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Performance Weight Adjustment for Iterative Cautious Control Design

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***Abstract*— Research in control design on the basis of identified models has lead to many iterative algorithms where performance weights in a control objective function are determined a-priori and remain fixed in order to evaluate performance improvement during iterations. This paper considers the case where performance weights are adjusted in iterative identification and control schemes where performance robustness is maintained during each iteration to yield a cautious control design. In order to adjust performance weights, a nominal model along with uncertainty description obtained from a system identification procedure are used to adjust the performance weighting function conjointly with robust control synthesis. A framework is developed for measuring performance comparison between iterations which allows adjustment of the performance weights during the iterative cautious control design.**

I. Introduction

Addressing the problem of approximate identification and model-based control generally requires iterative schemes [1] composed of separate model estimation and control design steps. Many early model-based iterative schemes focus on nominal $\mathcal{H}_\infty$ performance enhancement of a closed-loop transfer function, denoted by $T(P,C)$ indicating dependance on a plant model P and controller C, while subsequent degradation of achieved performance of the controller C applied to the plant P_0 is evaluated via the triangle inequality [2]. A requirement that performance degradation should be small places emphasis on a closed-loop relevant identification error and implies the need for performance robustness in both the identification and control design [3].

Model-based iterative schemes have been developed such that evaluation of achieved performance is bounded by the worst-case performance of a controller C evaluated over a set of models $\mathcal{P}$. Performance robustness can then be monitored at each stage of the iteration process such that a monotonic decrease in worst-case performance is guaranteed [4]. Monitoring performance robustness however only entails performance improvement if the closed-loop transfer function $T(P,C)$ is adjusted appropriately during iterations. One way to incorporate appropriate adjustments is through the explicit parametrization of $T(P,C,W)$ as a function of W, where W reflects the general notion of a performance weighting function. Obviously selecting a fixed weighting function W allows a comparison between $\|T(P_0,C_{i+1},W)\|_\infty$ and $\|T(P_0,C_i,W)\|_\infty$ as a measure of performance [4], whereas adjustment of W during subsequent identification and control design iterations would require a notion of performance improvement dependent on the choice of W.

Considering the limited knowledge available on the plant dynamics at the beginning of an iterative identification and control procedure, selecting a performance weighting W a-priori seems hardly realistic. Even for given model and uncertainty description, maximizing performance robustness by adjusting weights is considered standard in practice [5] and has been used to compare controllers of different structure based on achieved closed-loop performance [6]. Several concepts of weighing function adjustment have been used in iterative identification and control design schemes, although in a different way than proposed here. Under an $\mathcal{H}_2$ performance measure, both model-based [3] and model-free [7] controller tuning have been presented with consideration for performance robustness. For considering performance robustness with respect to modeling uncertainty in the $\mathcal{H}_\infty$ criteria, [8] proposed scalar weighting to influence the desired cross-over frequency for improved nominal performance while robust stability with respect coprime factor uncertainty was maximized in control synthesis. Adjustable transfer function performance weights were pursued in an iterative $\mathcal{H}_\infty$ loop-shaping procedure [9] where accurate frequency response measurements of the unknown plant are progressively shaped for each iteration of control design.

This paper summarizes a methodology for adjusting the performance weight to W_i throughout an iterative identification and control design procedure. Rather than focusing on the specific design of performance weights, the proposed methodology is first presented in a general way to allow selection of the performance weight W_i either by tuning parameters in a selected weight characterization [10], [11] or by advanced optimization methods [12], [13] alleviating much of the need for engineering intuition. The measure of performance is then specified in terms of parameters of the current weighting function W_i. While maintaining performance robustness, the weighting function is adjusted to maximize the measure of performance enabling a cautious control design for each iteration. Specifically, the task during of each identification step is on control oriented nominal model estimation and uncertainty overbounding in a coprime factor framework such that for each iteration the effects of model uncertainty have less impact on the control performance [14]. The control design step considers simultaneous performance weight adjustment and controller synthesis such that performance robustness is maximized for each iteration. Controller improvement is indicated by the weighting function itself, i.e. the controller satisfying the most aggressive performance weight has achieved better performance.

