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RECURSIVE LEAST SQUARES GENERALIZED FIR FILTER ESTIMATION FOR ACTIVE NOISE CANCELLATION

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Abstract: A feedforward control algorithm for active noise control based on the recursive estimation of a generalized finite impulse response (FIR) filter is presented in this paper. Recursive least square estimation (RLSE) with variable forgetting factors is applied to a commercial air ventilation silencer to provide the online estimation of the generalized finite impulse response (FIR) filter to obtain active noise compensation in an airduct. The advantage of the generalized FIR filters lies in the possibility to include prior knowledge of system dynamics in the tapped delay line of the filter. It is shown that a significant improvement in noise cancellation is obtained with the implementation of generalized FIR filter for feedforward ANC.

Keywords: Active Noise Control, Generalized FIR filter, Recursive Least Square

1. INTRODUCTION

In applications where external sound disturbances interfere with the environment, passive or active attenuation can be used to control sound emission. Passive noise control is effective at reducing high frequency sound components but requires large amounts of absorption material to reduce low frequent noise signals (Gentry *et al.*, 1997; Bernhard, 2000). Active noise control (ANC) can be used for sound reduction and can be particularly effective at lower frequency sound components. ANC allows for much smaller design constraints to achieve sound and noise suppression and has received attention in recent years in many active noise cancellation applications (Fuller and Von Flotow, 1995; Berkman and E.K., 1997; Cabell and Fuller, 1999; Meurers and Veres, 1999; Esmailzadeh *et al.*, 2002). The basic principle and idea behind ANC is to cancel sound by a controlled emission of a secondary opposite (out-of-phase) sound signal (Denenberg, 1992; Wang *et al.*, 1997).

In the situation of measurable sound disturbances with ignorable acoustic coupling, feedforward compensation provides an effective resource to create a controlled emission for sound attenuation. Algorithms based on recursive (filtered) Least Mean Squares (LMS) minimization (Haykin, 2001) can be quite effective for the estimation and adaptation of feedforward based sound cancellation (Cartes *et al.*, 2002). To facilitate an output-error based optimization of the feedforward compensation, a linearly parametrized finite impulse response (FIR) filter has been used for the recursive estimation and adaptation.

In this paper we adopt the framework of output-error based optimization of a linearly parametrized filters for feedforward sound compensation. However, the feedforward control algorithm presented here is based on the recursive least square (RLS) estimation of a generalized finite impulse response filter (Zeng and de Callafon, 2003). Generalized or orthogonal FIR models have been proposed in (Heuberger *et al.*, 1995) and exhibit the same linear parametrization as a standard FIR filter. Combined with a RLS estimation with variable forgetting factors (Landau, 1990), adaptive infinite impulse response (IIR) filter estimation can be obtained for feedforward sound compensation. The adaptive IIR filter requires less parameters to be estimated and provides a better approximation of the filter needed for feedforward compensation.

2. ACTIVE NOISE CONTROL

2.1 Analysis of feedforward compensation

In order to analyze the design of the feedforward compensator F, consider the schematic representation of a linear duct depicted in Figure 1. Sound waves from an external noise source are predominantly traveling from right to left and can be measured by a pick-up microphone at the inlet and an error microphone at the outlet.

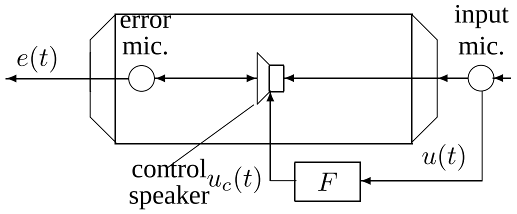

Fig. 1. Schematics of ANC system

The (amplified) signal $u(t)$ from the input microphone is fed into a feedforward compensator F that controls the signal $u_c(t)$ to the internal speaker for sound compensation.

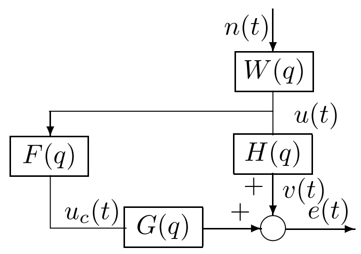

Fig. 2. Block diagram of ANC system with feedforward compensation

The block diagram corresponding to Figure 1 is given in Figure 2. Following this block diagram, the dynamical relationship between the discrete time sampled signals in the ANC system are characterized by difference equations, where the operator q is used to denote a unit sample delay $qu(t) = u(t+1)$. The measured sound disturbance $u(t)$ measured at the input microphone is characterized by

$$u(t) = W(q)n(t)$$

where $n(t)$ is a zero-mean filtered white noise signal with variance $E\{n(t)^2\} = \lambda$ and $W(q)$ is a (unknown) stable and stably invertible noise filter.

