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Scheduling Control for Periodic Disturbance Attenuation

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***Abstract*— In this paper, a scheduled controller is presented by the merging together of internal model-based control and linear parameter-varying control theory. The internal model is used to attenuate periodic disturbances and concepts from linear parameter-varying control theory are used to design the feedback gain that quadratically stabilizes the closed loop system during scheduling. An estimate of the frequency of the disturbance is used as the scheduling variable that updates the internal model. The order of the controller is kept low by making use of the separation principle and designing a reduced order observer for state feedback. A design example of a simple mass-spring-damper system demonstrates the effectiveness of the controller to cancel periodic disturbances with a time-varying frequency.**

I. INTRODUCTION

Systems, such as active noise control [1], [2], rotating machinery, and structural systems, are often systems are subjected to periodic disturbances with a time-varying frequency. One method of designing controllers to cancel periodic disturbances is by means of the internal model principle [3]. Francis and Wonham [3] showed that the purpose of the internal model principle is to place closed loop transmission zeros were the unstable poles of the disturbance is located. This placement of the closed loop transmission zeros gives a robust controller that asymptotically rejects periodic disturbances. Controllers designed upon this principle are called internal model-based (IMB).

The drawback of IMB controllers is the assumed exact knowledge of the disturbance frequency. To overcome this, two methods have been developed. One method is to design robust repetitive controllers [4], or robust IMB controllers, when the frequency of the disturbance varies only slightly from the nominal value. This method results in a controller that is stable and performs well with respect to small perturbations in the disturbance frequency. Steinbuch [4] showed that a robust repetitive control applied to a Compact Disk Drive was able to reject the periodic disturbance when the frequency of the disturbance is perturbed by 0.5%. Another method developed to accommodate larger fluctuations in the disturbance is called adaptive IMB control or adaptive repetitive control [5], [6]. In this scheme, an estimate of the disturbance frequency is used to adapt the controller. A variety of identification and adaptive control methods can be used to this end.

One option to deal with changes in the disturbance frequency is to design a scheduled controller. This can be accomplished with an H_∞ linear parameter-varying (LPV) controller [7]. The drawback, however, is the increase in complexity (order) of the controller. The resulting controller will be the order of the plant plus two times the order of the internal model. For repetitive control system, a specific class of internal model control systems [8], the order of the internal model can be very large. To overcome these difficulties, LQG (or H_2) control can be used to design a controller in which the complexity is reduced to the order of the plant plus the order of the internal model.

In this paper, a method for designing a low order controller is presented by using LPV theory [9], [10] and IMB control. The IMB controller presented in [11] is extended to accommodate time-varying frequencies. The internal model and feedback gain are scheduled by monitoring the frequency of the disturbance. The feedback gain is designed to quadratically stabilize the closed loop system. A design example is included to demonstrate the effectiveness of the proposed control strategy.

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II. DESIGN OF NON-SCHEDULED IMB CONTROLLER

A. Problem Description

In this section, we consider the design of an LQG (linear, quadratic and gaussian) controller in the IMB framework. Let the state space model of the plant $P(s)$ be given by

$$\begin{aligned} \dot{x}_p(t) &= A_p x_p(t) + B_u u(t) + B_w w(t) \\ y(t) &= C_p x_p(t) + D_{yw} w(t) \end{aligned}, \qquad (1)$$

where $u(t)$ is the controlled input, $w(t)$ is an iid random variable with zero mean and unity covariance, and $y(t)$ is the measurable output of the plant. The internal model $M(s)$, a model of the disturbance used in the control design, is given by

$$\begin{aligned} \dot{x}_m(t) &= A_m x_m(t) + B_m u_m(t) \\ y_m(t) &= I_m x_m(t) \end{aligned}.$$

Since the seperation principle [12] applies to LQG controllers, the control problem can be divided into two distinct problems. First, a state feedback controller is designed to minimize the L_2 norm of the optimization vector $z(t)$. Secondly, an observer is designed for $P(s)$ that minimizes the variance of the estimation error. Estimation of the internal model states is not needed since the internal model is used in the control design and the states $x_m(t)$ are used directly for feedback purposes. Finally, the unique connection of these two problems yields an IMB controller.