II. Limiting Identification and Control via Performance Robustness

At iteration i consider a set of models $\mathcal{P}_i$ parametrized by a nominal model $\hat{P}_i$ along with an upper bound on the

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modeling error U_i

$$\mathcal{P}_i(\hat{P}_i, U_i) = \left\{ P \mid P = F(\hat{P}_i, \Delta), \text{ with } \|\Delta U_i\|_\infty < 1 \right\} \quad \text{such that} \quad P_0 \in \mathcal{P}_i \tag{1}$$

where P_0 represents the unknown plant and F denotes a particular parametrization of the model set $\mathcal{P}_i$. Common choices for the parametrization of $\mathcal{P}_i$ with nominal model and norm bounded uncertainty include additive, multiplicative and comprime uncertainty descriptions [15].

Consider an initial controller C_i that internally stabilizes all $P \in \mathcal{P}_i$, then we can define a stable closed-loop transfer function $T(P, C_i, W)$ where W is a stable and stably invertable weighting filter such that $\|T(P, C_i, W)\|_\infty$ is bounded. For a given controller C_i and weighting function W the performance for all $P \in \mathcal{P}_i$ is defined by

$$J(P, C_i, W) = \|T(P, C_i, W)\|_\infty. \tag{2}$$

Consequently, a weighting filter W_i can be chosen such that

$$\sup_{P \in \mathcal{P}_i} J(P, C_i, W_i) \leq 1 \tag{3}$$

and constitutes a robust performance condition.

For a given pair (C_i, W_i), the condition (1) and (3) constitute a modeling or identification problem in which a set of models $\mathcal{P}_i$ needs to be found that satisfies

$$P_0 \in \mathcal{P}_i \quad \text{and} \quad \sup_{P \in \mathcal{P}_i} J(P, C_i, W_i) \leq 1 \tag{4}$$

refering to the estimation of an uncertainty set that satisfies a performance robustness condition. Once a set of models $\mathcal{P}_i$ is found, the pair (C_i, W_i) can be updated via a cautious control design that emphasizes performance robustness improvement. Since both C_i and W_i will change to (C_{i+1}, W_{i+1}) the value of the norm function $J(P_0, C_{i+1}, W_{i+1})$ does not suffice in characterizing performance robustness improvement. Instead the (normalized) evaluation of the performance robustness

$$\sup_{P \in \mathcal{P}_i} J(P, C_{i+1}, W_{i+1}) \leq 1 \tag{5}$$

indicates that performance robustness has been satisfied for the pair (C_{i+1}, W_{i+1}) while the adjustment of the weighting function W_{i+1} (with respect to W_i) now provides an indication of the performance improvement.

III. Iterative Weight Adjustment

A. Evaluating performance weight improvement

In model-based control design, performance robustness is specified by the required bound (3) in which W_i plays an important role in the computation of an optimal (robust) controller. To quantify performance in terms of weighting functions, the improvement of W_i during iterations need to be monitored. For monitoring performance weights at each iteration consider the following definition [16].

Definition 1: A pseudometric on a set X is a function $\{\Gamma(\cdot,\cdot) : X \times X \to [0, \infty)\}$ such that

$$\begin{aligned} &\Gamma(x, y) \geq 0 \quad \forall\, x, y \in X \\ &\Gamma(x, y) = \Gamma(y, x) \quad \forall\, x, y \in X \\ &\Gamma(x, z) \leq \Gamma(x, y) + \Gamma(y, z) \quad \forall\, x, y, z \in X. \end{aligned} \tag{6}$$

A pseudometric $\Gamma(x, y)$ can be interpreted as the distance from x to y, with a common example being any norm. When evaluating performance weights in an iterative identification and control framework, the second argument y in (6) can be fixed through all iterations. Denote the function $J_{pw}(W)$ as a pseudometric that acts on stable transfer functions (on $\mathcal{RH}_\infty$) to produce a positive real number

$$\{J_{pw}(W) : \; \mathcal{RH}_\infty \to [0, \infty)\} \tag{7}$$

where the second argument is the origin $J_{pw}(W) = \Gamma(W, 0)$ or some fixed desired performance weight $J_{pw}(W) = \Gamma(W, W_*)$. Since weighting functions are chosen to reflect design objectives, the evaluation criteria J_{pw} should support this choice and emphasize desired properties in the performance of the closed-loop system, for example bandwidth and disturbance rejection. Thus this framework, although general, still requires some level of engineering intuition to initialize the characterization of performance improvement in J_{pw} which allows the performance weight to be progressively tuned and monitored between iterations. For example

$$J_{pw}(W_{i+1}) \geq J_{pw}(W_i) \tag{8}$$

provides an ordering of the performance weighting functions in an iterative scheme in which both the controller C and the weighting function W are adjusted.