The error microphone signal $e(t)$ can be described by

$$e(t) = W(q)\left[H(q) + G(q)F(q)\right]n(t) \qquad (1)$$

where $H(q)$ is a stable filter in the 'primary path' and $G(q)$ is a stable filter in the 'secondary path' of the ANC system. Both $H(q)$ and $G(q)$ characterize the discrete time dynamic aspects of the sound propagation through the ANC system to the error $e(t)$ microphone signal. In case the transfer functions in (1) are known, an ideal feedforward compensator $F(q) = F_i(q)$ can be obtained in case

$$F_i(q) = -\frac{H(q)}{G(q)} \qquad (2)$$

is a stable and causal transfer function. The solution of $F_i(q)$ in (2) assumes full knowledge of $G(q)$ and $H(q)$. Moreover, the filter $F_i(q)$ may not be a causal or stable filter due to the dynamics of $G(q)$ and $H(q)$ that dictate the solution of the feedforward compensator $F_i(q)$. An approximation of the feedforward filter $F_i(q)$ can be made by an output-error based optimization that aims at finding the best causal and stable approximation $F(q)$ of the ideal feedforward compensator in $F_i(q)$ in (2).

2.2 Estimation of feedforward compensation

A direct adaptation of the feedforward compensator $F(q,\theta)$ can be performed by considering the parametrized error signal $e(t,\theta)$

$$e(t,\theta) = H(q)u(t) + F(q,\theta)G(q)u(t). \qquad (3)$$

Definition of the signals

$$y(t) := H(q)u(t),\ u_f(t) := -G(q)u(t) \qquad (4)$$

reduces (3) to

$$e(t,\theta) = y(t) - F(q,\theta)u_f(t) \qquad (5)$$

for which the minimization

$$\min_{\theta} \frac{1}{N}\sum_{t=1}^{N} e^2(t,\theta) \qquad (6)$$

to compute the optimal feedforward filter $F(q,\theta)$ is a standard output-error (OE) minimization problem in a prediction error framework (Ljung, 1999).

Using the fact that the variance of the inlet microphone signal $u(t)$ satisfies $\|u\|_2 = |W(q)|^2\lambda$, the minimization of (6) for $\lim_{N\to\infty}$ can be rewritten into the frequency domain expression

$$\min_{\theta} \int_{\pi}^{-\pi} |W(\mathrm{e}^{j\omega})|^2 |H(\mathrm{e}^{j\omega}) + G(\mathrm{e}^{j\omega})F(\mathrm{e}^{j\omega},\theta)|^2 \qquad (7)$$

using Parceval's theorem (Ljung, 1999). It can be observed that the standard output-error (OE) minimization problem in (6) can be used to compute the optimal feedforward filter $F(q,\theta)$, provided $y(t)$ and $u_f(t)$ in (4) are available.

The signals in (4) are easily obtained by performing a series of experiments. The first experiment is done with $F(q,\theta)=0$, so that the error microphone signal $e_1(t)$ satisfies

$$e_1(t) = H(q)u(t)$$

and obviously $y(t)$ in (4) is found by $y(t) = e_1(t)$. Subsequently, the input signal $u_f(t)$ can be obtained by pass the measured input microphone signal $u(t)$ from the first experiment through an estimated model $\hat{G}(q)$ of $G(q)$. Because $G(q)$ is fixed once the mechanical and geometrical properties of the ANC system in Figure 1 are fixed, an initial off-line estimation can be used to estimate a model for $G(q)$ to construct the filtered input signal $u_f(t)$.

Estimation of a model $\hat{G}(q)$ can be done with the standard open-loop identification technique by performing an experiment using the control speaker signal $u_c(t)$ as excitation signal and the error microphone signal $e(t)$ as output signal. Since $\hat{G}(q)$ is used for filtering purposes only, a high order model can be estimated to provide an accurate reconstruction of the filtered input signal via

$$\hat{u}_f(t) := \hat{G}(q)u(t) \tag{8}$$

that can be used in the adaptive and recursive optimization of the feedforward filter $F(q,\theta)$ in (5).

