An Iterative Learning Design for Repeatable Runout Cancellation in Disk Drives

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Abstract—**In this paper, we consider the iterative learning control (ILC) framework to design a reference signal that directly cancels periodic disturbances in a feedback measurement. Cancellation of periodic disturbances is useful in reducing undesirable repeatable tracking errors in applications such as the two-stage servo track writing process for disk drives. A general problem description is given for a linear discrete-time periodic system and convergence results for the learning system are derived. A learning filter is designed with the use of a finite-impulse response model approximation for the inverse of the closed-loop sensitivity such that convergence is achieved in learning a reference signal that provides cancellation with periodic perturbations affecting the system measurement. The ILC algorithm is applied to a disk drive system where experimental results demonstrate the effectiveness of the method in reducing periodic measurement disturbances.**

Index Terms—**Hard disk drive, iterative learning control (ILC), periodic disturbance, plant uncertainty.**

I. Introduction

THE challenges of compensating for periodic disturbances appears in various applications dealing with rotating machinery. For example, in hard disk drive (HDD) servomechanisms, disturbances consist of both a repeatable and nonrepeatable nature which appear in the position error signal (PES) of the head following a data track. The repeatable run-out (RRO) disturbance generally occurs at frequencies that are integer multiples of the frequency of rotation of the disk and is a considerable source of PES with respect to the center of the data track [1].

Numerous control design methods, categorized by either a feedback or feedforward structure, have been developed specifically for eliminating periodic disturbances. Generally these algorithms consider generating a control input whereby the system asymptotically tracks the periodic disturbance in the output. In the feedback category, methods such as repetitive control [2] and disturbance observer (DOB)-based control [3] have demonstrated effective compensation for repeatable disturbances. However, repetitive control tends to amplify nonrepeatable disturbances between the frequencies of the repeatable disturbances while DOB-based control can alter the closed-loop properties of the system. The feedforward category, including methods such as adaptive feedforward cancellation (AFC) [4], [2] and iterative learning control (ILC) [5], [6], have also shown successful application in the reduction of periodic disturbances. Unfortunately AFC methods require intensive computation when rejecting multiple disturbances which can be disadvantageous. Based on the internal model principle, ILC schemes first developed by [7], have been shown as the dual of repetitive control [8] and have demonstrated improved tracking performance for disk drives [9]. With ILC methods, control effort is focused at the frequencies of the periodic disturbances to improve the tracking performance of the system.

Improved performance of the feedback system when the output measurement is perturbed by periodic disturbances however, leads to undesirable repeatable tracking errors. As an example, progress in HDD servo track writers has led to a two stage servo track writing process where a master servo disk, created in stage one, is used as a reference from which the servo tracks on the remaining disks in the stack are written in stage two [10]. Repeatable run-out either written-in during the servo track writing process or resulting from mechanical disk assembly can be eliminated by considering them as sources of periodic measurement noise and canceling them via a modified reference signal. Thus, the objective of the servomechanism is not to follow the RRO error, but to follow a virtual perfectly circular track thereby reducing ac-squeeze of data track following. Learning algorithms designed for this purpose has been successfully applied to disk drive systems by [11] where knowledge of a nominal plant model and access to the control signal were required [12], where a conservative approximate model was developed from the closed-loop frequency response and [13], where an approximation of the closed-loop system was obtained from averaged signals. In these, the result was a reference signal called the zero acceleration path (ZAP) that cancelled the RRO allowing the read/write head of the disk drive to follow a virtual circular track with zero actuator acceleration.

Compared to [11]–[13], the motivation for this study is to generalize periodic disturbance cancellation results for systems without knowledge of a nominal plant model with access to the control signal or requiring special frequency–domain model approximation tools. This paper considers ILC methods for computing disturbance canceling reference signals (DCRS) that provide reduction of periodic measurement disturbances based on closed-loop frequency-response measurements. Section II outlines the general problem formulation for this study, provides a brief review of some of the extensively studied theory on ILC, and discusses a learning filter design methodology for satisfying convergence criteria. Section III describes the HDD application, the development of the learning filter for the experimental

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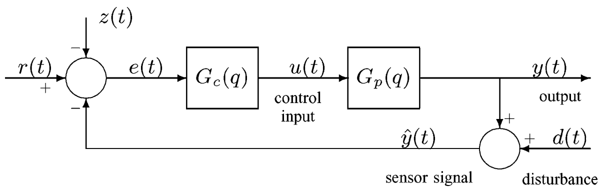

Fig. 1. Block diagram of feedback control system with periodic measurement disturbance $d(t)$.

disk drive drive servomechanism and presents the results for the DCRS experiment.

II. General Problem Formulation and ILC Configuration

A. System Description

A general block diagram description for a single-input–single-output (SISO) linear time invariant (LTI) discrete-time system in which periodic disturbances occur is presented in Fig. 1. Let $G_p(q)$ be the plant and $G_c(q)$ be the feedback compensator operating over a finite time interval t where $t = 1, \ldots, N$ and q is the shift operator. The reference $r(t)$, the disturbance $d(t)$ and the error signal $e(t)$ are periodic with period N. Due to the periodic nature of the signals, the assumption is made that at the end of each finite time interval the initial conditions of the system are resent.

Cancellation of the periodic measurement disturbance $d(t)$ is achieved by subtracting $z(t)$ at the reference. The disturbance must be learned from available measurements of the error signal $e(t)$, where the effect of the disturbance on the feedback error is given by

$$e(t) = -S(q)d(t)\ , \ \ S(q) = \frac{1}{1 + G_p(q)G_c(q)}$$

where $S(q)$ denotes the sensitivity function. Intuitively, the $z(t)$ that directly cancels the periodic disturbance would a filtered version of the error $e(t)$ where an appropriate filter is given by the inverse of the sensitivity function $z(t) = S^{-1}(q)e(t)$. In practice, however, perfect models for the closed-loop sensitivity function are not available and the effectiveness of $z(t)$ for canceling periodic disturbances depends upon the accuracy of the model $\hat{S}^{-1}(q)$. Furthermore, in servomechanisms where the closed-loop system contains at least an integrator for steady state tracking, a stable model for the sensitivity function generally lends to an unstable inverse making the signal $z(t)$ unbounded. Fortunately, ILC methods have been shown to implicitly find through iteration the stable approximation to the inverse of the system [14] which would result in a modified reference signal that cancels with the periodic disturbance.

