AN ADAPTIVE INTERNAL MODEL-BASED CONTROLLER FOR PERIODIC DISTURBANCE REJECTION

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Abstract: This paper details the design of an adaptive internal model-based controller to attenuate periodic disturbances in the presence of random noise. The internal model-based controller is designed in two steps, making use of the separation principle to design a controller with the desired properties that stabilizes the closed-loop system. Periodic disturbances appear as harmonics of a fundamental frequency. The fundamental frequency of the disturbance is estimated with a magnitude/phase-locked loop and is used to adjust the parameters of the controller, specifically the natural frequency of the internal model and the feedback gain. An active noise control study shows the ability of the proposed method to cancel time varying disturbances in an acoustic system.

Keywords: Feedback Control, Adaptive Control, Disturbance Rejection, Active Noise Control, Frequency Estimation

1. INTRODUCTION

Repetitive control has been a popular area of research for the last few years and can be viewed as an extension of the internal model principle. (de Roover *et al.*, 2000) discussed the connection between the internal model principle and repetitive controllers. Repetitive control (Hara *et al.*, 1988) is used to attenuate a specific class of periodic disturbances. Thus, it is seen that internal model-based (IMB) controllers are well suited to rejecting periodic disturbances (Hu, 1996).

IMB controllers are designed on the internal model principle (Francis and Wonham, 1976). The principle loosely states that if the controller contains the poles of the disturbance then the closed loop with have zeros at the location of the poles of the disturbance. This causes the complete asymptotic rejection of the disturbance.

The drawback of IMB controllers is the assumed exact knowledge of the disturbance frequency. To overcome this, two methods have been developed. One method is to design robust repetitive controllers (Steinbuch, 2002), or robust IMB controllers, when the frequency of the disturbance varies only slightly from the nominal value. This method results in a controller that is stable and performs well with respect to small perturbations in the disturbance frequency. (Steinbuch, 2002) showed that a robust repetitive control applied to a Compact Disk Drive was able to reject the periodic disturbance when the frequency of the disturbance is perturbed by 0.5%. Another method developed to accommodate larger fluctuations in the disturbance is called adaptive IMB control or adaptive repetitive control (Dotsch *et al.*, 1995; Yamada *et al.*, 2004). In this scheme, an estimate of the disturbance frequency is used to adapt the controller. A variety of identification and adaptive control methods can be used to this end.

In this paper, an adaptive IMB controller based upon gain scheduling (Åström and Wittenmark, 1995) is presented. The adaptive controller is an extension of the static controller presented in

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(de Roover *et al.*, 2000). The fundamental frequency of the disturbance is estimated with a magnitude/phase locked-loop (MPLL) (Wu and Bodson, 2003) and is used to adjust the internal model and feedback gain to reject the disturbance and maintain stability. This particular design was chosen for its computational speed and simplicity. The controller was developed for systems subjected to periodic disturbances with an unknown frequency or that are slowly time-varying. Such as acoustic, vibrational, and rotational systems. An active noise control (ANC) study demonstrates the effectiveness of this method to reject the undesired disturbance that varies 5.5% from its nominal value.

2. PROBLEM FORMULATION

Consider a linear discrete-time plant $P(q)$ that is subjected to a random disturbance $w(k)$, a periodic disturbance $d(k)$, and a control signal $u(k)$. The control problem is to find the control sequence $\{u(1), u(2), ..., u(k)\}$ that attenuates the periodic disturbance in the presence of the random disturbance $w(k)$ and the unknown or time-varying nature of the periodic disturbance $d(k)$. Additionally, it is desirable that the controller has optimal properties and is suited for adaptation since adaptation will be used to handle the time-varying nature of the periodic disturbance.

One method that is suitable for the given constraints is to design a controller based upon the internal model principle. This method takes the model of the periodic disturbance and appends it onto the plant via the internal model principle and it described in the following section.