B. Non-Scheduled Internal Model Structure

When the disturbance has a periodic nature, an oscillator (with no dampening) can be used as an internal model. One state space description of an oscillator is given by

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ y_m(t) \end{pmatrix} = \left[\begin{array}{cc|c} 0 & 1 & 0 \\ -\omega^2 & 0 & 1 \\ \hline 0 & 1 & 0 \end{array}\right] \begin{pmatrix} x_1(t) \\ x_2(t) \\ u_m(t) \end{pmatrix}, \qquad (2)$$

where ω is the frequency of the periodic disturbance. If the disturbance is comprised of several periodic signals then the internal model can be obtained with the series connection of several oscillators. Another valid state space representation of (2) is given by

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ y_m(t) \end{pmatrix} = \left[\begin{array}{cc|c} 0 & \omega & 0 \\ -\omega & 0 & 1 \\ \hline 0 & 1 & 0 \end{array}\right] \begin{pmatrix} x_1(t) \\ x_2(t) \\ u_m(t) \end{pmatrix}. \tag{3}$$

It is easily verified that these two systems, (2) and (3), are equivalent for the time-invariant case but are not equivalent for the time-varying situation, as will be shown in §III-C.

C. State Feedback Gain

Let the optimization vector $z(t)$ be defined as

$$z(t) = \begin{bmatrix} 0 & I_m \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_p(t) \\ x_m(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \alpha \end{bmatrix} u(t),$$

then series connection of the plant and internal model is given by

$$\begin{bmatrix} \dot{x}_p(t) \\ \dot{x}_m(t) \\ z(t) \end{bmatrix} = \left[\begin{array}{cc|c} A_p & 0 & B_u \\ B_m C_p & A_m & 0 \\ \hline 0 & I_m & 0 \\ 0 & 0 & \alpha \end{array}\right] \begin{bmatrix} x_p(t) \\ x_m(t) \\ u(t) \end{bmatrix}.$$

Define

$$\left[\begin{array}{c|c} A_s & B_s \\ \hline C_s & D_s \end{array}\right] = \left[\begin{array}{cc|c} A_p & 0 & B_u \\ B_m C_p & A_m & 0 \\ \hline 0 & I_m & 0 \\ 0 & 0 & \alpha \end{array}\right],$$

then the optimal state feedback control problem consists of finding the control sequence $\{u(0), u(1), ...\}$ such that the quadratic objective functional J_c

$$J_c = \int_0^\infty x(t)^T C_s^T C_s x(t) + u(t)^T D_s^T D_s u(t) dt$$

is minimized.

The optimal control sequence is given by the state feedback law

$$u(t) = -Kx(t),$$

where the optimal state feedback gain K is given by

$$K = (D_s^T D_s)^{-1} B_s^T P_c \tag{4}$$

and P_c is a solution to the following Riccati equation

$$A_s^T P + P A_s - PB(D_s^T D_s)^{-1} B^T P + C_s^T C_s = 0 .$$

D. Observer

The second step in the IMB control process is to design an observer. The observer is used to estimate the states $x_p(t)$ and does not estimate the states of the internal model $x_m(t)$. The idea behind this is the fact that the internal model states are available for feedback and therefore it is unnecessary to estimate them. Additionally, this precludes any possible interference the estimator would have with the internal model states, resulting in undesirable properties.