B. General procedure

To address the trade-off between performance objectives and a tolerance to uncertainties while utilizing available tools for robust performance control design, the control objective function $J(P, C, W)$ is restricted to being an $\mathcal{H}_\infty$-norm computation. A method for determining a controller that maximizes performance according to a weighted objective function but subject to robust performance constraints is formulated as follows.

Problem 1: Let a plant P_0 form a stable feedback connection with the currently implemented controller C_i. From data collected in closed-loop, estimate a set of models $\mathcal{P}_i$ where $P_0 \in \mathcal{P}_i$ and choose weighting function W_i such that

$$\sup_{P \in \mathcal{P}_i} J(P, C_i, W_i) \leq 1 \tag{9}$$

Subsequently evaluate the following iterative procedure.

(a) Given a set of models $\mathcal{P}_i$ design performance weight W_{i+1} and robust controller C_{i+1} to satisfy

$$(C_{i+1}, W_{i+1}) = \max_W J_{pw}(W) \quad \text{such that} \quad \sup_{P \in \mathcal{P}_i} J(P, C_{i+1}, W_{i+1}) \leq 1, \tag{10}$$

where

$$C_{i+1} = \arg\min_C \sup_{P \in \mathcal{P}_i} J(P, C, W_{i+1}). \tag{11}$$

(b) If the performance weight has improved, that is

$$J_{pw}(W_{i+1}) > J_{pw}(W_i), \tag{12}$$

then implement the controller C_{i+1} and collect (new) data from a closed-loop experiment. Estimate a set of models $\mathcal{P}_{i+1}$ such that $P_0 \in \mathcal{P}_{i+1}$ and

$$\sup_{P\in\mathcal{P}_{i+1}} J(P, C_{i+1}, W_{i+1}) < \sup_{P\in\mathcal{P}_i} J(P, C_{i+1}, W_{i+1}). \quad (13)$$

The iterations are terminated when the performance weight can not be improved via control design, that is when (12) is not satisfied, or when newly collected data does not provide information that enables reduction in model uncertainty (b).

In case the set of models $\mathcal{P}$ is characterized with a nominal model $\hat{P}$ and upper bound on the model uncertainty U such that $P_0 \in \mathcal{P}$, the formulation of Problem 1 generates an iterative sequence between simultaneous model-based control and performance weight synthesis (C, W) and model set identification $(\hat{P}, U)$. For the model-based control design of step (a), the condition $J_{pw}(W_{i+1}) \geq J_{pw}(W_i)$ in (10) enforces performance improvement imposed by the metric acting on the performance weighting function, while $\sup_{P\in\mathcal{P}_i} J(P, C_{i+1}, W_{i+1}) \leq 1$ is a performance robustness condition that guarantees $J(P_0, C_{i+1}, W_{i+1}) \leq 1$. Solving the argument for C_{i+1} in (11) is a robust control design problem. For the model set identification of step (b), condition (13) considers closed-loop relevant identification of $\mathcal{P}_{i+1}$ such that $P_0 \in \mathcal{P}$ and where the information contained in the new model set improves the robust performance measure under the weighting W_{i+1} and the new controller C_{i+1}.

The proposed iterative method in Problem 1 readily lends itself to the subset of objectives, uncertainty sets and performance weights that are numerically tractable for which standard tools are available [15]. The following sections provide an example that discusses the characterization of the model set as well as the simultaneous performance weight and control design which illustrates benefits in progressively adjusting performance weights while maintaining performance robustness.

IV. Characterization of Model Set

For evaluating performance robustness, a choice is made for the characterization of the model set $\mathcal{P}$ constructed from a nominal model $\hat{P}$ along with an upper bound U on the mismatch between $\hat{P}$ and the actual plant P_0. The theory of fractional representations [17] is used to characterize the model set by representing (possibly unstable) transfer functions as a ratio of two stable transfer functions because provides a framework for identifying a control relevant nominal model [18] and uncertainty description [14]. Any system P has a *right coprime factorization* (*rcf*) (N, D) over $\mathcal{RH}_\infty$ if there exist $X, Y, N, D \in \mathcal{RH}_\infty$ such that $P(z) = N(z)D^{-1}(z)$; $XN + YD = I$. Normalized coprime factors are defined such that $N^T(z^{-1})N(z) + D^T(z^{-1})D(z) = I$. Consider the dual-Youla model set representation of all systems P parametrized by a nominal model $\hat{P}_i$ with *rcf* $(\hat{N}_i, \hat{D}_i)$ that are internally stabilized by controller C_i with *rcf* $(N_{c,i}, D_{c,i})$.