3. INTERNAL MODEL-BASED CONTROL DESIGN

3.1 Description

In this section, we consider the design of an LQG (linear, quadratic and gaussian) controller in the IMB framework. Let the state space model of the plant $P(q)$ be given by

$$\begin{aligned} x_p(k+1) &= A_p x_p(k) + B_u u(k) + B_w w(k) \\ y(k) &= C_p x_p(k) + D_{yw} w(k) \end{aligned}, \quad (1)$$

where $u(k)$ is the controlled input, $w(k)$ is an *iid* random variable with zero mean and unity covariance, and $y(k)$ is the measurable output of the plant. The internal model $M(q)$, a model of the undesirable disturbance used in the control design, is given by

$$\begin{aligned} x_m(k+1) &= A_m x_m(k) + B_m y(k) \\ y_m(k) &= I_m x_m(k) \end{aligned}. \quad (2)$$

Since the seperation principle (Chen *et al.*, 1995) applies to LQG controllers, the control problem can be divided into two distinct problems. First, a state feedback controller is designed to minimize the L_2 norm of the optimization vector $z(k)$. Secondly, an observer is designed for $P(q)$ that minimizes the variance of the estimation error. The observation of the internal model states is not needed since the internal model is used in the control design and is not part of the physical system. Finally, the unique connection of these two problems yields an IMB controller.

3.2 Design of the State Feedback Gain

Let the optimization vector $z(k)$ be defined as

$$z(k) = \begin{bmatrix} 0 & I_m \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_p(k) \\ x_m(k) \end{bmatrix} + \begin{bmatrix} 0 \\ \alpha \end{bmatrix} u(k), \quad (3)$$

then series connection of the plant and internal model (without noise) is given by

$$\begin{bmatrix} x_p(k+1) \\ x_m(k+1) \\ z(k) \end{bmatrix} = \left[\begin{array}{cc|c} A_p & 0 & B_u \\ B_m C_p & A_m & 0 \\ \hline 0 & I_m & 0 \\ 0 & 0 & \alpha \end{array}\right] \begin{bmatrix} x_p(k) \\ x_m(k) \\ u(k) \end{bmatrix}. \quad (4)$$

Define

$$\left[\begin{array}{c|c} A_s & B_s \\ \hline C_s & D_s \end{array}\right] = \left[\begin{array}{cc|c} A_p & 0 & B_u \\ B_m C_p & A_m & 0 \\ \hline 0 & I_m & 0 \\ 0 & 0 & \alpha \end{array}\right], \quad (5)$$

the the optimal state feedback control problem consists of finding the control sequence $\{u(0), u(1), ...\}$ such that the quadratic objective functional J_c

$$J_c = \lim_{N\to\infty} \sum_{k=0}^{N} x(k)^T C_s^T C_s x(k) + u(k)^T D_s^T D_s u(k) \quad (6)$$

is minimized.

The optimal control sequence is given by the state feedback law

$$u(k) = Kx(k), \quad (7)$$

where the optimal state feedback gain K is given by

$$K = (B_s^T P_c B_s + \alpha^2)^{-1} B_s^T P_c A_s, \quad (8)$$

and P_c is a unique positive definite solution to the following Riccati equation

$$\begin{aligned} P_c &= A_s^T P_c A_s - A_s^T P_c B_s (B_s^T P_c B_s + \alpha^2)^{-1} B_s^T P_s A_s \\ &+ C_s^T C_s, \end{aligned} \quad (9)$$

3.3 Design of the Observer

The second step in the IMB control process is to design an observer. The observer is designed without the internal model. The idea behind this is the fact that the observer could possible invert some of the dynamics of the internal model. This would result in a controller with undesirable properties. Since the internal model will be placed into the controller, this could be viewed as a suboptimal observer that has poles in common with the internal model.