The observer problem for the plant $P(s)$ described in (1) is to find the gain L_p such that the cost function J_o is minimized.

$$J_o = \lim_{t\to\infty} E\{[x - \hat{x}]^T [x - \hat{x}]\}.$$

$\hat{x}$ is the estimated states of $P(q)$, x is the true states of $P(s)$, and E is the mathematical expectation. The estimator dynamics is given by

$$\dot{\hat{x}}(t) = (A_p - L_p C_p)\hat{x}(t) + L_p y(t). \tag{5}$$

where L_p is the steady state Kalman gain given by

$$L_p = (P_o C_p^T + B_w D_{yw}^T)(D_{yw} D_{yw}^T)^{-1} \tag{6}$$

and P_o is the solution to the following Riccati equation:

$$\begin{aligned} &A_p P_o + P_o A_p^T \\ &-(P_o C_p^T + B_w D_{yw}^T)(D_{yw} D_{yw}^T)^{-1}(C_p P_o + D_{yw} B_w^T) \\ &+B_w B_w^T = 0. \end{aligned}$$

E. IMB Controller

The Internal Model-Based (IMB) controller is defined as

$$C(s) = \left[\begin{array}{cc|c} (A_p - L_p C_p - B_u K_1) & -B_u K_2 & L_p \\ 0 & A_m & B_m \\ \hline -K_1 & -K_2 & 0 \end{array}\right], \tag{7}$$

where $[K_1 \quad K_2]$ is the state feedback gain from (4) and L_p is the Kalman gain from (6).

It can be observed from (7) that the eigenvalues of the controller contain the eigenvalues of the internal model. Therefore, it is internal model-based. Additionally, the order of the controller is the order of the internal model plus the order of the plant. A full order design method, based upon loop shaping [13], would result in a controller that is the order of the plant plus twice the order of the internal model.

This kind of IMB controller was suggested in [8], although it was not used adaptively. In [14] and [15] a repetitive control algorithm was developed and experimentally tested. The repetitive control algorithm is used with an observer that uses an error signal and a filtered input signal to estimate the states of the system. More specifically, the periodic part of the input signal is filtered out before the observer. Here, the internal model is completely taken out of the observer problem reducing the order of the resulting observer.

In [8] it was shown that the design of repetitive and learning controllers is an application of IMB control and can be realized through loop-shaping. In [16], it was shown that the design of learning and repetitive controllers are dual problems. Thus, the internal model-based control design methodology can be applied to many types of control problems.

III. Scheduling Control

A. Problem Description

In this section, we consider the design of an LPV controller in the IMB framework. Let the state space model of the plant $P(s)$ be given by

$$\begin{aligned} \dot{x}_p(t) &= A_p x_p(t) + B_u u(t) + B_w w(t) \\ y(t) &= C_p x_p(t) + D_{yw} w(t) \end{aligned},$$

where $u(t)$ is the controlled input, $w(t)$ is an iid random variable with zero mean and unity covariance, and $y(t)$ is the measurable output of the plant. The internal model $M(s, \theta)$, a model of the undesirable disturbance used in the control design, is given by

$$\begin{aligned} \dot{x}_m(t) &= A_m(\theta)x_m(t) + B_m y(t) \\ y_m(t) &= I_m x_m(t) \end{aligned}.$$

Since the states of the plant are not typically available, an observer can be used to estimate them. The novelty of this type of control formulation presented here, is that the time-varying part of the system lies in the internal model and therefore a non-scheduled observer can be used. A full order control design for the plant in series with the internal model would result in an observer that varies in θ. The observer that will be used for the scheduled controller is given by (5).

Similar to the non-adaptive controller the adaptive control design can be divided into three pieces. First, a quadratically stabilizing state feedback gain is found. Second, a fixed observer is designed. Finally, the combination of these two problems yields a scheduled IMB controller that is quadratically stable. Quadratic stability is defined in the following section.