B. Iterative Learning of the Reference Signal

Iterative learning control methods applied to systems have primarily focused on compensating for periodic disturbances via a modified control signal. Thus, the control system is redesigned with a feedforward component added to the system input [2], [15]. Although this ILC structure is most commonly studied in the literature, modification of the reference signal can achieve improved control performance for systems, where it is undesirable to reconfigure the control signal. The iterative learning of reference signals has been referred to as *cascaded* ILC [5]. The problem formulation from Section II-A motivates the use of this structure for determining the adjunct reference signal $z(t)$ that cancels with the periodic disturbance $d(t)$. A block diagram representation for the ILC system with cascaded structure is shown in Fig. 2. Here $z_i(t), \hat{r}_i(t), \hat{y}_i(t)$, and $e_i(t)$ denote the DCRS, modified reference signal, system output measurement, and error signal, respectively, where the subscript i indicates the ith iteration. The blocks labeled *MEM* denote memory arrays that store signals of the current iteration for use in the next learning iteration.

Notice from Fig. 2 that the ILC scheme is added outside the existing control loop through a modified reference signal, $\hat{r}_{i+1}(t) = r(t) - z_{i+1}(t)$. Here the ILC update law is classified under previous cycle learning (PCL) whereby the current iteration of the adjunct reference signal $z_{i+1}(t)$ is some form of signals from previous iterations, in particular

$$z_{i+1}(t) = z_i(t) + G_f(q)e_i(t) \tag{1}$$

where the initial DCRS $z_0(t) = 0$, $t \in [1, N]$ and $G_f(q)$ is the learning filter of the ILC update. The learning filter $G_f(q)$ is used to filter the previous iteration error signal $e_i(t)$ and will play a crucial role in the convergence of the ILC system. The learning convergence condition is derived by observing the feedback error evolution from one iteration to the next. The current iteration feedback error is given by

$$\begin{aligned} e_{i+1}(t) &= \hat{r}_{i+1}(t) - \hat{y}_{i+1}(t) \\ &= S(q)[r(t) - d(t) - z_{i+1}(t)] \\ &= S(q)[r(t) - d(t) - z_i(t) - G_f(q)e_i(t)] \\ e_{i+1}(t) &= [1 - S(q)G_f(q)]e_i(t). \end{aligned} \tag{2}$$

The transfer function $[1 - S(q)G_f(q)]$ represents the map from the error at iteration i to the error at iteration $i+1$. The frequency content of the error $e_{i+1}(t)$ from (2) is given by the Fourier transform

$$E_{i+1}(\omega) = [1 - S(e^{j\omega})G_f(e^{j\omega})]E_i(\omega). \tag{3}$$

The discrete frequency components in $e_{i+1}(\omega)$ are composed of the corresponding frequency components of $e_i(\omega)$ multiplied in the magnitude and shifted in phase by the magnitude and phase of $[1 - S(e^{j\omega})G_f(e^{j\omega})]$, respectively. Thus imposing the magnitude condition

$$|1 - S(e^{j\omega})G_f(e^{j\omega})| < 1 \tag{4}$$

for all frequencies ω up to the Nyquist frequency assures that the amplitudes of the frequency components decay monotonically with every iteration. This suggests convergence to zero error, however, as shown in [16], this reasoning is not rigorous providing only a sufficient condition for convergence which applies to the parts of the trajectory where steady-state response has been achieved.

The approximate convergence condition (4) indicates monotonic decay for steady-state error trajectories and is related to an exact Euclidian norm monotonic convergence condition for

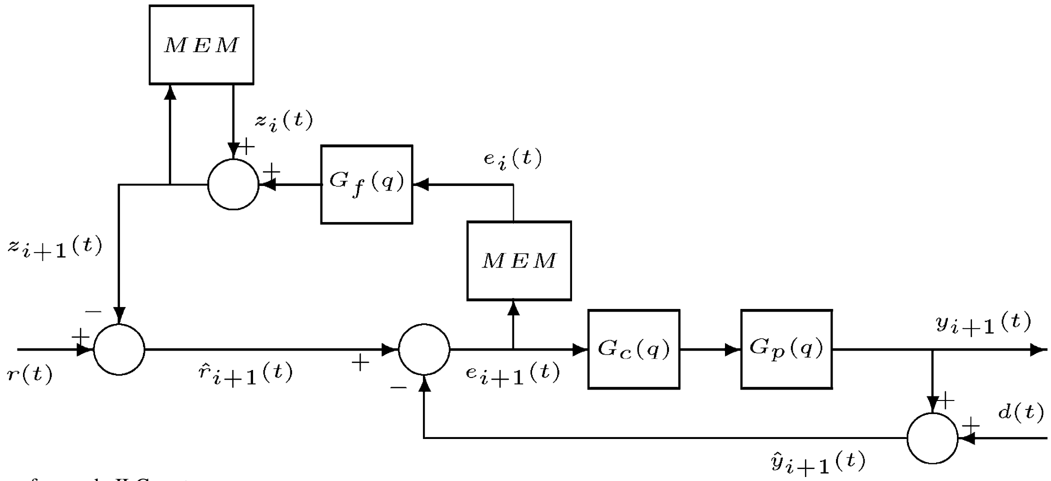

Fig. 2. Illustration of cascade ILC system.

nonsteady-state trajectories. Consider the size of the error signal e_i measured by

$$\|e_i\|_2^2 \doteq \int_0^\infty \|e_i(t)\|^2 dt \tag{5}$$

where $\|e\|^2 = e^T e$ denotes the Euclidian norm of signal e. Then from Parseval's theorem ([17]), we have that

$$\sup_{\|e_i\|_2} \frac{\|e_{i+1}\|_2}{\|e_i\|_2} = \|1 - S(e^{j\omega})G_f(e^{j\omega})\|_\infty \leq \rho < 1 \tag{6}$$

where $\|\cdot\|_\infty$ denotes the $\mathcal{H}_\infty$ norm. This guarantees the monotonic convergence of the entire error trajectory. Various iterative learning control algorithms have been developed to satisfy this rigorous condition, see [16] for an overview and where it is shown that (4) is sufficient for (6) at the equally spaced discrete frequencies.