The observer problem for the plant $P(q)$ described in (1) is to find the gain L_p such that the cost function J_o is minimized.

$$J_o = \lim_{k\to\infty} E\{[x-\hat{x}]^T[x-\hat{x}]\}. \qquad (10)$$

$\hat{x}$ is the estimated states of $P(q)$, x is the true states of $P(q)$, and E is the mathematical expectation. The estimator dynamics is given by

$$\hat{x}(k+1) = (A_p - L_pC_p)\hat{x}(k) + L_py(k). \qquad (11)$$

where L_p is the steady state Kalman gain given by

$$L_p = (D_{yw}^T B_w + A_pP_oC_p^T)(C_pP_oC_p^T + D_{yw}D_{yw}^T)^{-1}, \quad (12)$$

and P_o is the solution to the following Riccati equation:

$$\begin{aligned} P_o &= A_pP_oA_p^T - (D_{yw}^T B_w + A_pP_oC_p^T) \\ &\quad \times (C_pP_oC_p^T + D_{yw}D_{yw}^T)^{-1} \\ &\quad \times (C_pP_oA_p^T + B_w^T D_{yw}) + B_wB_w^T. \end{aligned} \qquad (13)$$

3.4 IMB Controller

The internal model based controller is defined as

$$C(q) = \left[\begin{array}{cc|c} (A_p - L_pC_p - B_uK_1) & -B_uK_2 & L_p \\ 0 & A_m & B_m \\ \hline -K_1 & -K_2 & 0 \end{array}\right], \quad (14)$$

where $[K_1 \quad K_2]$ is the state feedback gain from (8) and L_p is the Kalman gain from (12).

It can be observer from (14) that the eigenvalues of the controller contain the eigenvalues of the internal model. Therefore, it is internal model-based. Additionally, the order of the controller is the order of the internal model plus the order of the plant. A full order design method, based upon loop shaping (Zhou, 1998), would result in a controller that is the order of the plant plus twice the order of the internal model.

This kind of IMB controller was suggested in (de Roover and Bosgra, 1997), although it was not used adaptively. In (Hätönen *et al.*, 2003) and (Freeman *et al.*, 2004) a repetitive control algorithm was developed and experimentally tested. The repetitive control algorithm uses an observer that does not observer the states of the internal model, the periodic part of the signal is removed with a filter, and a state feedback gain to control the system. The main difference in these two methods is the use of the internal model as part of the feedback and the order of the system.

4. ADAPTATION OF THE INTERNAL MODEL-BASED CONTROLLER

4.1 Adaptation Strategy

For acoustic, vibration, and similar systems it is often possible to measure the frequency of the disturbance. This measurement can be made, for example, with a tachometer for rotational systems. For some systems like acoustic or vibrational systems a microphone or accelerometer can be used to estimate the frequency of the disturbance. Additionally, the disturbances often appear in multiples of the fundamental frequency resulting in only one frequency that has to be identified.

In the situation that the disturbance cannot be measured, then the input and output signals must be used to estimate the fundamental frequency of the disturbance.

Once the frequency of the disturbance is known the internal model can be adjusted to reject the disturbance and the control gain K can be updated to maintain stability of the closed loop system. The observer is fixed since the dynamics of the disturbance are changing and not the acoustic system. This creates a simple adaptive controller that is parameterized in term of the frequency of the disturbance.

The internal model $M(q)$ and feedback gain K can be adjusted in two different ways. First, if there is enough computational recourses on the digital controller and time varying disturbances are present then solving a Riccati equation every N samples to determine the feedback gain is an option. Conversely, if computational speed is a concern then calculating the feedback gain off-line and using a lookup table or a curve fit to calculate the feedback gain is a viable alternative to the computationally intensive method of solving Riccati equations. The lookup table option is used in the ANC study presented in this paper.

