTIMIZATION

Next to providing a parametrization of all stabilizing controllers, the Youla parametrization in (1), and its dual form for uncertainty representation, provides an another advantage for controller adaptation: all closed-loop transfer function are linear in the controller perturbation Q of (1). This can be seen as follows.

Consider the output sensitivity function or transfer function S from the input disturbance d to the output signal or PES e in Figure 1 given by

$$S = 1/(1+G_\Delta C) \quad (5)$$

Assuming the nominal actuator model G is stable, allows substitution of $C=C_Q$ given in (2) to modify the sensitivity function S in (5) to S_Q

$$S_Q = (1-G_\Delta Q)/(1+G_\Delta C) = S(1-G_\Delta Q) \quad (6)$$

and indicating that the closed-loop transfer function that models the disturbance rejection (sensitivity function) is *linear* in the controller perturbation parameter Q. A similar result can also be obtained when using the full freedom in right coprime factorization, allowing the nominal actuator model also to be unstable.

The Youla parametrization allows affine optimization techniques to, for example, minimize the 2-norm or ∞-norm of the (weighted) sensitivity function S as a function of Q. Moreover, direct minimization of the variance of the PES e can be used for a 2-norm, allowing direct data-based closed-loop tuning of a minimum variance controller [8] computed via the controller perturbation parameter Q. For that purpose, the transfer function $Q(q)$ must also parametrized in a linear affine form and here Q(q) is chosen to be parametrized via a nth tap discrete-time Finite Impulse Response (FIR) filter

$$Q(q,\theta) = \sum_{k=1}^{n} \theta_k q^{-1} \quad (7)$$

where q denotes the time shift operator and θ_k the parameters to be estimated in the FIR filter. The parametrization of the controller $C_Q(q)$ in (2) and $Q(q,\theta)$ in (7) provides a rich class of controllers that does not necessarily require a specific parametrization of noise models to tune the feedback controller [9].

5. CLOSED-LOOP AND DATA-BASED TUNING

For computational purposes, the variance of the PES e given in Figure 1 is given by

$$\mathrm{var}\{e\} = \sum_{t=1}^{N} e^2(t) \quad (8)$$

and computed over a final time interval of N data points. To minimize the variance of the PES e as depicted in Figure 1 and to use the affine optimization result given in (6) and (7) we need access to specific closed-loop signals for direct data-based tuning of the controller perturbation parameter $Q(q,\theta)$. Assuming disturbances that influence the variance of the PES e occur as an additive output disturbance d as indicated in Figure 1, we see that

$$e(t) = S_Q(q)d(t) \quad (9)$$

and substitution of S_Q in (9) yields

$$e(t,\theta) = S(q)d(t) - Q(q,\theta)G_\Delta(q)S(q)d(t) \quad (10)$$

so that minimization of the 2-norm of the time domain signal $e(t,\theta)$ using the affine parametrization in (7) requires access to the two closed-loop signals

$$\begin{aligned} &y(t) = S(q)d(t) \text{ and} \\ &v(t) = G_\Delta(q)S(q)d(t) = G_\Delta(q)y(t) \end{aligned} \quad (11)$$

The signal y(t) given in (11) is readily available, as this is the PES when the initial controller C is implemented in the feedback loop or $Q(q,\theta)=0$. For the computation of the signal $v(t)$ we need the (perturbed) actuator model $G_\Delta(q)$ that can be approximated by the nominal model $G(q)$ that is used also for the computing the perturbation of the controller C to C_Q given in (2). The availability of the signals $y(t)$ and $v(t)$ defined in (11) now allows the minimization of the variance of the PES as function of the parameter θ of the FIR filter $Q(q,\theta)$ via

$$\hat{\theta} = \arg\min_\theta \sum_{t=1}^{N} [y(t) - Q(q,\theta)v(t)]^2$$

and becomes a standard Least Squares (LS) optimization for the affine parametrization of $Q(q,\theta)$ given in (7). The LS optimization can be implemented recursively [8] to facilitate adaptive tuning of the controller, but special conditions on the rate of change in the parameters θ has to be imposed to guarantee stability of the resulting time varying system [2].