C. Learning Filter Design

Although little information about the closed-loop system is required to satisfy learning convergence conditions, it is intuitive that knowledge of the sensitivity function $S(q)$ is beneficial when designing the learning filter $G_f(q)$ in order to satisfy (6). Observe the error propagation given by (2) written as a function of the error prior to the first iteration of the learning algorithm $e_0(t)$

$$e_{i+1}(t) = [1 - S(q)G_f(q)]^{i+1} e_0(t). \tag{7}$$

Equation (7) indicates that convergence in the iteration domain is expedited for all frequencies up to the Nyquist frequency when the upper bound ρ from (6) is close to zero. This poses the desired design problem for the learning filter, summarized in the following.

Problem 1: Let the size of the error signal $e_i(t)$ at iteration i be given by (5). Let the learning filter be parametrized by $G_f(q,\theta)$. To compute the optimal learning filter, consider the optimization

$$\arg\min_\theta \|1 - S(q)G_f(q,\theta)\|_\infty \quad \text{subject to: } G_f(q,\theta) \in \mathcal{RH}_\infty \tag{8}$$

which will provide monotonic convergence of $\|e_i\|_2$ provided that (6) holds.

In Problem 1, the learning filter $G_f(q,\theta)$ is parametrized by a fixed parameter θ. Stability of the learning filter is required to insure that the signals within the learning system remain bounded. The above problem intuitively suggests that, over the entire frequency range, expedited convergence of the error $\|e_i\|_2$ is achieved with a learning filter designed as the inverse sensitivity function. From this intuition rewrite (8) into standard model-matching form as

$$\arg\min_\theta \|G_f(q,\theta) - S^{-1}(q)\|_\infty. \tag{9}$$

Assuming no restrictions on the structure of $G_f(q,\theta)$, an equivalent problem formulation is given by the Hankel approximation problem also known as the Nehari extension problem for which there are well-known algorithms for determining a solution [18]. These algorithms, however, are computationally expensive and require accurate knowledge of the inverse sensitivity function for all frequencies ω up to the Nyquist frequency.

Avoiding computationally expensive methods, in [11] a design was presented that assumed knowledge of an open-loop nominal plant model as well as access to the control signal $u(t)$ in order to construct an approximate model $\hat{S}^{-1}(q)$. This method also requires additional filtering in order to maintain stability of the learning system. Typically for servomechanisms, a stable model that accurately describes the sensitivity function is not guaranteed to be stably invertible. However, there are several approaches for directly identifying a stable approximation to the system inverse, one of which uses standard identification algorithms to fit a model to the inverse of the measured frequency response $\hat{S}^{-1}(e^{j\omega})$ [19], [12].

To address the direct modeling of $\hat{S}^{-1}(q)$ recognize that the learning system only deals with the periodic disturbance $d(t)$, which acts only at frequencies $\omega_k \in \Omega$ [rad/s] where

$$\Omega = \left\{ \omega_k \mid \omega_k = \frac{k}{N} 2\pi f_s, \quad k = 1, \ldots, \frac{N}{2} \text{ (for } N \text{ even)} \right. \\ \left. \text{or } k = 1, \ldots, \frac{N-1}{2} \text{ (for } N \text{ odd)} \right\} \tag{10}$$

with sampling frequency f_s [Hz]. That is, the disturbance acts at integer multiples of the first harmonic frequency up to the Nyquist frequency of the discrete-time system . Suppose the frequency content of the error (3) is decomposed into its discrete frequency components $e(\omega_k)$. Since there is no periodic disturbance effect between the frequencies ω_k, the convergence condition (6) can be relaxed to the approximate convergence condition (4) and rewritten as

$$|1 - S(e^{j\omega_k})G_f(e^{j\omega_k}, \theta)| \leq \rho < 1 \qquad \forall\, \omega_k \in \Omega \qquad (11)$$

to indicate the discrete frequencies of interest, $\omega_k \in \Omega$. Furthermore, (3) implies that the rate at which each individual frequency component of the error $e(\omega_k)$ converges in the iteration domain depends upon the magnitude $|1 - S(e^{j\omega_k})G_f(e^{j\omega_k}, \theta)|$ evaluated at the frequencies ω_k [16]. This motivates the following proposition.

Proposition 1: Let the disturbance $d(t)$ be periodic with period N and given by the discrete Fourier transform

$$D(\omega_k) = \sum_{t=1}^{N} d(t)e^{-j\omega_k t} \qquad (12)$$

which only has frequency content at $\omega_k \in \Omega$ defined in (10). Consider the ILC system described by Fig. 2; then the minimization in Problem 1 can be solved via

$$\theta = \arg\min_{\tilde{\theta}} \max_{\omega_k \in \Omega} |1 - S(e^{j\omega_k})G_f(e^{j\omega_k}, \tilde{\theta})| \quad \text{subject to: } G_f(q, \theta) \in \mathcal{RH}_\infty \qquad (13)$$

which is sufficient for monotonic convergence of $\|e_i\|_2$ provided that (11) holds.