3. GENERALIZED FIR FILTER

In general, the OE minimization of (6) is a non-linear optimization but reduces to a convex optimization problem in case $F(q,\theta)$ is parametrized linearly in the parameter θ. Linearity in the parameter θ is also favorable for on-line recursive estimation of the filter, as converge to optimal and unbiased feedforward compensators is obtained irrespective of the coloring of the noise on the data (Ljung, 1999). A linear parametrization of $F(q,\theta)$ can be obtained by using a FIR filter

$$F(q,\theta) = \sum_{k=0}^{N} \theta_k q^{-k} \tag{9}$$

but many parameters θ_k are required to approximate an optimal feedforward controller for a complex ANC with many lightly damped resonance modes. To improve these aspects, generalized FIR filters can be used.

To improve the approximation properties of the feedforward compensator in ANC, the linear combination of tapped delay functions q^{-1} in the FIR filter of (9) are generalized to

$$F(q,\theta) = D_0 + \sum_{k=0}^{N} \theta_k V_k(q)$$

where D_0 is a direct feedtrough term and $V_k(q)$ are generalized (orthonormal) basis functions (Heuberger *et al.*, 1995) that contain knowledge of the dynamics of the optimal feedforward controller for the ANC system. The basis functions are a generalization of $V_k(q) = q^{-k}$ used in a FIR filter and guarantee the causality and stability of the feedforward compensator for implementation purposes. For details on the construction of the functions $V_k(q)$ one is referred to (Heuberger *et al.*, 1995). A short overview of the properties is given here.

Let (A,B) be the state matrix and input matrix of an input balanced realization with a McMillan degree $n>0$, and with $rank(B)=m$. Then matrices (C,D) can be constructed according to

$$\begin{aligned} C &= UB^*(I_n + A^*)^{-1}(I_n + A) \\ D &= U[B^*(I_n + A^*)^{-1}B - I_m] \end{aligned}$$

where $U \in \mathbb{R}^{m\times m}$ is any unitary matrix. This yields a square $m\times m$ inner transfer function $P(q) = D + C(qI - A)^{-1}B$, where (A,B,C,D) is a minimal balanced realization. $(\cdot)^*$ indicates complex conjugate transpose of a matrix.

As $P(q)$ is a analytic outside and on the unit circle, it has a Laurent series expansion

$$P(q) = \sum_{k=0}^{\infty} P_k q^{-k}$$

which yields a set of orthonormal functions P_k (Heuberger *et al.*, 1995). Orthonormality of the set P_k can be seen by z-transformation of P_k :

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} P_i(\mathrm{e}^{j\omega})P_k^T(\mathrm{e}^{-j\omega})\mathrm{d}\omega = \begin{cases} I & i=k \\ 0 & i\neq k \end{cases}$$

Define $V_0(q) := (qI-A)^{-1}B$ and

$$V_k(q) = (qI-A)^{-1}BP^k(q) = V_0(q)P^k(q) \tag{10}$$

then a generalized FIR filter can be constructed that consists of a linear combination of the basis functions $D_0 + \sum_{k=0}^{N}\theta_k V_k(q)$. This yields a generalized FIR filter that can be augmented with standard delay functions

$$F(q) = q^{-n_k}\left[D_0 + \sum_{k=0}^{N}\theta_k V_0(q)P^k(q)\right] \tag{11}$$

to incorporate a delay time of n_k time steps in the feedforward compensator. A block diagram of the generalized FIR filter $F(q)$ in (11) is depicted in Figure 3 and it can be seen that it exhibits the same tapped delay line structure found in a conventional FIR filter, with the difference of more general basis functions $V_k(q)$.

An important property and advantage of the generalized FIR filter is that knowledge of the (desired) dynamical behavior can be incorporated in the basis function $V_k(q)$. If a more elaborate choice for the basis function $V_k(q)$ is incorporated, then (11) can exhibit better approximation properties for a much smaller number of parameters N than used in a conventional FIR filter. Consequently, the accuracy of the optimal feedforward controller will substantially increase. In

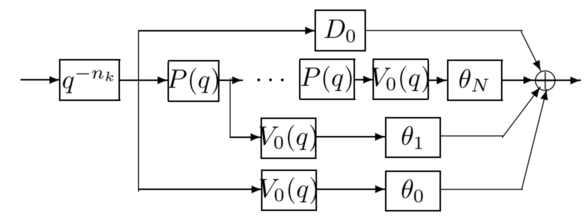

Fig. 3. Basic structure of generalized FIR filter

the next section we will elaborate on the choice of the basis function $V_k(q)$ and the use of the generalized FIR filter in the role of ANC based on feedforward compensation.

