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MODELING AND CONTROL OF LATERAL TAPE MOTION FOR IMPROVED PERFORMANCE

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Introduction

The current state of the art for controlling lateral tape motion is a well tuned PID controller that is able to reject all low frequent disturbances. In this paper, we seek to improve this control design by modeling the source of the disturbances in lateral tape motion LTM and designing a controller to reject these specific disturbances instead of all low frequent disturbances. This will enable us, through the well know Bode sensitivity integral [1], to design a controller with better performance at high frequencies thus improving the position error signal PES. From measured date it is shown that the disturbances are time-varying sinusoids and filtered white noise. The time-varying nature is due to the change in radius of the spools that occurs during shuttling of the tape. In this paper, we develop an analytic model of the time-varying sinusoids and compare the model to the measured data. Based upon this improved model of the disturbances a controller is design that targets these time-varying sinusoids by using known LQG [2] theory for linear time-varying systems.

Model of Periodic Disturbances in LTM

The following are assumed:

(A1) The speed of the tape is constant.
(A2) The radius of the spool changes continuously as a function of time and evenly around the spool.
(A3) The thickness of the tape is much smaller then the smallest radius of the spool.

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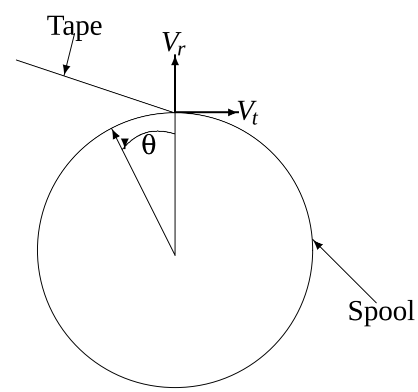

Figure 1. Spool with constant tape speed.

The speed of the the tape is given by

$$s = |v| = |\dot{r}\hat{e_1} + r\dot{\theta}\hat{e_2}|, \tag{1}$$

where $\hat{e}_1$ and $\hat{e}2$ are unit vectors in the radial and tangential directions respectively. By (A3) the thickness of the tape is very small in comparison to the radius of the empty spool. This implies that $\dot{r}$ will be very small as compared to $r\dot{\theta}$. Thus we have,

$$v \approx r\dot{\theta}, \tag{2}$$

in the e_2 direction.
By (A2), the radius of the spool as a function of time can be written as

$$r_e(t) = r_0 + \frac{\tau}{2\pi}\theta_e(t) \tag{3}$$

for the empty spool and

$$r_f(t) = R - \frac{\tau}{2\pi}\theta_f(t) \tag{4}$$

for the full spool. R is the size of the spool when completely full, r_0 is the size of the spool when empty, τ is the thickness of the tape, and $\theta \in [0,\infty)$ is the angle of the spool. The angle of the spool can also be written as $\bar{\theta} + n$ where $\bar{\theta} \in [0, 2\pi]$ and n is the number of revolutions.
The velocity can be integrated to get

$$\int_{t_0}^{t} v dt = (r_0\theta_e(t) + \frac{\tau}{4\pi}\theta_e^2(t)) \tag{5}$$

$$= p(t) \tag{6}$$

where $p(t)$ is the position of the tape and it is assumed that $\theta(t_0) = 0$. Rearranging and taking a derivative gives

$$\dot{\theta}_e = f(\theta_e, \tau, r, v) \tag{7}$$

$$= \frac{v(t)}{\left(\sqrt{r_0^2 + \frac{\tau}{\pi}p(t)}\right)} \tag{8}$$

where we dropped the negative sign since we are rotating in a positive θ direction. By $(A1)$ we have $p(t) = Kt$, which gives

$$\dot{\theta}_e = \frac{K}{\left(\sqrt{r_0^2 + \frac{\tau}{\pi}Kt}\right)}, \tag{9}$$

where K is the speed of the tape. The full spool is obtained similarly and the solution can be written as

$$\dot{\theta}_f = \frac{K}{\left(\sqrt{r_0^2 - \frac{\tau}{\pi}Kt}\right)}, \tag{10}$$

where the minus sign is due to the decreasing radius of the spool. From this point one the subscripts will be dropped and $\dot{\theta}$ will be used instead.
Any wobble due to imperfections in the manufacturing process will manifest themselves in a periodic manner since the same path will be traced each time the spool rotates fully around. The traced path can be fit will a Fourier series, since it is periodic, and therefore there will be a fundamental frequency given by $\dot{\theta}$ and higher harmonics that must be rejected.

If the angle of the drive is known then we can solve directly for $\dot{\theta}$ as a function of θ, we can solve for θ as a function of time, and an equivalent discrete time version can be solved.

Current Control Design

The baseline control design is a PID controller with a fixed notch that is able to reject low frequent disturbances. However, due to the fundamental limitations in feedback control, like the Bode's Integral, the removal of the low frequent disturbances causes the sensitivity function to exceed unity in magnitude at higher frequencies. This amplifies disturbances in this higher frequency band and degrades the performance of the system.

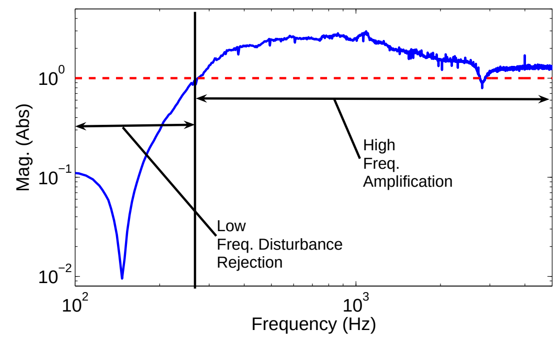

Figure 2. Baseline sensitivity function.