Proof: The $\mathcal{H}_\infty$-norm of the transfer function (6) is simply the peak value of $|1 - S(e^{j\omega})G_f(e^{j\omega}, \theta)|$ evaluated as a function of frequency, that is $\|e_{i+1}\|_2 = \sup_\omega |1 - S(e^{j\omega})G_f(e^{j\omega}, \theta)|\ \|e_i\|_2$. Since the signal $e_i(t)$ is a sampled periodic signal with period N, the properties of the discrete-time Fourier transform are such that $e_i(e^{j\omega}) = 0$, $\forall\, \omega \neq \omega_k$. Result (13) immediately follows from (8), where the supremum over ω is replaced by the maximum over the finite frequency range $\omega_k \in \Omega$. ■

The min–max problem (13) needs only be solved over a finite set of frequency points, which if parametrized linearly in the parameter θ, can be solved via linear programming. By imposing a finite-impulse response (FIR) model structure on the learning filter $G_f(q, \theta)$, a solution to (13) can be provided by imposing and is summarized in the following proposition.

Proposition 2: Let $G_f(q, \theta)$ be parametrized by the FIR filter

$$G_f(q, \theta) = \sum_{k=0}^{N-1} \theta_k q^{-k} \quad \text{such that} \quad \sum_{k=0}^{N-1} \theta_k = 0 \qquad (14)$$

where $\theta \in \mathbb{R}^N$. Then the minimizing parameter vector θ of (13) is given by

$$\theta = \overline{\theta} - \frac{1}{N}\sum_{k=0}^{N-1} \overline{\theta}_k \quad \text{where} \quad \overline{\theta} = \mathcal{F}^{-1}\left\{\hat{S}^{-1}(e^{j\omega_k})\right\} \qquad (15)$$

where $\hat{S}(e^{j\omega_k})$ are the frequency response measurements of the closed-loop sensitivity function at the frequencies ω_k, and $\mathcal{F}^{-1}\{\cdot\}$ denotes the inverse discrete Fourier transform (IDFT).

Proof: Let $\overline{\theta} = \mathcal{F}^{-1}\left\{\hat{S}^{-1}(e^{j\omega_k})\right\}$ then using the properties of IDFT it immediately follows that $G_f(e^{j\omega_k}, \theta) = \hat{S}^{-1}(e^{j\omega_k})$, where $\theta = \overline{\theta} - (1/N)\sum_{k=0}^{N-1} \overline{\theta}_k$ removes the mean of the parameter vector to satisfy $\sum_{k=0}^{N-1} \theta_k = 0$. Obviously, $\max_{\omega_k \in \Omega} |1 - S(e^{j\omega_k})G_f(e^{j\omega_k})| = 0$ and $G_f(e^{j\omega_k})$ will be a minimizing solution to (13). ■

Although a learning filter $G_f(q, \theta)$ that satisfies (11) is not unique, the FIR parametrization imposed on $G_f(q, \theta)$ in (14) allows for straight-forward implementation of the update law (1) in a DSP environment. The added condition on the sum of the parameters in (14) equating to zero guarantees that the filter $G_f(q, \theta)$ has a discrete-time dc gain of zero, since no compensation for dc components is required. In [12], the simple DFT/IDFT method is mentioned as the best choice for designing the learning filter, however, was not investigated for perceived reasons of computational complexity in performing and storing complex valued operations in the IDFT operations. Instead, a low order and conservative design for the learning algorithm is suggested, which increases the iterations required for converging to a satisfactory performance level. In [13], an alternative to the DFT/IDFT method was proposed; however, it requires more signal averaging and thus more computation time. In Section II-D, the IDFT computations are shown to require only real valued computations, significantly reducing the computational complexity in determining the parameters (15) of the nonconservative FIR filter design.

D. Implementation

The FIR learning filter $G_f(q, \theta)$ proposed in (14) provides an additional implementation advantage since the ILC update law (1) becomes a linear regression in the parameters θ and the error signal of the previous iteration $e_i(t)$

$$z_{i+1}(t) = z_i(t) + \theta_0 e_i(t) + \cdots + \theta_{N-1} e_i(t - N - 1). \qquad (16)$$

At each iteration, the linear regression update law (16) requires N multiplication and $N+1$ addition computations of the shifted error signal which can be implemented efficiently in a digital signal processor (DSP) environment. Additionally, parameters θ need only be computed once for a given time-invariant closed-loop system. The two steps for designing the learning filter according to Proposition 1 are as follows:

1) estimate the frequency response of the closed-loop sensitivity function $\hat{S}(e^{j\omega_k})$;
2) compute the parameters θ of $G_f(q, \theta)$ according to (15).

Using frequency response magnitude and phase measurements of the sensitivity function at frequencies $\omega_k \in \Omega$, the IDFT (15) in the second step can be calculated without performing complex valued operations typically required with Fourier transform computations.

Proposition 3: Given closed-loop sensitivity function frequency response measurements of magnitude $\mathrm{M}(\omega_k) = |\hat{S}(e^{j\omega_k})|$ and phase $\Phi(\omega_k) = \angle\, \hat{S}(e^{j\omega_k})$ at frequencies $\omega_k \in \Omega$ defined by (10). The IDFT of (15) is computed without complex valued operations, as shown by (17)–(18) at the bottom of the page, where $m = 0, \ldots, N-1$ for both cases.

Proof: Let N be even and let the sensitivity function frequency response measurements $\hat{S}(e^{j\omega_k})$ evaluated at ω_k have magnitude and phase, $\mathrm{M}(\omega_k)$ and $\Phi(\omega_k)$, respectively. The IDFT is defined by

$$x(m) = \frac{1}{N}\sum_{p=0}^{N-1} X(p)e^{j2\pi pm/N}, \qquad \text{where } m = 0, \ldots, N-1. \tag{19}$$

Setting the dc gain $X(0) = 0$, then some simple algebra leads to

$$x(m) = \frac{1}{N}\left\{ \sum_{k=1}^{(N/2)-1} \left[X(k)e^{j2\pi km/N} + X(N-k)e^{j2\pi(N-k)m/N}\right] + X\left(\frac{N}{2}\right)e^{j\pi m}\right\}. \tag{20}$$

In (20), replace $x(m)$ with θ_m and $X(k)$ with $\hat{S}^{-1}(e^{j\omega_k}) = (1/\mathrm{M}(\omega_k))e^{-j\Phi(\omega_k)}$ for $k \leq N/2$ and $\hat{S}^{-1}(e^{j\omega_k}) = (1/\mathrm{M}(\omega_{N-k}))e^{j\Phi(\omega_{N-k})}$ for $k > N/2$, where the second equality is the complex conjugate of the first equality in reverse order. From the properties of complex numbers the imaginary parts cancel out in the summation to get (17). The proof for N odd is analogous. ■

E. Robustness and an Iteration Update Parameter

The learning filter designed from either a nominal model $\hat{S}(e^{j\omega})$ or frequency response measurements $S(e^{j\omega})$ needs to be robust against modeling or measurement errors. A robust convergence criteria can be used to guarantee convergence of the ILC algorithm in the presence of such uncertainties. Different approaches have been considered for adding robustness to ILC update algorithms, see e.g., [20]–[22]. Similar to [20], the learning filter can be designed such that the convergence condition (11) is robustly satisfied and the result is summarized in the following.