4.2 Adapting the Internal Model and Feedback Gain

Let the fundamental frequency of the disturbance be called ω_n, then the internal model $M(q,\omega_n)$ is updated for each value of ω_n and the state space representation is given by

$$\begin{aligned} x_m(k+1) &= A_m(\omega_n(k))x_m(k) + B_my_p(k) \\ y_m(k) &= I_mx_m(k) \end{aligned}. \quad (15)$$

It can be observed from (8) that changes in the in the internal model cause a change in the feedback gain $K(\omega_n)$ that stabilizes the closed loop system. However, changes in $M(q,\omega_n)$ do not affect the observer gain defined in (12). As a result, the adaptive controller is given by

$$C(q,\omega_n) = \left[\begin{array}{cc|c} (A_p - L_pC_p - B_uK_1(\omega_n)) & -B_uK_2(\omega_n) & L_p \\ 0 & A_m(\omega_n) & B_m \\ \hline -K_1(\omega_n) & -K_2(\omega_n) & 0 \end{array}\right], \quad (16)$$

where $K(\omega_n)$ and $A_m(\omega_n)$ are calculated for each value of ω_n. Figure 1 shows the adaptive IMB controller connected to $P(q)$. Notice that only the internal model and feedback gain vary as functions of ω_n.

To create a look-up table, $M(q, \omega_n)$ and stabilizing control gain $K(\omega_n)$ are calculated off-line for several different values of ω_n.

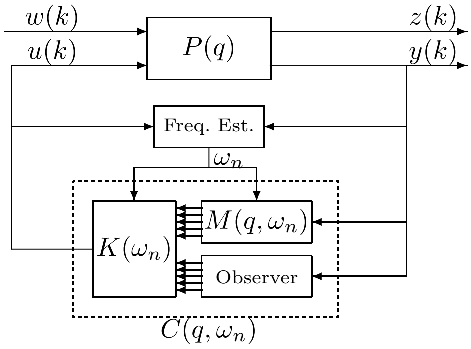

Fig. 1. The adaptive internal model-based controller $C(q, \omega_n)$ connected to $P(q)$.

4.3 *Estimating the Fundamental Frequency of the Disturbance*

There are several methods available to estimate the frequency of the periodic disturbances. Notch filtering (Handel and Nehorai, 1994), disturbance observers, recursive least squares, Kalman filtering, phase-locked loops (Bodson, 2005) and extensions of the aforementioned can all be utilized to estimate the fundamental frequency of the disturbance. A more recent algorithm developed by (Brown and Zhang, 2003) uses the states of the internal model and the output of the plant to update the frequency of the internal model.

For the purposes of this paper, the magnitude/phase-locked loop MPLL will be used to estimate the frequency of the disturbance. This method is simple to implement and is superior to PLL in that the error signal converges to zero when the estimation parameters converge to their nominal values (Wu and Bodson, 2003).

The discrete-time MPLL (Guo and Bodson, 2003) is shown is Figure 2. For a signal $d(k)$ comprised of one sinusoidal component, the MPLL estimates the magnitude, frequency, and phase of the signal. This is accomplished in two coupled loops. In the magnitude loop the error signal $\varepsilon(k)cos(\hat{\alpha}(k))$ is integrated to obtain the estimate of the magnitude $\hat{m}(k)$. This gives

$$\hat{m}(k) = \frac{K_m \Delta T}{q-1} \varepsilon(k) cos(\hat{\alpha}(k)), \qquad (17)$$

where $\varepsilon(k)$ is the error given by

$$\varepsilon(k) = d(k) - \hat{d}(k), \qquad (18)$$

and $\hat{\alpha}(k)$ is the estimated phase. The phase and frequency are estimated in the frequency loop with

$$\begin{aligned} \hat{\alpha}(q) &= \frac{K_f(q-a)}{q-1} \hat{\omega}(k) \\ \hat{\omega}(k) &= -\frac{K_w \Delta T}{q-1} \varepsilon(k) sin(\hat{\alpha}(k)) \end{aligned} \quad . \qquad (19)$$

The estimated signal $\hat{d}(k)$ is used to update the error equation and is given by

$$\hat{d}(k) = \hat{m}(k)\cos(\alpha(k)). \qquad (20)$$

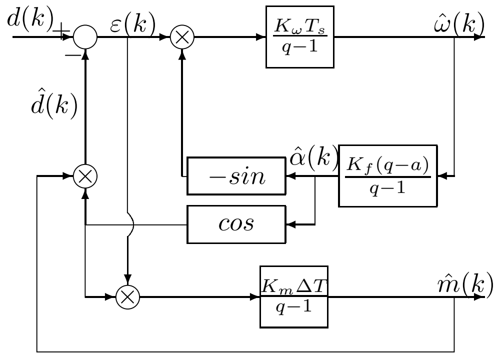

Fig. 2. Magnitude/phase-locked loop.