Figure 2 shows the magnitude of the baseline sensitivity function. Notice that this controller is able to reject the low frequency disturbances but amplifies some of the higher frequency disturbances as a result.

Controller for Improved Performance

In this section, we describe a discrete time control scheme based upon the internal model principle [3] and LQG theory. The internal model principle is used to cancel the time-varying periodic disturbances. LQG theory is used to design the state feedback and observer gains to minimize the affect of random noise upon the system. Finally, LPV theory can be used to create a sub-optimal controller that is efficiently implemented online by making use of quadratic stability but is beyond the scope of this paper.
It was shown in [4] that the LTI controller given by

$$C(q) = \left[\begin{array}{cc|c} A_p - L_pC_p - B_uK & 0 & L_p \\ L_mC_p & A_m(k) & -L_m \\ \hline -K & C_m & 0 \end{array}\right], \tag{11}$$

is an internal model-based controller and has an interpretation of a learning controller when the appropriate internal model is chosen. $A_m(k)$ and C_m are given (time-varying) matrices, called the internal model, and are used to model the time-varying periodic disturbance with frequency given by $\dot{\theta}$.

The job of the control engineer is to find the gains K, L_p, and L_m so that the closed loop system is stable and performs well. This can be accomplished in two steps: 1. Find the state feedback gain for the plant. 2. Find the time-varying observer gains L_p and L_m for the series connection of the internal model and plant.

Step 1. From standard LQR results, the state feedback gain is given by

$$K^* = (\mathcal{B}_u^T P_c \mathcal{B}_u + \mathcal{D}_{zu}^T \mathcal{D}_{zu})^{-1} \mathcal{B}_u^T P_c \mathcal{A} \tag{12}$$

where P_c satisfies

$$P_c = A_p^T P_c A_P - A_p^T P_c B_u (B_u^T P_c B_u + D_{zu}^T D_{zu})^{-1} B_u^T P_c A_p + Q$$

and Q is chosen by the designer to push the states of the closed loop to the origin and R is chosen to minimize control effort.

Step 2. The Kalman predictor for this system is

$$\hat{x}(k+1) = (\mathcal{A}(k) - \mathcal{L}(k)\mathcal{C}_p)\hat{x}(k) + \mathcal{L}_m u_m(k) + \mathcal{L}(k) y_p(k) \tag{13}$$

and the error system is

$$\tilde{x}(k+1) = (\mathcal{A}(k) - \mathcal{L}(k)\mathcal{C}_p)\tilde{x}(k) + w(k) - \mathcal{L}(k)v(k). \tag{14}$$

We arrive at the following Ricatti difference equation for the Kalman predictor gain

$$\begin{aligned} P_{k+1} &= \mathcal{A}(k) P_k \mathcal{A}(k)^T \\ &\quad - \mathcal{A}(k) P_k \mathcal{C}_p^T (\mathcal{C}_p P_k \mathcal{C}_p^T + V)^{-1} \mathcal{C}_p P_k \mathcal{A}(k)^T \\ &\quad + W \\ P_0 &= \mathcal{A}(0) \mathbb{X}(0) \mathcal{A}(0)^T + W \end{aligned} \tag{15}$$

and the predictor gain is

$$\mathcal{L}(k) = \mathcal{A}(k) P_k \mathcal{C}_p^T (\mathcal{C}_p P_k \mathcal{C}_p^T + V)^{-1}. \tag{16}$$

where

$$\begin{aligned} L &:= \begin{bmatrix} L_p \\ L_m \end{bmatrix} \quad \mathcal{A} := \begin{bmatrix} A_p & B_u C_m \\ 0 & A_m \end{bmatrix} \\ \mathcal{C}_p &:= \begin{bmatrix} C_p & 0 \end{bmatrix} \qquad \mathcal{C}_m := \begin{bmatrix} C_m & 0 \end{bmatrix}. \end{aligned} \tag{17}$$

V and W are chosen by the engineer to obtain an observer with good properties in a similar manner to R and Q.

To visualize what this controller is accomplishing, let the frequency of the disturbance be constant. The resulting sensitivity function is shown in Fig. 3. Notice that only specific low frequency disturbances are rejected and therefore there is no amplification at higher frequencies. Compare this figure to Fig. 2. From this figure it is clear that this controller is essentially placing time-varying notches where needed to cancel the disturbances.

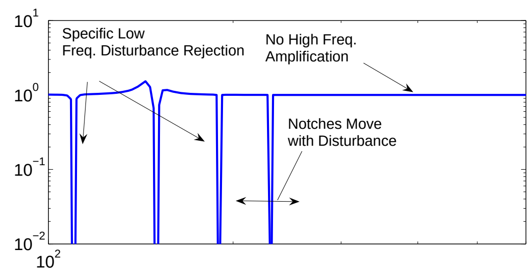

Figure 3. Improved sensitivity function.

Conclusions

This paper presented a new method of designing a controller for rejecting time-varying periodic disturbances that appear in lateral tape motion. A model for the frequency of the disturbance was derived and used to design a time-varying controller that, unlike the current PID based controller, is able to target specific disturbances. Targeting specific disturbances results in no over-amplification at higher frequencies, which is a problem with PID based controllers.

References

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