Proposition 4: Let the uncertainty on the measurements $S(e^{j\omega_k})$ be characterized by $\mathcal{S}(e^{j\omega_k})$

$$\mathcal{S}(e^{j\omega_k}) = \Big\{S(e^{j\omega_k}) \,|\, \hat{S}(e^{j\omega_k}) - \underline{\beta}_k < |S(e^{j\omega_k})| < \hat{S}(e^{j\omega_k}) + \overline{\beta}_k; \; \angle\hat{S}(e^{j\omega_k}) - \underline{\varphi}_k < \angle S(e^{j\omega_k}) < \angle\hat{S}(e^{j\omega_k}) + \overline{\varphi}_k; \; \forall\omega_k \in \Omega\Big\} \tag{21}$$

where $\underline{\beta}_k$, $\overline{\beta}_k$ and $\underline{\varphi}_k$, $\overline{\varphi}_k$ are frequency-dependent parameters that overbound the uncertainty in magnitude and phase, respectively. Then the ILC convergence criteria (11) for periodic disturbances converges robustly provided

$$\max_{S\in\mathcal{S}} |1 - S(e^{j\omega_k})G_f(e^{j\omega_k})| < 1 \qquad \forall\omega_k \in \Omega. \tag{22}$$

In Proposition 4, the uncertainty set (21) encompasses various uncertainty descriptions, additive, multiplicative, etc., which can be represented as enclosed regions $\mathcal{S}(e^{j\omega_k})$ centered around the nominal frequency response $\hat{S}(e^{j\omega})$ in the complex plain [23]. The parameters $\underline{\beta}_k = \overline{\beta}_k$ and $\underline{\varphi}_k = \overline{\varphi}_k$ are then the magnitudes and phases of the uncertainty overbound evaluated at the frequencies $\omega_k \in \Omega$.

Additional robustness can be incorporated into the learning system by introducing an iteration update parameter γ_i. The ILC update algorithm (1) becomes

$$z_{i+1}(t) = z_i(t) + \gamma_i G_f(q,\theta)e_i(t) \tag{23}$$

where γ_i is a function of the iteration number and indicates how much of an adjustment the learning update is allowed to make at each iteration. The learning convergence condition (22) then requires evaluation at each iteration i of the algorithm.

$$\max_{S\in\mathcal{S}} |1 - \gamma_i S(e^{j\omega_k})G_f(e^{j\omega_k},\theta)| < 1 \qquad \forall\omega_k \in \Omega. \tag{24}$$

In light of the learning filter design methodology presented in Section II-C, any $\gamma_i \neq 1$ provides a conservative update, thus providing additional robustness in the learning algorithm against uncertainty in the sensitivity function nominal model [22]. This, however, reduces the convergence rate of the algorithm over all frequencies.

Additionally, the update parameter $\gamma_i = 0$ may also be implemented to stop the ILC algorithm once a desired performance criteria has been met at iteration i. The robustness may be useful

For N even :

$$\theta_m = \frac{1}{N}\left\{ \sum_{k=1}^{(N/2)-1} \left[\frac{1}{\mathrm{M}(\omega_k)}\cos\left(-\Phi(\omega_k) + \frac{2\pi km}{N}\right) + \frac{1}{\mathrm{M}(\omega_k)}\cos\left(\Phi(\omega_k) + \frac{2\pi(N-k)m}{N}\right)\right] + \frac{1}{\mathrm{M}(\omega_{N/2})}\cos\left(-\Phi(\omega_{N/2}) + \pi m\right)\right\} \tag{17}$$

For N odd :

$$\theta_m = \frac{1}{N}\left\{ \sum_{k=1}^{(N-1)/2} \left[\frac{1}{\mathrm{M}(\omega_k)}\cos\left(-\Phi(\omega_m) + \frac{2\pi km}{N}\right) + \frac{1}{\mathrm{M}(\omega_k)}\cos\left(\Phi(\omega_k) + \frac{2\pi(N-k)m}{N}\right)\right]\right\} \tag{18}$$

TABLE I
EXPERIMENTAL DISK DRIVE SPECIFICATIONS

Spindle motor speed	$4200rpm(70Hz)$
Number of data sectors, N	120
Servo sampling frequency, f_s	$8.4KHz$
Track pitch	$8\mu in$

in maintaining the long-term stability of the learning algorithm (as i gets large) as wells as providing cautious updates when the data of the closed-loop sensitivity function $\hat{S}(e^{j\omega_k})$ is not accurate enough for fast convergence.

III. Experimental Results

A. Application to HDD

The closed-loop servomechanism of disk drives operates by the feedback of the position error signal (PES) $e(t)$, which is perturbed by disturbances that are a combination of the repeatable and nonrepeatable run-out. Typically, one would like to design the control system such that the read/write head follows the center of the data track. However, in certain HDD applications, such as two-stage servo track writing and read/write head testing, the objective is to follow a virtual perfect circle track. This is equivalent to assuming periodically perturbed measurements of the PES and providing cancellation of the periodic disturbances via the disturbance canceling reference signal DCRS.

The experimental system consists of a 2.5-in form-factor magnetic disk drive with specifications presented in Table I. The disk drive servo processor is replaced by a DSP and host computer that allow access to the PES and have the capacity to input modified reference signals to the system.

