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MODELING AND ROBUST CONTROL FOR HARD DISK DRIVES

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1. INTRODUCTION

As track density in hard disk drives increases, the requirements for actuator servo system track following become more strict (Chew, 1995). For improved track following performance the bandwidth of the servo system should be increased, however this is limited by sampling frequency and mechanical resonances of the head/disk assembly.

This paper demonstrates modeling and control designs that reflect the performance objectives for hard disk drive actuators in terms of of desired control bandwidth. Control-relevant modeling is accomplished by using information of appropriate weighting functions applied in the control design. The control design is based on standard $\mathcal{H}_\infty$ optimization and requires an upper bound on the characterization of model uncertainty as well as specification of a performance weighting function that directly relates to the closed-loop transfer functions of interest.

2. CONTROL-RELEVANT MODELING

2.1 Estimation of a nominal model

In designing an optimal servo controller for disturbance rejection and improved track following, it is preferable to estimate a set of models $\mathcal{P}$ for which the difference between the designed performance and the achieved performance of the controller implemented on the real system is minimized. Bandwidth and disturbance rejection performance are characterized by the sensitivity function

$$S = (1 + PC)^{-1} \tag{1}$$

where P denotes a dynamical model of the voice coil motor (VCM) with flexibilities of the E-block and suspension and C denotes the VCM servo controller. The desired performance characterized by the shape of the sensitivity function is captured in the weighting function W_S, which is specified in terms of bandwidth ω_b and maximum disturbance amplification M_s requirements in the form

$$W_S = \left(\frac{s/\sqrt[k]{M_s} + \omega_b}{s + \omega_b \sqrt[k]{\epsilon}} \right)^k, \quad \text{for} \quad k \geq 1 \tag{2}$$

where ϵ determines the level of steady-state error rejection.

A link between modeling and control is established by considering a minimization of the difference between the designed performance and the achieved performance in terms of the sensitivity function (1) providing a control-relevant identification given by (Van Den Hof and Schrama, 1995)

$$\|W_S(1 + P_0C)^{-1} - W_S(1 + PC)^{-1}\|_\infty. \tag{3}$$

Identification of a model P from the control-relevant criteria (3) can be written into a weighted additive difference between the actual VCM dynamics P_0 and the nominal model P via

$$\|W_S(1 + P_0C)^{-1}(P_0 - P)C(1 + PC)^{-1}\|_2 \tag{4}$$

motivated by the fact that $\mathcal{L}_2$ approximation tends to $\mathcal{L}_\infty$ approximation (Caines and Baykal-Gürsoy, 1989).

The weighted additive difference between P_0 and the nominal model P can be written in terms of frequency domain criterion. When using experimental frequency domain data $P_0(\omega_k)$, where $\omega_k,\ k = 1, 2, \ldots, N$ refers to a frequency domain grid, the two-norm criterion in (4) can be approximated by

$$\sum_{k=1}^{N} |(P_0(\omega_k) - P(\omega_k))W(\omega_k)|^2$$

where the frequency weighting $W(\omega_k)$ given by

$$W(\omega_k) = \frac{W_S(\omega_k)}{1 + C(\omega_k)P_0(\omega_k)} \cdot \frac{C(\omega_k)}{1 + C(\omega_k)P(\omega_k)}. \tag{5}$$

As a result, computation of a finite order discrete-time model $P(q)$ can be found with the (non-linear) Least Squares optimization techniques

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available in (de Callafon R.A. and Van Den Hof, 1996).

From (5) it can be observed that the frequency weighting requires information of the actual sensitivity function $(1+CP_0)^{-1}$, the sensitivity function $(1+CP)^{-1}$ based on the model to be estimated and the desired performance W_S based on a desired shape of the sensitivity function. Data of the actual sensitivity $(1+CP_0)^{-1}$ is easily constructed from closed-loop experiments using the controller C or can be computed from the open-loop frequency response data $P_0(\omega_k)$ and the controller frequency response $C(\omega_k)$. The weighting $(1+CP)^{-1}$ can only be computed once the frequency response of a model $P(\omega_k)$ is available. However, an iterative update of the frequency weighting can be used to update the weighting $(1+CP_i)^{-1}$ on the basis of a model P_i. The desired performance W_S can be fixed with ideal parameters $\omega_b^*,\ M_s^*,\ \epsilon^*$ even though these parameters may not yield a control design satisfying the performance robustness requirements discussed in Section 3.