B. LPV Systems and Quadratic Stability

Consider the following LPV system:

$$\begin{aligned} G: \ \dot{x}(t) &= A(\theta)x(t) + Bw(t) \\ y(t) &= Cx(t), \end{aligned} \qquad (8)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $A(\cdot)$ is an affine function of θ, and $\theta \in \Theta$ is a bounded and continuously varying parameter in time, $B \in \mathbb{R}^{n \times n_w}$ and $w(t) \in \mathbb{R}^n_w$ is a disturbance. Let the parameter set Θ be defined by

$$\begin{aligned} \Theta &:= Co\{\xi_1, \xi_2, ..., \xi_N\} \\ &= \{\textstyle\sum_{i=1}^N \alpha_i(t)\xi_i : \alpha_i(t) \geq 0, \sum_{i=1}^N \alpha_i(t) = 1\}, \end{aligned} \qquad (9)$$

where $Co\{\cdot\}$ denotes the convex hull, then the LPV system is denoted a polytopic LPV system [7]. For the polytopic LPV system (PLPV) described in (8) and (9) quadratic stability is defined as follows.

Definition 1 (Quadratic Stability): The PLPV system $\dot{x}(t) = A(\theta)x(t)$, where $\theta \in \Theta$, is Quadratically Stable if $\exists$ a single positive definite $P \in \mathbb{S}^n$ such that

$$A(\xi_i)^T P + PA(\xi_i) < 0, \quad \forall i. \qquad (10)$$

Definition 2: The H_2 norm for an exponentially stable LPV system G is defined as

$$\|G\|_2^2 := \lim_{t \to \infty} \mathbb{E}\frac{1}{t}\int_0^t y(\tau)^T y(\tau) d\tau,$$

when $x(0) = 0$ and $w(t)$ is a zero-mean white noise process with an identity power spectrum density matrix [17].

The following lemma is useful for proving quadratic stability of systems.

Lemma 1: Consider the block matrix $Q(\theta)$, where

$$Q(\theta) = \begin{bmatrix} Q_{11}(\theta) & 0 \\ Q_{21}(\theta) & Q_{22}(\theta) \end{bmatrix}.$$

Suppose the matrices $Q_{11}(\theta)$ and $Q_{22}(\theta)$ are quadratically stable, as defined in Definition 1, and $Q(\theta)$ is continuous and bounded, then $Q(\theta)$ is quadratically stable.

Proof: The proof of this lemma can be found in [7]. ∎

C. Time-Varying Internal Model Structure

As indicated in §II-B, for the time-varying case the two time-invariant representations, (2) and (3), are not the same [18]. To see this fact, consider the homogeneous solution to (2), given by

$$x_1(t) = A\cos(\omega t + \phi)$$

where the magnitude A and the phase ϕ are determined by the initial conditions. Define

$$\alpha(t) := (\omega t + \phi),$$

and

$$\frac{d}{dt}\alpha(t) := \omega_d(t),$$

then taking derivatives gives

$$\begin{aligned} \dot{x}_1(t) &= -A\sin(\alpha(t))(\omega_d(t)) \\ \ddot{x}_1(t) &= -A\cos(\alpha(t)))(\omega_d(t))^2 - A\sin(\alpha(t))(\dot{\omega}_d(t)) \\ &= -x_1(t)(\omega_d(t))^2 + \dot{x}_1\dot{\omega}_d(t)\tfrac{1}{\omega_d(t)} \end{aligned}.$$

The second term in the last equality is zero for the time-invariant case. This is were the difference between the two case lies; the rate of change of ω_d affects the time-varying representation. A time varying realization of the above is given by

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ y(t) \end{pmatrix} = \left[\begin{array}{cc|c} 0 & 1 & 0 \\ -\omega_d(t)^2 & \dot{\omega}_d(t)\frac{1}{\omega_d(t)} & 1 \\ \hline 0 & 1 & 0 \end{array}\right] \begin{pmatrix} x_1(t) \\ x_2(t) \\ u(t) \end{pmatrix}, \qquad (11)$$

and another is given by

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ y(t) \end{pmatrix} = \left[\begin{array}{cc|c} 0 & \omega_d(t) & 0 \\ -\omega_d(t) & 0 & 1 \\ \hline 0 & 1 & 0 \end{array}\right] \begin{pmatrix} x_1(t) \\ x_2(t) \\ u(t) \end{pmatrix}. \qquad (12)$$

It can be seen that the two representations given in (2) and (3) are the time invariant versions of (11) and (12). Since $\dot{\omega}_d$ might not be available, the preferred time varying representation is (12).