The HDD system can be represented by the general LTI discrete time system described in Section II-A, where the finite time interval $t = 1, \ldots, N$ corresponds to the number of sectors on the disk. The PCL learning system of Fig. 2 generates the current iteration signal based on information from previous iterations. Under the assumptions of ILC, at the end of each finite time interval the initial conditions of the system are resent, thus the information must be repeatable. However, residual random noise always exists in practical systems, and it often leads to lower performance or even divergence of the learning system [16]. To overcome this, the PES is measured over sufficiently many disk rotations, and an average is taken over the sector interval. The repeatable component remains, whereas the random noise of zero mean is reduced, leaving only the periodic disturbance effect on the PES, as shown in Fig. 3.

The periodic disturbance includes written-in RRO effects such as the eccentricity of the track and AC-squeeze that contribute to the PES. Measurement of the PES occurs once at each data sector along the disk. As a result, the disturbance only has an effect on the PES at integer multiples of the frequency of rotation of the disk. Closed-loop disturbance rejection or amplification is determined by the sensitivity function. Measurements $S(e^{j\omega})$ of the sensitivity function along with possible perturbations in the amplitude and phase are compared with the Bode plot or a fifth-order model $\hat{S}(q)$ in Fig. 4.

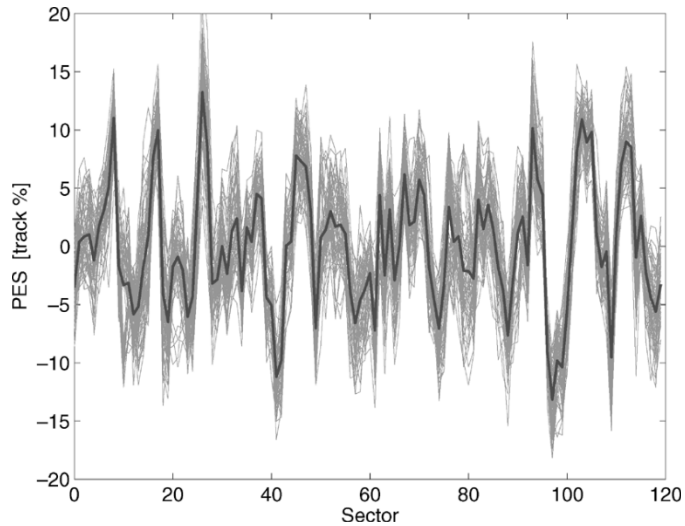

Fig. 3. Sector plot of the original PES (grey) and the averaged PES (dark).

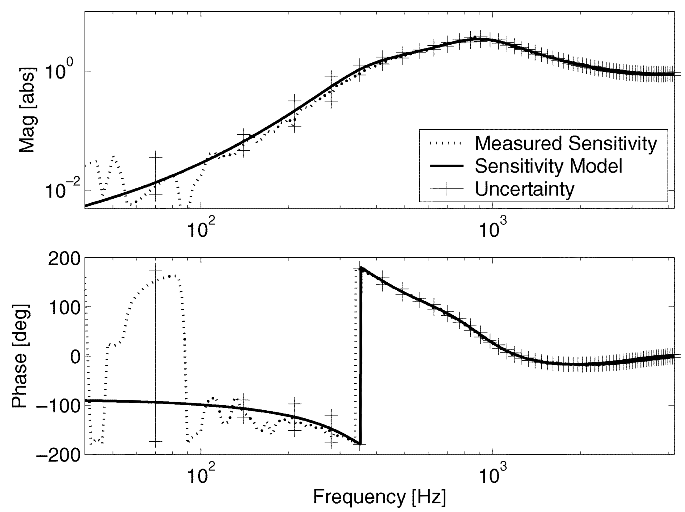

Fig. 4. Measured closed-loop sensitivity frequency response (dotted line) with perturbation (vertical lines) and fifth-order model $\hat{S}(e^{j\omega})$ (solid line).

The disk drive sensitivity function model $\hat{S}(q)$ is not stably invertible, and thus its inverse cannot be used directly as a learning filter design. Motivated by the procedure in Section II-C, the model is used to generate closed-loop frequency response $\hat{S}(e^{j\omega_k})$ at the frequencies of the periodic disturbance ω_k, where $\omega_k = [70, 140, \ldots, 4200]$ Hz are the integer multiples of the frequency of rotation up to the Nyquist frequency.

Since the learning filter is designed from the frequency response of a model of the sensitivity function, any perturbation or error between the nominal model and the measurements results in a violation of the learning convergence condition. Robustness in the ILC algorithm, specifically at lower frequencies where the frequency response measurements are not reliable, can be introduced through filtering in order to guarantee (22). This additional filtering can be implemented by modifying the frequency response data at the desired frequencies of ω_k prior to evaluating the parameters of the FIR model. The coefficients θ are generated according to Proposition 2 and using the real-valued computation (17). The FIR model $G_f(q, \theta)$ describes a stable system

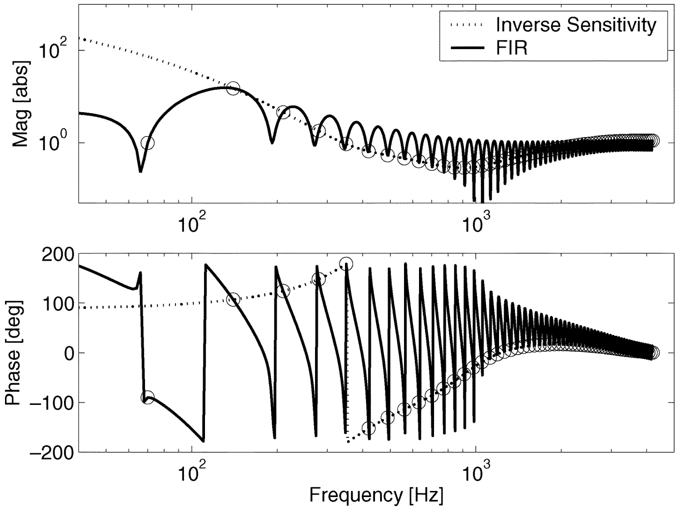

Fig. 5. Frequency response of FIR model $G_f(q,\theta)$ approximation (solid line) to $\hat{S}^{-1}(e^{j\omega})$ (dotted line) and the intersection of the FIR model with $\hat{S}^{-1}(e^{j\omega})$ at ω_k (o).