$$\mathcal{P}_i := \Big\{P | P = (\hat{N}_i + D_{c,i}\Delta_R)(\hat{D}_i - N_{c,i}\Delta_R)^{-1} \text{ with } \Delta_R \in \mathcal{RH}_\infty \text{ and } \|\Delta_R\|_\infty < 1\Big\} \quad (14)$$

To apply standard results for robust analysis and control design, the uncertainty description (14) is written in the linear fractional transformation (LFT) framework [15]. Consider the input/output signals u, y for any model $P \in \mathcal{P}_i$ described by (14), then perturbations on the nominal model can represented by a feedback connection with norm bounded uncertainty $\Delta \in \mathcal{RH}_\infty$ as an upper LFT

$$\mathcal{F}_u(Q, \Delta) := Q_{22} + Q_{21}\Delta(I - Q_{11}\Delta)^{-1}Q_{12} \quad (15)$$

and depicted in Figure 1. The LFT representation for the set $\mathcal{P}_i$ is obtained by defining $\Delta = \Delta_R U^{-1}$ and considering the map from $col(d, u)$ to $col(z, y)$,

$$\mathcal{P}_i = \Big\{P | P = \mathcal{F}_u(Q, \Delta) \text{ with } \Delta \in \mathcal{RH}_\infty\ ,\ \|\Delta\|_\infty < 1, \text{ where } Q = \begin{bmatrix} U_i\hat{D}_i^{-1}N_{c,i} & U_i\hat{D}_i^{-1} \\ D_{c,i} + \hat{P}_iN_{c,i} & \hat{P}_i \end{bmatrix}\Big\}. \quad (16)$$

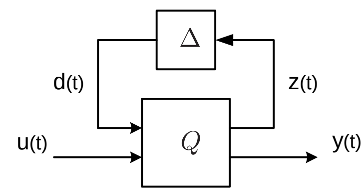

Fig. 1. Linear fractional transformation diagram for robust performance analysis/synthesis.

Model set identification, for initializing and performing subsequent iterations of steps (b) in Problem 1, consists in estimating the set $\mathcal{P}$ to reduce the effects of uncertainty on the control design. Low-order models can be approximated by separate estimation of a nominal model $\hat{P}_i$ and an upper bound on the uncertainty Δ characterized by filter U_i.

A. Estimation of nominal model and uncertainty

Methods for identifying control-relevant nominal model and uncertainty over bound from closed-loop experiments have been proposed in [18], [4] and are briefly reviewed here for completeness. Consider the mapping from reference signals $col(r_0, r_1)$ onto input/output $col(y, u)$ given by the transfer matrix $T(P, C)$

$$T(P, C) = \begin{bmatrix} P \\ I \end{bmatrix} (I + CP)^{-1} \begin{bmatrix} C & I \end{bmatrix} \quad (17)$$

where signals from the achieved closed-loop system of P_0 in feedback with currently implemented controller C_i satisfy

$$\begin{bmatrix} y \\ u \end{bmatrix} = T(P_0, C_i) \begin{bmatrix} r_0 \\ r_1 \end{bmatrix} + \begin{bmatrix} I \\ -C_i \end{bmatrix} (I + P_0C_i)^{-1} v. \quad (18)$$

For identification purposes, it is presumed that the signals u, y are measured and that r_0, r_1 are uncorrelated with the noise v. Access to both (normalized) *rcf* and an uncertainty characterization can be obtained by considering the auxiliary signals

$$\begin{aligned} x &:= (\hat{D}_x + C_i\hat{N}_x)^{-1} \begin{bmatrix} C_i & I \end{bmatrix} \begin{bmatrix} y \\ u \end{bmatrix} \\ z &:= (D_{c,i} + \hat{P}_xN_{c,i})^{-1} \begin{bmatrix} I & -\hat{P}_x \end{bmatrix} \begin{bmatrix} y \\ u \end{bmatrix} \end{aligned} \quad (19)$$