A periodic disturbance with a frequency that lies in a closed interval can be formulated as a PLPV system. In fact, this type of system will only have two vertices. This observation will be used in the following sections to create an PLPV system.

IV. Design of Scheduled IMB Controller

A. Quadratically Stable Feedback Gain

Let the optimization vector $z(t)$ be defined as

$$z(t) = \begin{bmatrix} 0 & I_m \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_p(t) \\ x_m(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \alpha \end{bmatrix} u(t),$$

assuming the states are available, then series connection of the plant and internal model is given by

$$\begin{bmatrix} x_p(k+1) \\ x_m(k+1) \\ z(t) \end{bmatrix} = \left[\begin{array}{cc|cc} A_p & 0 & B_u & B_w \\ B_m C_p & A_m(\theta) & 0 & 0 \\ \hline 0 & I_m & 0 & 0 \\ 0 & 0 & \alpha & 0 \end{array}\right] \begin{bmatrix} x_p(t) \\ x_m(t) \\ u(t) \\ w(t) \end{bmatrix}.$$

Define

$$\left[\begin{array}{c|cc} A_s(\theta) & B_{su} & B_{sw} \\ \hline C_s & D_{su} & D_{sw} \end{array}\right] = \left[\begin{array}{cc|cc} A_p & 0 & B_u & B_w \\ B_m C_p & A_m(\theta) & 0 & 0 \\ \hline 0 & I_m & 0 & 0 \\ 0 & 0 & \alpha & 0 \end{array}\right], \quad (13)$$

and

$$H_{wz}(s,\theta) = C_s(sI - A_s(\theta))^{-1} B_{sw} + D_{sw},$$

then the control problem is to find a state feedback controller such that the feedback connection is quadratically stable and $\|H_{wz}(s,\theta)\|_2$ is minimized.

Proposition 1: Consider the continuous time polytopic LPV plant $P_s(s,\theta)$ with a state space realization given by (13) and a state feedback control given by

$$u(t) = -(\sum_{i=1}^{N} \alpha_i(t) K_i) x(t). \quad (14)$$

$P_s(s,\theta)$ is quadratically stabilized by the state feedback control law (14) iff K_i can be written as

$$K_i = L_i P^{-1},$$

where $P \in \mathbb{S}^n$ and $L_i \in \mathbb{R}^{p\times n}$ are matrices that satisfy

$$PA_s^T + A_s P - B_{su} L_i - L_i^T B_{su}^T < 0 \;\; \forall i, \quad P > 0.$$

Proof: The proof follows directly from the definition of quadratic stability applied to the closed loop system and defining

$$L_i := K_i P.$$

■

For the control design, it is very useful to be able to evaluate a norm of the system so that a controller out of the set of quadratically stabilizing controllers can be chosen. In this paper, H_2 norm was chosen for design purposes since the separation principle applies.

Lemma 2: Consider a positive definite matrix P s.t.

$$A_s(\theta)P + PA_s(\theta)^T + B_{sw}B_{sw}^T < 0,$$

then $H_{wz}(s,\theta)$ is quadratically stable and

$$\|H_{wz}(s,\theta)\|_2^2 < tr(C_s P C_s^T).$$

Proof: Follows directly from [17, lemma 1] and definition 1. ■

Proposition 2: Consider the continuous time polytopic LPV plant $P_s(s,\theta)$ with a state space realization given by (13). If there exists a positive definite $P \in \mathbb{S}^n$, $L_i \in \mathbb{R}^{p\times n}$ $\forall i$, and $W \in \mathbb{S}^r$ such that