5. ACTIVE NOISE CONTROL STUDY

5.1 *System Description*

To create a realistic ANC study, experimental data was obtained from the acoustic system depicted in Figure 3. The acoustic system consists of a fan mounted inside an acoustic duct. At the end of the duct, a feedback microphone and four speakers (connected in parallel so as to act as one large speaker) are located for ANC.

A high order ARX model $G_o(q)$ was fit to the time series data between the speakers and microphone. The adaptive controller was designed from a low order OE model G(q). Figure 4 shows the frequency response of $G_o(q)$ and $G(q)$.

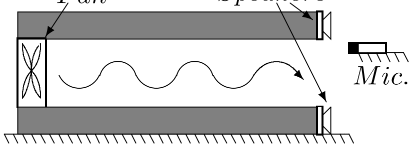

Fig. 3. Acoustic system used for active noise control study.

Figure 5 shows that fan noise has two distinct components: periodic and non-periodic. In open loop, the noise from the fan $v(k)$ was recorded by the feedback microphone as the fan changed speeds from 910-960 Hz. This signal was used to simulate the disturbance input to the closed loop system. A low order AR model of the non-periodic component of the fan noise $H(q)$, shown in Figure 5, was fit to the time series data obtained from the fan. $H(q)$ fits the broadband noise from the fan. The periodic part of the fan noise $M(q)$ is modeled with lightly damped oscillators. For purposes of the control design, $M(q)$ is placed inside the feedback loop instead of being added to $H(q)$.

The feedback signal $y(k)$, the output of the high order plant, is given by

$$\begin{aligned} y(k) &= \left[\, G_o(q) \;\; 1 \,\right] \begin{pmatrix} u(k) \\ v(k) \end{pmatrix} \\ &= [P_o(q)] \begin{pmatrix} u(k) \\ v(k) \end{pmatrix} \end{aligned}, \tag{21}$$

and is used for the simulation. The plant $P(q)$ used for the control design is given by

$$P(q) = \left[\, G(q) \;\; H(q) \,\right]. \tag{22}$$

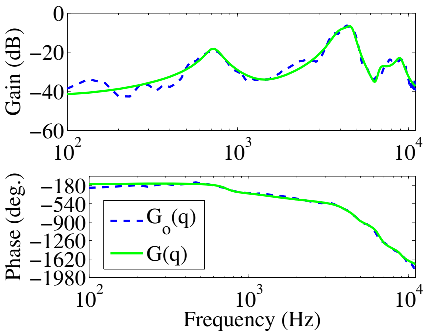

Fig. 4. Frequency response of $G_o(q)$ and $G(q)$.

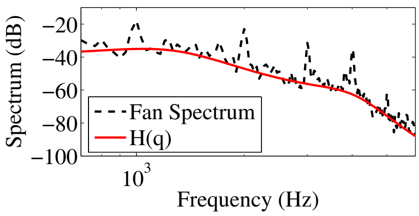

Fig. 5. Spectral content of the fan noise and $H(q)$.