where the notation $\hat{P}_x$ with (normalized) *rcf* $(\hat{N}_x, \hat{D}_x)$ will be used to denote either an accurate auxiliary model or the nominal model used in control design, $x = i$. For estimating a nominal model, the input/output signals of the feedback connection $T(P_0, C_i)$ from the auxiliary signal x are written

$$\begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} N_{0,F} \\ D_{0,F} \end{bmatrix} x + \begin{bmatrix} S_{out} \\ -C_i S_{out} \end{bmatrix} v \tag{20}$$

where

$$\begin{bmatrix} N_{0,F} \\ D_{0,F} \end{bmatrix} = \begin{bmatrix} P_0(I + C_i P_0)^{-1}(I + C_i \hat{P}_x)\hat{D}_x \\ (I + C_i P_0)^{-1}(I + C_i \hat{P}_x)\hat{D}_x \end{bmatrix}, \tag{21}$$
$$S_{out} = (I + P_0 C_i).$$

Approximately normalized *rcf* estimates $(\hat{N}_i, \hat{D}_i)$ for (21) are obtained from multivariable output error [18] or even constrained ARX [19] prediction error optimization algorithms [20].

The uncertainty weight U_i in (16) is used to bound Δ_R in (14) by estimating a frequency dependent upper bound for Δ_R such that $P_0 \in \mathcal{P}_i$. Frequency dependent uncertainty bounds can be determined using a model error modeling approach based on prediction error framework [21]. Consider auxiliary signals x, z in (19) generated using the estimated nominal model from (21), $\hat{P}_x = \hat{P}_i$, then identify a consistent model $\hat{R}_i$ for the dual-Youla uncertainty $\Delta_{R,i}$ via

$$z = \Delta_{R,i} x + D_{c,i}(I + P_0 C_i)^{-1} v. \tag{22}$$

Confidence intervals on the model error model $\hat{R}_i$ provide a frequency dependent upper bound with specified probability. Spectral overbounding methods such as [22] can be used to estimate low-order stable and stably invertible uncertainty weight U_i.

V. Design for robust performance

To develope a robust controller, a specific choice for $J(P, C, W)$ must be made. The performance objective function used in this paper is taken to be a weighted-sensitivity

$$J(P, C, W) = \|W S_{in}\|_\infty, \quad S_{in} = (1 + CP)^{-1}. \tag{23}$$

This is a specific case for the widely applicable weighted four-block problem, i.e. a weighted version of (17), and is used particularly for developing intuition behind adjusting weighting functions in an iterative identification and control design scheme.

A. Weighted sensitivity robust performance

The dual-Youla uncertainty structure (14) allows any $\Delta_R \in \mathcal{RH}_\infty$ while preserving internal stability of the current feedback system, with controller C_i operating. To analyze the performance robustness for a new controller C applied to any model $P \in \mathcal{P}_i$, the closed loop system is written in the LFT framework.

Lemma 1: Consider the set $\mathcal{P}_i$ defined in (14) and a controller C such that the transfer matrix $T(P, C)$ is well-posed for all $P \in \mathcal{P}_i$. Then

$$\mathcal{P}_i = \{P | T(P, C, W) = \mathcal{F}_u(M, \Delta_R) \text{ with } \Delta_R \in \mathcal{RH}_\infty, \ \|\Delta_R\|_\infty < 1\} \tag{24}$$

where the entries of M are given by

$$M = \begin{bmatrix} -U_i(\hat{D}_i + C\hat{N}_i)^{-1}(C - C_i)D_{c,i} & U_i(\hat{D}_i + C\hat{N}_i)^{-1}C \\ W(I + \hat{P}_i C)^{-1}(I + \hat{P}_i C_i)D_{c,i} & W(I + \hat{P}_i C)^{-1} \end{bmatrix}. \tag{25}$$

Proof: Create a feedback connection of Q in (16) with a controller C where $u = r_1 + C(r_0 - y)$. Consider the map from $col(d, v)$ onto $col(z, y)$, which can be verified from Figure 2. ■

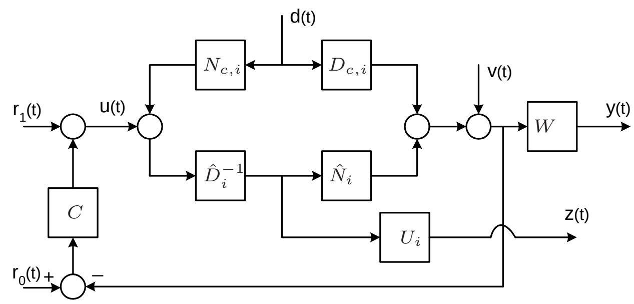

Fig. 2. Diagram for robust analysis/synthesis for dual-Youla uncertainty structure and weighted sensitivity performance.