It should be noted that the LS optimization aims at minimizing the variance of the PES resulting in large control signals for the minimum variance controller [8]. To avoid large control signal, an additional penalty on the control signal

$$u(t) = C_Q(q)S(q)d(t) \tag{12}$$

can be imposed by including a (filtered version) of *u(t)* in (12) in the minimization of (11). Alternatively, control signals can be limited by adding an additional fixed filter stable and stably invertible *F(q)* into the controller perturbation

$$Q_F(q,\theta) = Q(q,\theta)F(q) \tag{13}$$

and possibly including the inverse of *F(q)* in the filtering of *y(t)* as used in the LS optimization.

6. EXAMPLE

Practical implementations of REACT have been reported in [5] and at the current MIPE'09 conference [6]. To illustrate the main concepts in this paper, a simulation example is used to illustrate the power of the REACT algorithm, as it allows direct tuning of the feedback controller with respect to disturbances. For the example in this paper we consider the Zero Order Hold (ZOH) equivalent of a continuous-time 4th order servo actuator model sampled at 10 kHz

$$G(s) = \frac{K\omega_1^2\omega_2^2}{(s^2+2\beta_1\omega_1 s+\omega_1^2)(s^2+2\beta_2\omega_2 s+\omega_2^2)}$$

with (14)

$K = 100$, $\omega_1 = 10$ rad/s, $\omega_2 = 10^3$ rad/s, $\beta_1 = \sqrt{1/2}$ and $\beta_2 = 0.1$

that models a low frequency friction or flex cable mode and a high frequent poorly damped resonance mode. The actuator model is controlled by a 1st order discrete-time Proportional Derivative (PD) controller sampling at 10kHz and given by

$$C(q) = 10\cdot\frac{q-0.995}{q-0.95} \tag{15}$$

creating a feedback loop with a gain margin of 9dB at approx. 823 rad/s and a phase margin of 55.8 deg at approx. 200 rad/s.

For simulating the effect of REACT, a low-pass filtered unit variance white noise $\varepsilon(t)$ together with a sinusoidal signal of 10Hz are used to create an additive disturbance

$$d(t) = L(q)\varepsilon(t) + 0.1\sin(2\pi\cdot 10t) \tag{16}$$

where the low-pass filter $L(q)$ is a 4th order discrete-time Butterworth filter with a cutoff frequency of 125 rad/s. Simulating the performance of the initial controller $C(q)$ in (15) in feedback with the ZOH equivalent of $G(s)$ in (14) yields the disturbance *d(t)* and PES *e(t)* depicted in Figure 2. Although there is

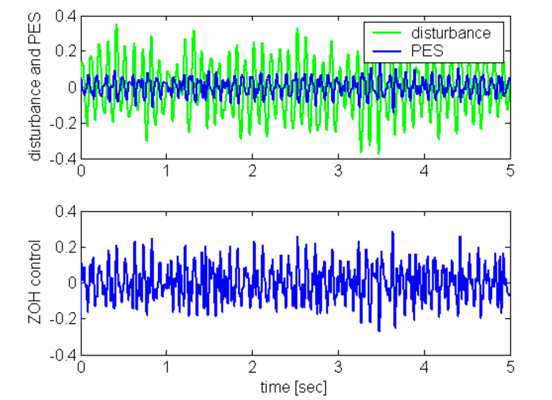

Fig. 2: Simulation results for rejection of disturbance *d(t)* given in (16) for the initial controller *C(q)* given in (15). Top figure: actual disturbance *d(t)* and PES *e(t)*. Bottom figure: control signal *u(t)*.

disturbance attenuation, the existing PD controller $C(q)$ in (15) has obviously not been optimized for the non-repeatable and low frequent periodic disturbances seen in the feedback loop.

To optimize the feedback controller using the REACT algorithm, the controller $C(q)$ in (15) is perturbed to $C_Q(q)$ in (2) with only a 10th order FIR filter $Q(q,\theta)$ in (7). To limit the control signal *u(t)* during the variance minimization of REACT, the filter *F(q)* in (13) is chosen as a low-pass 4th order discrete-time Butterworth filter with a cutoff frequency of 1000π rad/s. Simulating the performance of the perturbed controller $C_Q(q)$ in (2) in feedback with the ZOH equivalent of $G(s)$ in (14) yields the end result depicted in Figure 3.