4. ESTIMATION OF GENERALIZED FIR FILTER

4.1 Construction of basis functions

To facilitate the use of the generalized FIR filter, the basis function $V_k(q)$ in (10) have to be selected. A low order model for the basis functions will suffice, as the generalized FIR model will be expanded on the basis of $V_k(q)$ to improve the accuracy of the feedforward compensator. With no feedforward compensator in place, the signal $y(t)$ is readily available via

$$y(t) := H(q)u(t) \qquad (12)$$

and an initial low order IIR model $\hat{F}(q)$ of the feedforward filter $F(q)$ can be estimated with the signals available from (8) and (12) using the OE-minimization

$$\hat{F}(q) = F(q, \hat{\theta}),\ \hat{\theta} = \min_{\theta} \frac{1}{N} \sum_{t=0}^{N} \varepsilon^2(t, \theta) \qquad (13)$$

of the prediction error

$$e(t, \theta) = y(t) - F(q, \theta)\hat{u}_f(t)$$

where $\hat{u}_f(t)$ is given in (8). The initial low order IIR model $\hat{F}(q)$ can be used to generate the basis functions $V_k(q)$ of the generalized FIR filer of the feedforward compensator $F(q)$. An input balanced state space realization of the low order model $\hat{F}(q)$ is used to construct the basis function $V_k(q)$ in (10).

With a known (initial) feedforward $F(q, \hat{\theta})$ in place, the signal $y(t)$ can be generated via

$$y(t) := H(q)u(t) = e(t) + F(q, \hat{\theta})u_f(t) \qquad (14)$$

and requires measurement of the error microphone signal $e(t)$, and the filtered input signal $u_f(t) = G(q)u(t)$ that can be simulated by (8). Since the feedforward filter is based on the generalized FIR model, the input $\hat{u}_f(t)$ is also filtered by the tapped delay line of basis functions. A new filtered input signal $\bar{u}_k(t)$ can be defined as

$$\bar{u}_k(t) = V_k(q)\hat{G}(q)u(t) \qquad (15)$$

With the signal $y(t)$ in (14), $\hat{u}_f(t)$ in (8), $\bar{u}_k(t)$ in (15) and the basis function $V_k(q)$ in (10) from the initial low order model in (13), the system dynamics can be rewritten as a linear regression form

$$y(t) = \phi^T(t)\theta, \quad \theta = [\theta_1, ..., \theta_n]^T \qquad (16)$$

where $\phi^T(t) = [\bar{u}_1^T(t), ..., \bar{u}_n^T(t)]$ is the available input data vector and θ is the parameter vector to be estimated of the generalized FIR feedforward compensator.

4.2 Recursive estimation

The objective is to identify (estimate) the values of the parameters θ in (16) such that the feedforward controller minimizes the error signal $e(t)$. The parameters θ can be identified with the available input-output data up to time t by a standard recursive least square (RLS) algorithm (Haykin, 2001). It is known that RLS algorithm at steady-state operation exhibits a windup problem if the forgetting factor remains constant, which will deteriorate the estimation results. As a result, a variable forgetting factor (Landau, 1990) be employed to prevent this problem from occurring. The parameters ϑ can be estimated by RLS algorithm with variable forgetting factor through two steps in each sample time:

(1) Compute the gain vector $k(t)$ and the parameters $\hat{\theta}(t)$ at the current sample time

$$k(t) = \frac{P(t-1)\phi(t)}{\lambda_1(t) + \phi^T(t)P(t-1)\phi(t)} \qquad (17)$$

$$\xi(t) = y(t) - \hat{\theta}^T(t-1)\phi(t) \qquad (18)$$

$$\hat{\theta}(t) = \hat{\theta}(t-1) + k(t)\xi^*(t) \qquad (19)$$

(2) Update the inverse correlation matrix $P(t)$ and the forgetting factor $\lambda_1(t)$

$$\begin{aligned} P(t) = \lambda_1(t)^{-1}P(t-1) - \\ \lambda_1(t)^{-1}k(t)\phi^T(t)P(t-1) \end{aligned} \qquad (20)$$

$$\lambda_1(t) = \lambda_0\lambda_1(t-1) + 1 - \lambda_0; 0 < \lambda_0 < 1 \quad (21)$$

where the typical values can be: $\lambda_1(0) = 0.95 \sim 0.99$; $\lambda_0 = 0.95 \sim 0.99$.