2.2 Estimation of additive uncertainty bounds

To facilitate the design of a robust performing feedback controller, the open-loop frequency domain data $P_0(\omega_k)$ can be used to account for modeling and approximation errors made by considering possible variations in the nominal response modeled by $P(q)$. Consider a set of models $\mathcal{P}$ consisting of a nominal model P along with an upper bound allowable additive model perturbation W_A

$$\mathcal{P} = \{P \mid P + \Delta W_A,\ \|\Delta\|_\infty \leq 1\} \qquad (6)$$

such that the real system, represented by P_0, is contained in the model set $P_0 \in \mathcal{P}$.

A frequency dependent upper bound is available via a model error model identification between the control signal and prediction error residual (Ljung, 1999). A control-relevant frequency dependent upper bound can be obtained such that

$$\|\Delta\|_\infty \leq \delta(\omega) \text{ with prob. } \geq \alpha. \qquad (7)$$

Subsequently spectral over bounding routines (Bayard and Yam, 1994) can be used to construct a limited complexity stable and stably invertible weighting filter W_A that over bounds $\delta(\omega)$.

3. ROBUST CONTROL DESIGN

Performance considerations are well suited under the worst-case $\mathcal{H}_\infty$-norm based optimal control which allows robustness issues to be incorporated into the servo design in the form of uncertainty models (Zhou et al., 1996). Control design such that $\|W_S S\|_\infty < 1$ guarantees that the sensitivity function is bounded above by the desired shape W_S^{-1}. Minimization of this cost function alone is not practical as it leads to infinite controller gains. In practice it is useful to also consider a bound on the transfer function between the disturbance and control signal $\|W_A CS\|_\infty < 1$, yielding a bound on control energy and considering robust stability with respect to additive plant uncertainty. Combining stability robustness and performance, a performance robustness condition can be constructed.

Consider a performance weight of the form (2) with M_s fixed, then a mixed sensitivity optimization problem for finding a stabilizing controller to achieve maximum bandwidth ω_b can be proposed as

$$\max |\omega_b| \ \text{ s.t. } \left\| \begin{matrix} W_S S \\ W_A CS \end{matrix} \right\|_\infty < 1. \qquad (8)$$

The above optimization may be implemented as an outer loop around standard mixed-sensitivity $\mathcal{H}_\infty$ control design (Skogestad and Postlethwaite, 1996).

3.1 $\mathcal{H}_\infty$ synthesis

The combination of weighting functions and design specifications requires the design specifications (8) can be performed via standard $\mathcal{H}_\infty$-synthesis (Zhou et al., 1996). The model and weighting functions are combined in a generalized plant

$$\begin{bmatrix} z_1 \\ z_2 \\ y \end{bmatrix} = \begin{bmatrix} W_S & -W_S P \\ 0 & W_A \\ 1 & -P \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \qquad (9)$$

where z_1, z_2 are weighted signals, y, u are the input/ouput of the controller and w is the disturbance. Standard $\mathcal{H}_\infty$ methods synthesis a controller C such that $\|T_{zw}\|_\infty$ is minimized, where T_{zw} denotes the transfer function from w to z. Noting that $u = Cy$ the performance robustness design specification (8) is recovered from (9). Although the limited controller complexity has been addressed via low-order control-relevant model estimation, a reduction of the controller may be required where closed-loop balanced model reduction methods (Obinata and Anderson, 2001) account for the performance objectives.

4. HDD SERVO DESIGN APPLICATION

The experimental system consists of a 2.5" disk drive with 120 sectors and rotational speed at $4200rpm$ giving it a servo sampling frequency f_s of $8.4kHz$. The disk drive servo processor is replaced by a DSP and host computer that allow access to parameters in a general control transfer function. The feedback control law for the existing drive is given by the discrete-time PID controller

$$u(t) = k_p e(t) + k_i \sum_{i=0}^{t} e(i) + k_d[e(t) - e(t-1)] \qquad (10)$$

and has an initial bandwidth of $600Hz$. The objective is to increase the bandwidth but restrict the complexity of the feedback controller to a general second-order transfer function so that fair comparison can be made with the PID controller with the same order

$$C(z) = \frac{b_0(q+b_1)(q+b_2)}{(q+a_1)(q+a_2)} \qquad (11)$$

where q denotes time-shift operation.