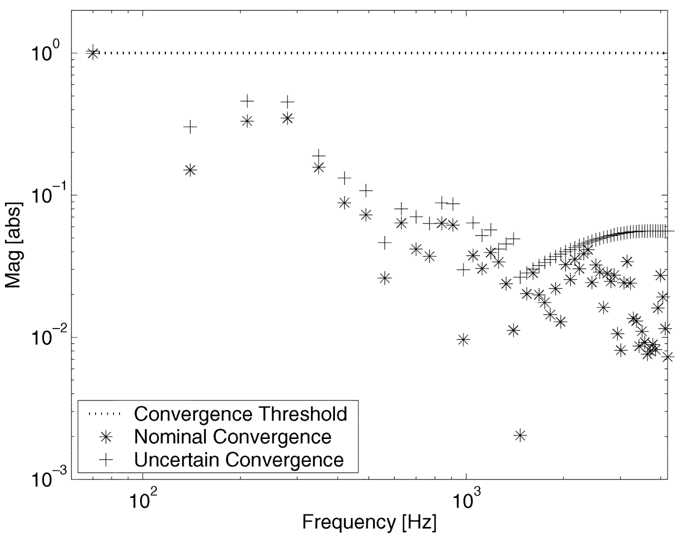

Fig. 6. Convergence condition (11) based on measurements $S(e^{j\omega_k})$ (∗) and worst case convergence condition (22) based on $\mathcal{S}(e^{j\omega_k})$ (+).

that exactly intersects the frequency response of $\hat{S}^{-1}(e^{j\omega_k})$ at the frequencies ω_k, except at 70 Hz, as shown in Fig. 5.

Frequency response measurements in disk drives are typically unreliable well below the bandwidth of the controller because of the low frequent disturbance rejection properties of the closed-loop system. The frequency response of the learning filter $G_f(e^{j\omega_k},\theta)$ has been modified at $\omega_k = 70$ Hz in order to conservatively guarantee the learning convergence condition (22), presented for the disk drive in Fig. 6. Note that the conservative design for $G_f(e^{j\omega_k},\theta)$ at $\omega_k = 70$ Hz will yield a slower convergence of the error at that frequency.

Additional, robustness in the ILC algorithm can be provided at each iteration through the iteration update parameter γ_i, where the convergence rate of the error at all frequencies is effected equally [22]. With the exception of some low-frequency data, the experimental disk drive was sufficiently modeled by $\hat{S}(q)$ to allow nonconservative update parameter $\gamma_i = 1$ for all i. Thus, evaluation of the robust convergence condition (22) is required only once prior to the start of the ILC algorithm.

B. Application of the ILC Algorithm

The original PES, $e_0(t)$ presented in Fig. 3, is used to determine the first iteration of the DCRS $z_1(t)$. The DCRS $z_1(t)$ is applied to the system as a adjunct reference signal that cancels some of the periodic measurement disturbance as seen in Fig. 7. Significant reduction in the PES is demonstrated after one iteration. Further iteration of the DCRS according to the ILC update law (16) demonstrates an improved performance with each iteration of the modified reference signal for canceling the periodic measurement disturbance in the PES. Fig. 7 shows the DCRS through the fourth iteration and the resulting reductions in the periodic measurement disturbance on the PES.

A close look at the progression of the DCRS in Fig. 7 shows very little change from the first iteration to the fourth, and yet these subtle changes have a significant impact on the resulting PES. Fig. 8 shows the maximum PES over the sectors as a function of iteration where $G_f(q,\theta)$ is designed as an approximation to the disk drive inverse sensitivity. As the size of the error approaches zero in the iteration domain $\|e_i\|_2 \rightarrow 0$, the DCRS also displays convergence $z_{i+1}(t) \approx z_i(t)$ to the DCRS that directly cancels the effect of the periodic measurement disturbance in the PES. The error increase in later iterations of the ILC algorithm results from working at the quantization level of the DSP. An iteration update parameter $\gamma_i = 0$, for example at $i = 5$, could have been used to stop the ILC algorithm once a sufficient performance level has been met.

The effectiveness of the DCRS for canceling periodic disturbances at multiple frequencies is observed in the spectral content of the PES. A comparison between the spectrum of the original PES and the PES with the fourth iteration of the DCRS applied is shown in Fig. 9. The periodic disturbance at 70 Hz, which corresponds to the large eccentricity of the disk with respect to the center of rotation, is reduced to the quantization level of the experimental HDD system prior to implementation of the ILC algorithm. The remaining 70-Hz disturbance below the quantization was not targeted by the learning system since this is well within the bandwidth of feedback loop. All other frequency components of the periodic disturbance have been significantly reduced in four iterations.

It is also important to mention that the nonrepeatable disturbances in the disk drive system were neither amplified nor decreased by the application of the ILC algorithm. This results from the *cascaded* ILC structure where modification of the reference signal does not affect the properties of the closed-loop transfer function. Addition of the ILC algorithm provides reduction of periodic measurement disturbances without the need for redesigning the feedback controller.