Provided that M is internally stable, robust performance of a controller C applied to all models $P \in \mathcal{P}_i$ is defined as $\|\mathcal{F}_u(M, \Delta_R)\|_\infty \leq 1$ for all Δ_R such that $\|\Delta_R\|_\infty < 1$ for all Δ_R such that $\|\Delta_R\|_\infty < 1$ and evaluated by computing structured singular value μ with respect to structured uncertainty $\mathbf{\Delta}$.

Definition 2: For $M \in \mathbb{C}^{n \times n}$, $\mu_{\mathbf{\Delta}(M)}$ is defined as

$$\mu_{\mathbf{\Delta}}(M) := \left(\min_{\Delta \in \mathbf{\Delta}} \{\overline{\sigma}(\Delta) : \det(I - M\Delta) = 0\} \right)^{-1} \tag{26}$$

unless no $\Delta \in \mathbf{\Delta}$ makes $(I - M(j\omega)\Delta(j\omega))$ singular, in which case $\mu_{\mathbf{\Delta}} := 0$.

Consider an uncertainty structure $\mathbf{\Delta}$ compatible with the transfer matrix M with a fictitious full block uncertainty Δ_S representing the $\mathcal{H}_\infty$ performance specification at the weighted sensitivity channel,

$$\mathbf{\Delta} = \{\text{diag}(\Delta_S, \Delta_R), \ \Delta_S, \Delta_R \in \mathcal{RH}_\infty \ \|\Delta_S\|_\infty < 1, \ \|\Delta_R\|_\infty < 1\}. \tag{27}$$

The feedback system $\mathcal{F}_u(M, \Delta)$ with $\Delta \in \mathbf{\Delta}$, $\|\Delta\|_\infty < 1$ satisfies performance robustness if and only if [15]

$$\sup_{\omega \in \mathbb{R}} \mu_{\mathbf{\Delta}}(M(j\omega)) \leq 1. \tag{28}$$

Generally μ_Δ is approximated by computing an upper bound over finite frequency grid which results in convex optimization over a set of positive scaling matrices $\mathcal{D}$,

$$\mu_{\mathbf{\Delta}}(M) \leq \inf_{D \in \mathcal{D}} \overline{\sigma}(DMD^{-1}), \tag{29}$$

where $\mathcal{D} = \{D \mid D\mathbf{\Delta} = \mathbf{\Delta}D\}$. For the uncertainty structure (27) containing two full blocks (Δ_S, Δ_R) the computation of $\mu_{\mathbf{\Delta}}$ in (29) is exact. This formulation allows the possibility to evaluate the (worst-case) performance of a controller C applied to a set of models $\mathcal{P}$ in a non-conservative way. Note that in case $C = C_i$, robust stability with respect to the dual-Youla uncertainty structure (14) is trivially satisfied since

B. μ-synthesis

The robust performance control design of step (a) in Problem 1 can be formulated in the μ-synthesis framework as

$$C_{i+1} = \left\{ C \mid \max_{(W)} J_{pw}(W) \text{ s.t. } \arg\min_{C} \mu_{\mathbf{\Delta}}(M) \leq 1 \right\} \tag{30}$$

such that the worst-case performance of (23) for all $P \in \mathcal{P}_i$ characterized by (14) is optimized. For application of standard μ-synthesis results the transfer matrix M is represented as a lower LFT feedback connection with the controller $M = \mathcal{F}_u\left(\mathcal{F}_l(G, C), \text{diag}(\Delta_S, \Delta_R)\right)$ where

$$G = \begin{bmatrix} W_i & 0 & 0 \\ 0 & U_i & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & (D_{c,i} + \hat{P}_i N_{c,i}) & \hat{P}_i \\ 0 & \hat{D}_i^{-1} N_{c,i} & \hat{D}_i^{-1} \\ 1 & (D_{c,i} + \hat{P}_i N_{c,i}) & \hat{P}_i \end{bmatrix}. \tag{31}$$

The μ-synthesis problem is typically evaluated using the upper bound (29) and solving the $\mathcal{H}_\infty$ problem

$$\min_{C} \inf_{D \in \mathcal{D}} \left\| D G D^{-1} \right\|_\infty \tag{32}$$

iteratively for the scaling matrix D and controller C, a process known as the D-K iteration for which standard solutions exist [15]. Performance weight adjustment and robust control design are implemented in the following problem to obtain the pair (C_{i+1}, W_{i+1}).