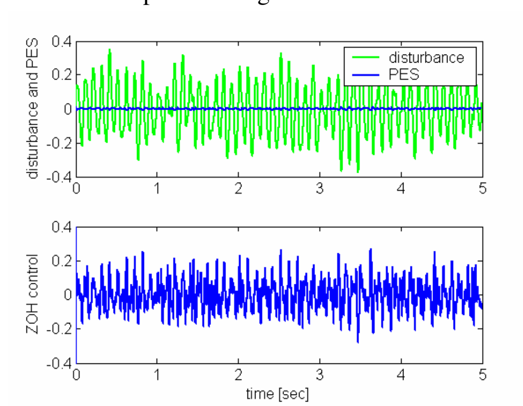

Fig. 3: Simulation results for rejection of disturbance *d(t)* given in (16) for the optimized REACT controller $C_Q(q)$ using a $n=10$ order FIR filter in (7) .

It can be seen from the simulation that REACT improves the variance of the PES *e(t)*, at only a small increase of the control signal *u(t)*. A comparison of the Bode plots of the initial controller *C(q)* in (15) and the controller $C_Q(q)$ in (2) is given in Figure 4.

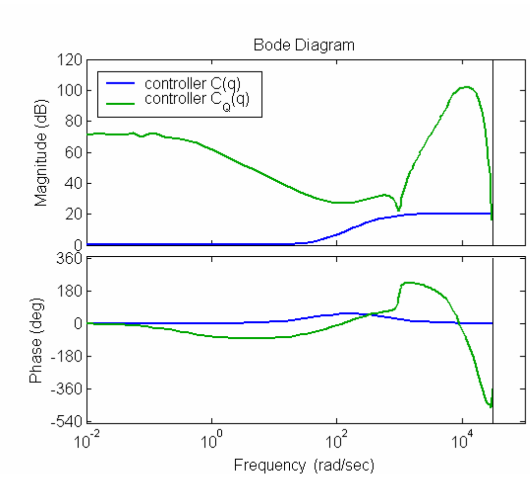

Fig. 4: Amplitude (top) and phase (bottom) Bode plot of the initial controller *C(q)* and the optimized controller $C_Q(q)$ found via REACT.

From Figure 4 it can be observed that the controller $C_Q(q)$ found after the optimization has increased the overall gain to provide non-repeatable disturbance rejection and created additional gain at low frequencies to target the low frequency disturbances.

The improvement in disturbance rejection can also be seen in Figure 5 in which a comparison is made between the sensitivity functions *S(q)* in (5) and $S_Q(q)$ in (6). The additional gain at low frequencies in $C_Q(q)$ emulates an integrator that was missing in the PD controller *C(q)* and now creates a feedback loop with a gain margin of 8.25dB at approx. 3500 rad/s and a phase margin of 52.3

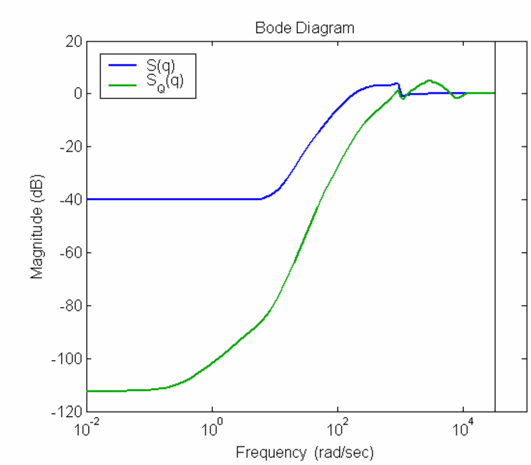

Fig. 5: Amplitude Bode plot of the initial sensitivity function *S(q)* in (5) and the optimized sensitivity function $S_Q(q)$ in (6) found via REACT.

deg at approx. 865 rad/s. The advantage is that the resulting controller $C_Q(q)$ was found by automatic tuning based on data obtained from the closed-loop system using the initial controller *C(q)*.

7. SUMMARY AND CONCLUSIONS

An initial (PID) servo control algorithm can be augmented with a Youla parametrization based perturbation to formulate a self-tuning algorithm for a servo controller. For actuator dynamics that can be modeled by a stable transfer function, the parametrization is formulated as a feedback loop that uses the actuator model and a free, but stable, perturbation transfer function given by an Finite Impulse Response (FIR) filter. The parameters of the FIR filter can be found by an affine optimization based on closed-loop data to optimally tune the perturbed feedback controller. Uncertainty on the actuator model can be incorporated by bounding the allowable controller perturbation to provide a Robust Estimation and Adaptive Controller Tuning (REACT) to disturbance spectra to minimize PES variance in high performance servo systems in data storage applications.

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