$$\begin{gathered} \left[\, PA_s(\xi_i)^T + A_s(\xi_i)P - L_i^T B_{su}^T - B_{su}L_i + B_{sw}B_{sw}^T \,\right] < 0 \\ \begin{bmatrix} W & CP - D_{su}L_i \\ (CP - D_{su}L_i)^T & P \end{bmatrix} > 0 \\ tr[W] < \gamma, \quad \forall i \end{gathered} \quad (15)$$

then $P_s(s,\theta)$ is quadratically stabilized by the state feedback controller

$$u(t) = -(\sum_{i=1}^{N} \alpha_i(t) K_i) x(t), \quad K_i = L_i P,$$

such that $\|H_{wz}(s,\theta)\|_2^2 < \gamma$, $\forall\, \theta \in \Theta$.

Proof: Quadtratic stability of the closed loop system is implied via the first LMI in (15) and the implied positive definiteness of P from the second LMI. $\|H_{wz}(s,\theta)\|_2^2 < \gamma$ follows from Lemma 2 and application of the Schur complement technique on the 2^{nd} LMI. ■

It should be observer from the above Proposition that quadratic stability not only guarantees robustness properties, it also simplifies the synthesis of the controller by requiring that $\dot{P} = 0$ as compared to the purely H_2 optimal case [17], but as a consequence the upper bound of $\|H_{wz}(s,\theta)\|_2$ given in Proposition 2 may be conservative.

B. Scheduled IMB Controller

The scheduled controller is composed of the observer and scheduled feedback gain from the previous sections. The resulting controller quadratically stabilizes the closed loop system, since the separation principle applies. The scheduled controller is given by

$$C(s,\theta) = \left[\begin{array}{cc|c} (A_p - L_pC_p - B_uK_1(\theta)) & -B_uK_2(\theta) & L_p \\ 0 & A_m(\theta) & B_m \\ \hline -K_1 & -K_2 & 0 \end{array}\right]. \quad (16)$$

Notice that the only part of the controller that is scheduled is the *internal model and feedback gain*. Figure 1 shows the scheduled controller connected to the plant in feedback. Notice that the observer dynamics are not scheduled, this gives the controller a unique structure that simplifies the design and analysis greatly.

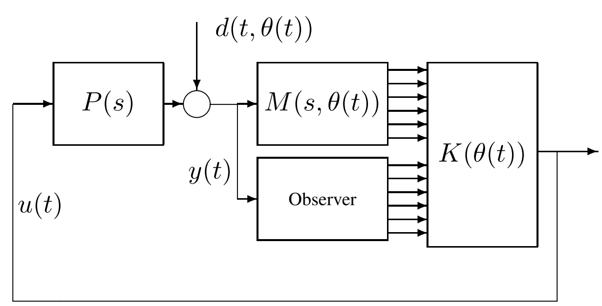

Fig. 1. Scheduled internal model-based controller connected in feedback to the plant $P(s)$.

Proposition 3: Consider the feedback connection of (1) and (16). Assume that (1) is quadratically stabilized by the state feedback control law

$$u(t) = -K(\theta)x(t),$$

and L_p is a stabilizing observer gain, then the closed loop system is quadratically stable.

Proof: Rearranging the closed loop $T_{cl}(s,(\theta))$ in terms of the error dynamic of the plant estimator $\tilde{x}_p(k)$ gives

$$T_{cl}(s,\theta) = \left[\begin{array}{llll|l} A_p - B_uK_1(\theta) & -B_uK_2(\theta) & B_uK_1(\theta) & & B_w \\ B_mC_p & A_m(\theta) & 0 & & B_mD_{yw} \\ 0 & 0 & A_p - L_pC_p & & L_pD_{yw} \\ \hline 0 & I_m & 0 & & 0 \\ -\alpha K_1(\theta) & -\alpha K_2(\theta) & \alpha K_1(\theta) & & 0 \end{array}\right],$$

and by lemma 1 the closed loop is quadratically stable. ∎

V. Design Example

A. System Description

To demonstrate the effectiveness of the proposed controller a simple mass-spring-damper system from will be used. An illustration of the system is shown in Figure 2. The