5.2 Adaptive Control Results

An adaptive IMB controller was designed to cancel the first four harmonics of the fan noise. The resulting coefficients of $K(\omega_n)$ are shown in Figure 6 versus ω_n. $K(\omega_n)$ is a vector with a length of 24. Each of the coefficients change smooth as a function of ω_n, this supports the use of look-up tables for the adaptive controller. At approximately 6 *sec.* the fan speed is switched from 910 to 960 Hz. Figure 7 shows the amount that the spectrum of $y(k)$ was reduced by the adaptive controller, only the first harmonic is shown for graphical reasons. It can be seen from this figure that the adaptive IMB controller rejects the periodic part of the disturbance and is able to adjust to step-like changes in the fundamental frequency of the disturbance. The slow response

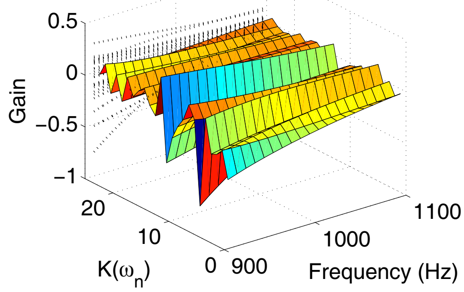

Fig. 6. The coefficients of the gain matrix K versus frequency.

of the frequency estimator causes the ramp that appears near 6 *sec.*. It can be seen from Figure 9, that the MPLL method lags behind the actual speed of the fan.

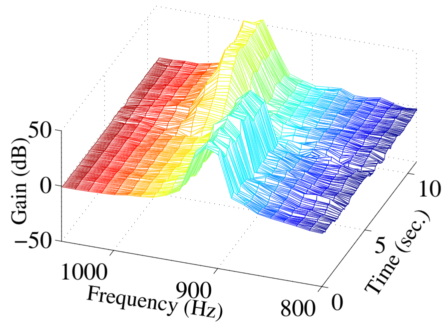

Fig. 7. The reduction of the output spectrum.

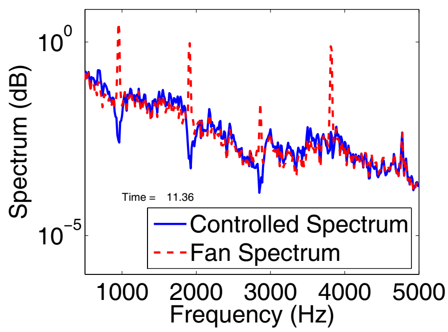

Fig. 8. The spectrum of the output before and after control is applied.

The spectrum of $y(k)$ before and after the adaptive controller is applied for an ANC study time of 11.36 *sec.* is shown in Figure 8. This figure shows the ability of the adaptive controller to cancel the first four harmonics of the disturbance.

5.3 Frequency Estimation Results

To obtain an accurate frequency estimate, the MPLL method was used to track the sixth harmonic of the fan noise. Since the controller was designed to cancel the first four harmonics, the sixth was left in tact. Additionally, a 4^{th} order butterworth bandpass filter was used to remove noise from the signal before it was sent to the MPLL. Figure 9 compares the MPLL method versus tracking the peak of the absolute value of the fast fourier transform. The MPLL method was able to track the change in fan speed with a very small stead state error. However, as seen in Figure 7, the convergence rate of the MPLL affected the performance of the adaptive controller.

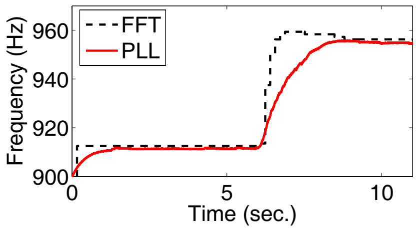

Fig. 9. Comparison of MPLL and FFT methods for frequency estimation.

6. CONCLUSIONS

In this paper, an adaptive controller was constructed from a family of internal model-based controllers that were parameterized with respect to the fundamental frequency of the disturbance. Only the feedback gain and internal model are varied as function of the fundamental frequency; the observer is fixed since the plant dynamics are time invariant. Lookup tables and a magnitude/phase-locked loop were used to implement the adaptive controller in an active noise control study.

The active noise control study demonstrates the ability of the adaptive controller to reject periodic, time-varying disturbances. The controller was designed to cancel the first four harmonics of the disturbance from the fan noise. The fundamental frequency of the disturbance was estimated by tracking the sixth harmonic of the output spectrum.

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