A control-relevant third-order nominal model is obtained from an estimation with identification criteria (4), shown in Figure 1. Using the nominal model, a second-order upper bound on the additive uncertainty is obtained with probability 99.9% by identifying a model error model and estimating a low-order spectral over bound.

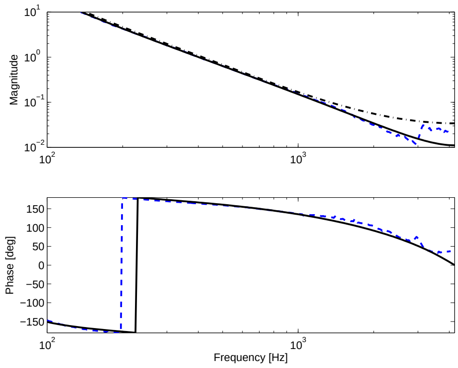

Fig. 1. Frequency response of the measured plant (dashed-line) and a nominal model (solid-line) with additive uncertainty (dash-dotted-line).

The performance weight W_S with structure given by (2) was with fixed maximum allowable disturbance amplification M_s. The bandwidth of the performance weight was maximized in an outer loop around a standard $\mathcal{H}_\infty$ controller synthesis problem (Zhou et al., 1996). The controller designed as in Section 3 lead to an eighth-order controller which was reduced to a second-order controller via closed-loop balanced model reduction techniques (Obinata and Anderson, 2001) presented in Figure 2. The full order controller has similar integral gain as the original PID, but provides higher gain and more phase margin around the closed-loop bandwidth. The closed-loop balanced controller reduction gives a controller which approximates the full-order controller around the closed-loop bandwidth sacrificing some integral gain at low frequencies.

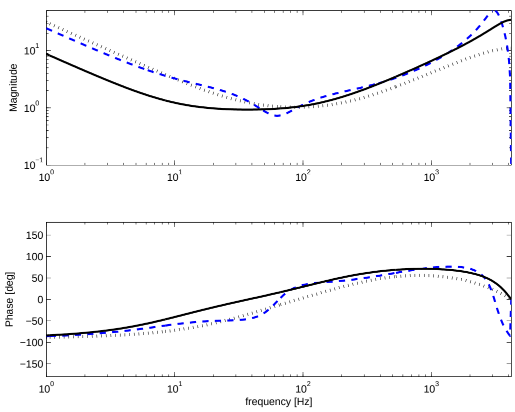

Fig. 2. Controller frequency response of original PID (dotted-line), full-order $\mathcal{H}_\infty$ controller (dashed-line) and second-order $\mathcal{H}_\infty$ controller (solid-line).

Although the designed performance has been improved via $\mathcal{H}_\infty$ control techniques, the improvement in achieved performance should be the driving factor for the new controller. Frequency response measurements of the disk drive sensitivity functions operating under feedback with PID controller and $\mathcal{H}_\infty$ controller are compared in Figure 3. The achieved frequency responses demonstrate that larger bandwidths are possible with $\mathcal{H}_\infty$ control design techniques at the price of slightly more complex servo control law.

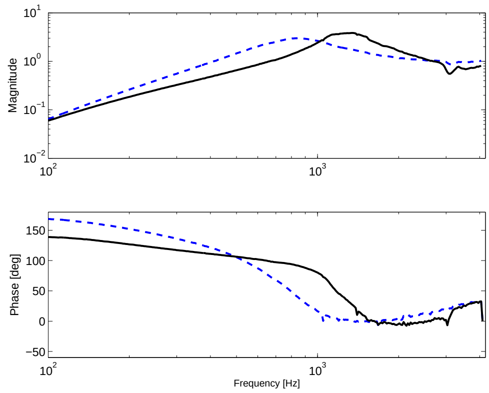

Fig. 3. Measured disk drive sensitivity functions resulting from servo controllers PID (dashed-line) and reduced-order $\mathcal{H}_\infty$ (solid-line).

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