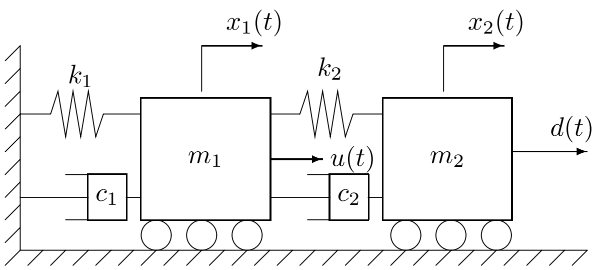

Fig. 2. Simple mass-spring-damper system.

control signal $u(t)$ is the force applied to the first mass, the disturbance $d(t)$ is the force applied to the second mass, and the measured output $y(t)$ is the position of the second mass. For the simulation the following parameters were used: m_1=m_2=1, k_1=k_2=100, c_1=c_2=1. In addition, it will also be assumed that there is measurement error $v(t)$, modeling errors $w(t)$, and disturbance is composed of a single sinusoidal disturbance with a varying frequency. A state space realization of the system to be used in the control design is given by

$$\begin{array}{l} \dot{x}(t) = Ax(t) + B_uu(t) + B_ww(t) \\ y(t) = Cx(t) + v(t) \end{array}, \tag{17}$$

where

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -200 & 100 & -2 & 1 \\ 100 & -100 & 1 & -1 \end{bmatrix},$$

$$B_u = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad B_w = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 80 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$$

Notice that (17) does not include the periodic disturbances. The periodic disturbances will be rejected by designing a controller to stabilize the series connection of the plant with the appropriate internal model.

The internal model used to design the controller is given by

$$\begin{pmatrix} \dot{x}_{d1}(t) \\ \dot{x}_{d2}(t) \\ y_m(t) \end{pmatrix} = \left[\begin{array}{cc|c} 0 & \omega_d(\theta) & 0 \\ -\omega_d(\theta) & 0 & 1 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right] \begin{pmatrix} x_{d1}(t) \\ x_{d2}(t) \\ e(t) \end{pmatrix},$$

where $e(t)$ is an external input.

B. Non-Scheduled IMB Controller

A non-scheduled internal model-based controller was designed based upon the theory presented in this paper. The IMB controller was designed to eliminate periodic disturbances with the same frequency as the dominant resonance mode in the transfer function from the disturbance to the output $T_{yd}(s)$. From the singular value plot shown in Figure 4, it can be seen that the dominant resonance mode is near 6 rad/sec, and therefore the system will be sensitive the periodic disturbances near this frequency (in open loop).

C. Scheduled IMB Controller

Using the design methodology from the previous sections a scheduled IMB controller was designed to reduce the affect of the periodic disturbance on the output of the plant. The controller (composed of the observer, internal model, and feedback gain) is changed on-line to cancel the periodic disturbance $d(t)$ with a time-varying frequency. The scheduled controller is given by (16), with

$$L_p = \begin{bmatrix} 238.43 \\ 162.58 \\ 70.304 \\ 8215.4 \end{bmatrix},$$

and

$$\begin{array}{l} K_1 = [\, 4635.5 \;\; 28729 \;\; 116.11 \;\; 1704.7 \;\; -15972 \;\; 15547 \,] \\ K_2 = [\, 4628 \;\; 28696 \;\; 115.97 \;\; 1701.8 \;\; -13962 \;\; 15250 \,]. \end{array} \tag{18}$$

It can be seen from (18), that the controller gains are very similar. For this problem, it might be feasible to design a robust LPV controller that performs well.