Problem 2: Robust performance control design (11) is achieved for $\mathcal{P}_i$ characterized by (14) and objective function (23) via the following optimization.

$$(C_{i+1}, W_{i+1}) = \max_{W} J_{pw}(W) \quad \text{s.t.} \quad \min_{C} \inf_{D \in \mathcal{D}} \left\| D G D^{-1} \right\|_\infty \leq 1. \tag{33}$$

The above problem was explored in [12], [13] using frequency domain and state-space oprtimization methods to iteratively find performance weights and controllers. One can also use parametrized functions J_{pw} in order to maximize performance with respect to variables that characterize system level performance.

C. Performance weight selection

Generally intuition and experience connect properties of control objective weights with desired performance of the closed-loop system. For a class of control design problems in which low frequency disturbance rejection is desired, bandwidth and disturbance amplification around the bandwidth are important considerations. For the weighted sensitivity objective function (23), properties such as bandwidth and maximum disturbance amplification are available through a standard performance weight of the form

$$W = \left(\frac{s/\sqrt[k]{M_s} + \omega_b}{s + \omega_b \sqrt[k]{\epsilon}} \right)^k, \quad \text{for} \quad k \geq 1 \tag{34}$$

where k indicates the desired slope of the transition between low-frequency and high-frequency performance and $\epsilon > 0$ determines the level of steady-state error rejection such that W has no imaginary axis poles for straight forward incorporation in the robust control design framework [15]. In this standard form there exist methods for adjusting the weighting function according to increased bandwidth ω_b and reduced maximum peak sensitivity M_s parameters [5]. An intuitive indication of performance improvement through iterative identification and control designs is given in terms of these parameters.

Proposition 1: For evaluating the performance weight improvement of (34) in terms of bandwidth ω_b and maximum peak sensitivity M_s consider the following function.

$$J_{pw}(W) = |\xi \omega_b + (1 - \xi) M_s^{-1}| \quad \text{for } \xi \in [0, 1] \tag{35}$$

where ξ remains fixed for all iterations and indicates the relative emphasis between bandwidth requirements ω_b and maximum absolute value M_s.

Proof: The evaluation criteria (35) satisfies conditions for pseudometric functions (6) with $J_{pw}(W) = \Gamma(W, 0)$. ■

Improved performance under the criteria (23), corresponding to closed-loop low-frequent disturbance rejection, typically results from high bandwidth ω_b and low peak sensitivity M_s. Iteratively increasing ω_b and decreasing M_s provides an indication that performance has improved if larger values of the evaluation criteria (35) are achieved for each iteration. The optimization problem becomes [10], [11]

$$\max_{\omega_b, M_s} J_{pw}(W) \quad \text{s.t.} \quad \mu_{\mathbf{\Delta}}(M) \leq 1, \tag{36}$$

where W is given by (34). If in (35) the parameter $\xi = 1$, performance of the iterative identification and control designs are compared based on the maximum achievable closed-loop bandwidth implemented as an outer-loop around μ-synthesis such that $\mu_{\mathbf{\Delta}} < 1$ [5].

VI. EXAMPLE

To illustrate the benefits for adjusting the performance weight through iterative identification and control design, the proposed algorithm of Problem 1 is initialized through the first iteration for a plant with double integrator, small resonance modes around the cross-over frequency and two large resonance modes at higher frequencies. The model set $\mathcal{P}$ is composed of a 6^{th} order nominal model constructed from estimated normalized coprime factors as well as the associated modeling error characterized by the dual-Youla uncertainty with 3^{rd} order uncertainty over bound filter U. The results are shown in Figure 3.

Given nominal model and uncertainty over bound, the optimal performance weight [13] is computed, shown in Figure 4. Of course to use the optimal weight for control synthesis requires a high-order fit resulting in a highly complex, possibly unstable controller which stabilizes the closed-loop with $\mu_{\mathbf{\Delta}} = 1$. However, a 3^{rd} order over bound of the form (34) is designed such that the resulting controller is stable and guarantees robust performance.