IV. Conclusion

In this paper, iterative learning control for the design of reference signals has been developed for the reduction of periodic measurement disturbances. The method is illustrated with experimental results on a hard disk drive read/write head tester. As with most control design, with ILC one has to address the issues of complexity of computation and implementation. The contributions of this paper are to discuss the design of the learning filter and demonstrate experimentally the effectiveness of ILC

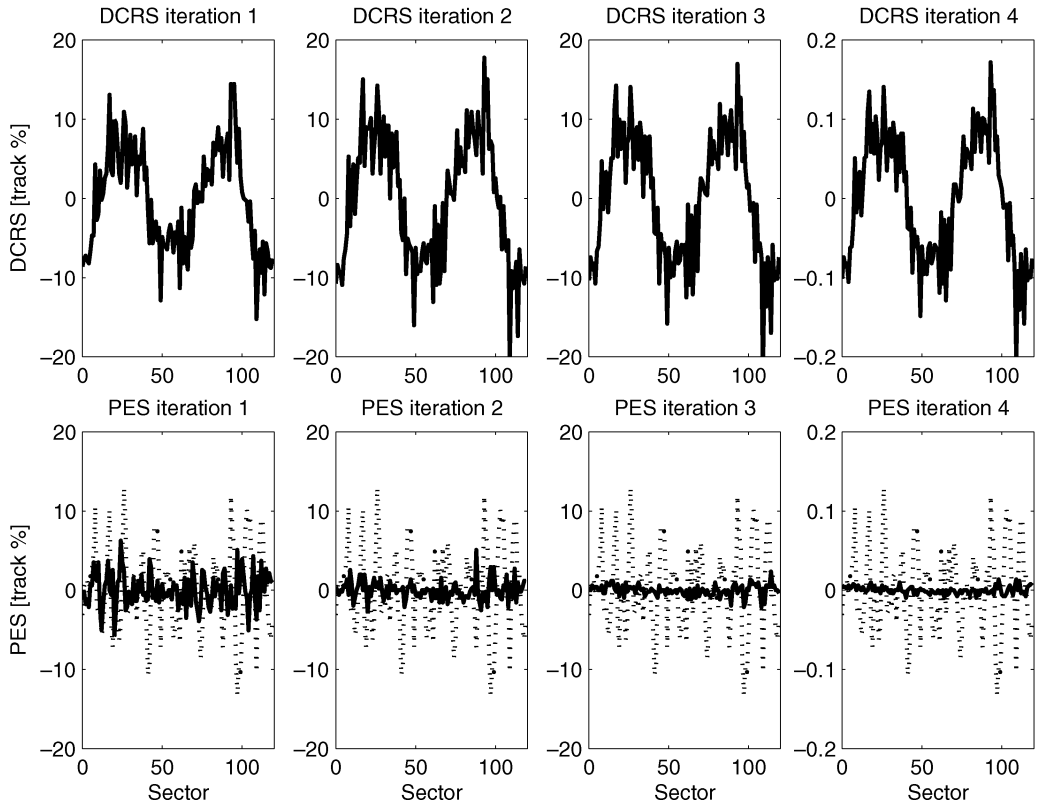

Fig. 7. Iteration 1–4 with resulting DCRS $z_i(t)$ (top plot) and the resulting PES $e_i(t)$ (solid line, bottom plot) compared to the original PES $e_0(t)$ (dotted line).

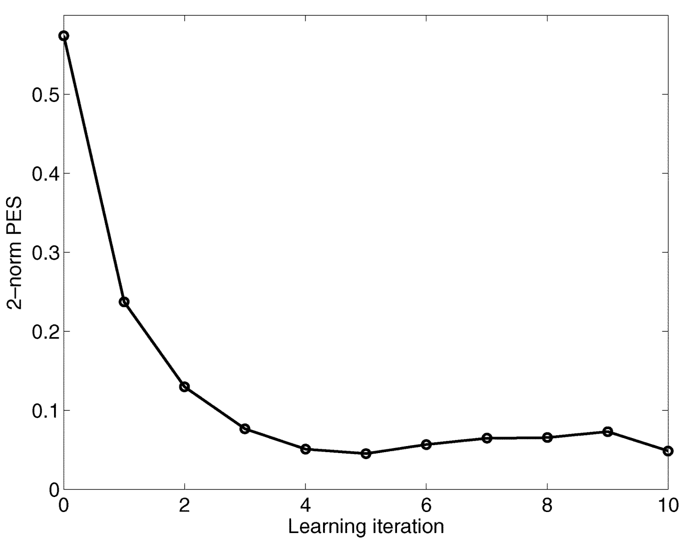

Fig. 8. Convergence of the norm of the PES $\|e_i(t)\|_2$ provided by the ILC algorithm.

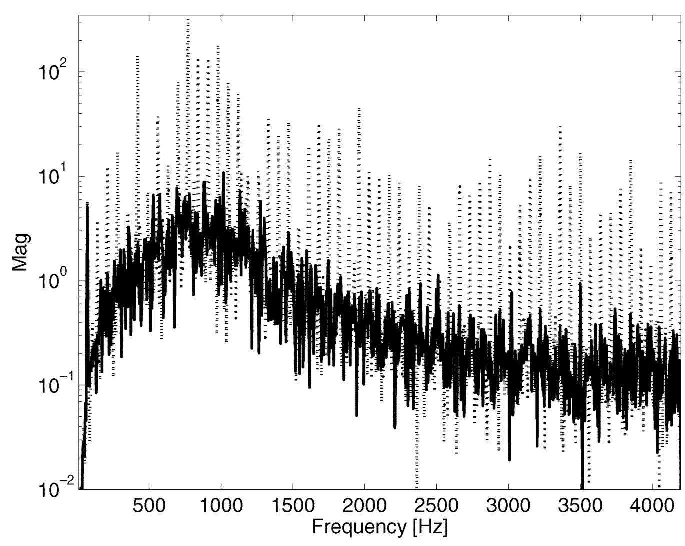

Fig. 9. Spectrum of the PES with DCRS $e_4(t)$ applied (solid line) compared with the spectrum of the original PES $e_0(t)$ (dotted line).

reference design methods for improving control performance. Although knowledge of the system is not required for the ILC algorithm to work, approximations of the system closed-loop frequency response are used to design learning filters as FIR model approximations of the system inverse providing for fast rates of convergence in the learning system.

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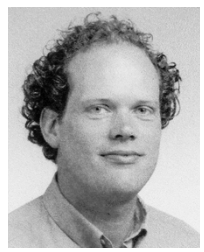

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