D. Simulation

For the simulation, the force disturbance on the second mass, $d(t)$, was set to

$$\begin{array}{l} d(t) = \sin(\omega(t)t) \\ \omega(t) = 3 + 3t \end{array}.$$

$\theta(t)$ was set to

$$\theta(t) = -3t\xi_1 + (1-3t)\xi_2,$$

where $\xi_1 = 3\frac{rad}{s}$ and $\xi_2 = 33\frac{rad}{s}$. Figure 3 shows the disturbance $d(t)$, closed loop system with a non-scheduled IMB controller, and the closed loop system with a scheduled IMB controller when $d(t)$ is applied. It can be seen from this figure, that the scheduled controller is able to attenuate the periodic disturbance while the frequency is varying. From Figure 4, it can be seen that non-scheduled IMB controller is able to completely reject disturbances only at 6 rad/sec and therefore performs poorly at neighboring frequencies.

Additionally, the non-scheduled controller asymptotically rejects the disturbance at 6 rad/sec, and since the frequency changes quickly the non-scheduled controller is not able to reject the disturbance completely. It is also noticeable that both controllers perform very well at higher frequencies. This is due to small gain of $T_{yd}(s)$ at high frequencies, as shown in Figure 4.

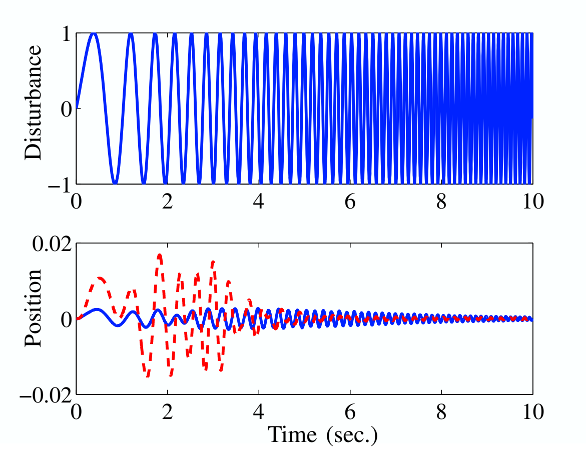

Fig. 3. Top: Disturbance signal $d(t)$. Bottom: Closed loop system with scheduled IMB control (Solid) and closed loop system with non-scheduled IMB control (Dashed).

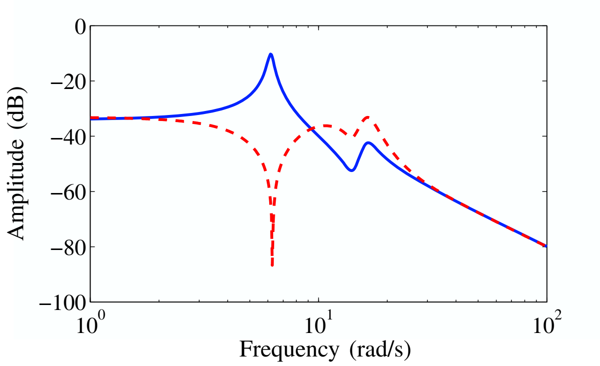

Fig. 4. Singular value plot of $T_{yd}(s)$ (Solid) and the closed loop system with the non-scheduled controller (Dashed).

VI. Conclusions

The control design presented in this paper merges together internal model-based theory and LPV theory. A scheduled controller is found by varying the internal model, and LPV theory is used to find the feedback gain that stabilizes the system while the scheduling takes place. More specifically, a quadratically stabilizing feedback law was found for the series connection of the plant with the internal model. An observer was designed to estimate the states of the plant. Finally, the scheduled controller was constructed by connecting the observer and internal model together with a stable feedback gain.

An example problem was included to demonstrate the effectiveness of the scheduled controller as compared to a non-scheduled controller designed upon the internal model principle. The example consisted of a simple mass-spring-damper system that was subjected to time varying periodic disturbances that range 83.3% from its nominal value. The scheduled controller used the frequency of the disturbance as the scheduled variable and changed the internal model and feedback gain to reject the disturbance and is shown to be superior to the non-scheduled controller. The control design presented in this paper can be applied to various systems that experience periodic disturbances such as active noise control, structural systems, and rotating machinery.

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