Adjusting the weighting function around a μ-synthesis control design allows for increased bandwidth subject to robust performance constraints, demonstrated in Figure 5. The performance evaluation criteria J_{pw} in (35) with $\xi = 0.9$ increased over 30% from the beginning performance weight to the final.

VII. CONCLUSIONS

The framework presented in this paper provides performance robustness improvement for cautious control designs based on iterative identification and control procedures. A

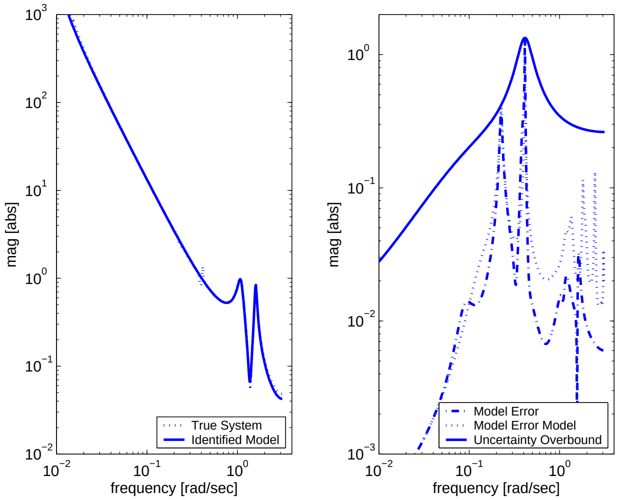

Fig. 3. Left: Nominal model constructed from 6^{th} order normalized coprime factors. Right: Dual-Youla characterization of modeling error (dash-dot-line), 20^{th} order ARMAX model error model (dotted-line) with 3^{rd} order uncertainty weight over bound (solid-line).

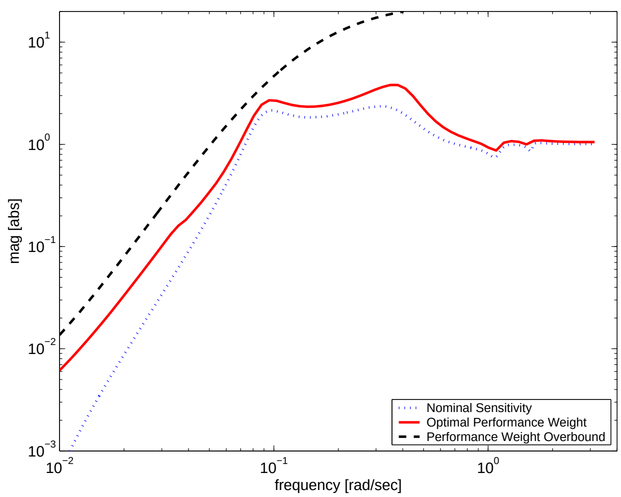

Fig. 4. Nominal Sensitivity (dotted-line) and performance weights: optimal (solid-line) and over bound (34) (dash-line).

performance weighting function is adjusted in conjunction with control synthesis to maximize performance robustness. Since both controller and weighting function change during the iterative procedure, the achieved performance is compared between iterations on the basis of the performance weight while the performance robustness is monitored.

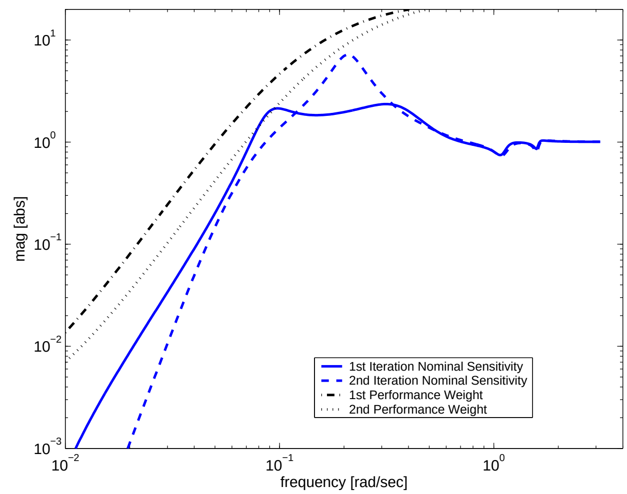

Fig. 5. Weight adjustment during first control design. Beginning nominal sensitivity (solid-line) and robust performance weight (dash-dot-line). Final nominal sensitivity (dashed-line) and robust performance weight (dotted-line).

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