Relationship (21) leads to a forgetting factor that asymptotically tends towards 1. The recursive least square minimization will be

$$J(t) = \sum_{i=1}^{t} \lambda_1(i)^{t-i}[y(i) - \hat{\theta}(t)^T\phi(i)]^2 \qquad (22)$$

The algorithm is initialized by setting

$$\hat{\theta}(0) = 0, \quad P(0) = \delta^{-1}I$$

a typical value for δ choose in this paper is $\delta = 0.001$.

From (17), we can see that even though the input data vector $\phi(t)$ is zero at some time t, the gain vector $k(t)$ does not increase because $\lambda_1 \neq 0$. A zero or small input data vector $\phi(t)$ can occur when the sound disturbance $u(t)$, measured by the inlet microphone in Figure 1, is small. In that event, the recursive estimation routine will be robust in the presence of lack of excitation from the sound disturbance. As a result, $\hat{\theta}(t) = \hat{\theta}(t-1)$, and the parameters $\theta(t)$ of the generalized FIR filter at the current sample time t remain constant when only a small or no inlet disturbance signal $u(t)$ is being measured. An additional advantage of the usage of a variable forgetting factor $\lambda_1(t)$ computed by (21) is a rapid decrease of the inverse correlation matrix. In general this results in an accelerating convergence by maintaining a high adaptation at the beginning of the estimation when the parameters θ are still far from the optimal value.

5. IMPLEMENTATION OF FEEDFORWARD ANC

5.1 *Modelling of the system dynamics*

For the experimental verification of the proposed feedforward noise cancellation, the ACTA silencer depicted in Figure 4 was used. The system is an open-ended airduct located at the System Identification and Control Laboratory at UCSD that will be used as a case study for the ANC algorithm presented in this paper. Experimental data and real time digital control is implemented at a sampling frequency of 2.56kHz and experimental data of the error and input microphone were gathered for the initialization of the feedforward controller.

Fig. 4. ACTA airduct silencer located in the System Identification and Control Laboratory at UCSD

In order to create the filtered input digital $\hat{u}_f(t)$ in (8) and $\bar{u}(t)$ in (15), a 21th order ARX model $\hat{G}(q)$ which can pick most main resonance modes of $G(q)$ was estimated for filtering purposes.

The filtered input signal $\hat{u}_f(t)$ and the observed error microphone signal $y(t)$ sampled at 2.56kHz were used to estimate a low (4th) order IIR model $F_f(q,\theta)$ to create the basis function $V_k(q)$ in (10) for the generalized FIR filter parametrization of the feedforward controller. During the estimation of the low order model $\hat{F}(q)$ also an estimate of the expected time delay n_k in (11) was performed and was found to be $n_k = 16$. The identification results of the 4th order IIR model $F_f(q,\theta)$ is shown in Figure 5. From Figure 5 it can be observed that the 4th order model $F_f(q,\theta)$ picks two resonance modes of $\frac{H(q)}{G(q)}$. The reason only 4th order model $F_f(q,\theta)$ is estimated is that $F_f(q,\theta)$ is only used to create the basis function, and a high accurate model is not necessary.

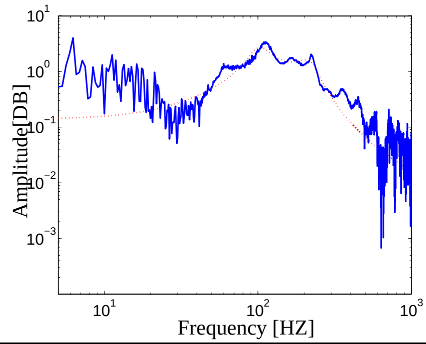

Fig. 5. Amplitude of spectral estimate of $-\frac{H(q)}{G(q)}$ (solid) and 4th order parametric model $F_f(q,\theta)$ (dotted)

5.2 *Implementation of feedforward ANC*

After initialization, the information of the filter $\hat{G}(q)$, the basis function $V_k(q)$ and the time delay n_k was used to perform a recursive estimation of the generalized FIR filter based feedforward compensator $F(q)$. To illustrate the effectiveness of the recursive generalized FIR feedforward compensator, data has been generated over 1.5 seconds, where a sound disturbances is generated into the air-duct during the first half second and the last half seconds, and is turned off in between. For the generalized FIR filter only $N = 5$ parameters $\theta_i,\ i = 1,\dots,5$ in (16) were estimated for the construction of the feedforward compensator. With a 4th order basis function $V_k(q)$, each parameter $\theta_i \in R^{1\times 4}$ and this amounts to IIR feedforward compensator of order 20.

The performance of the generalized FIR filter is confirmed by the estimates of the spectral contents of the microphone error signal $e(t)$ plotted in Figure 6. The spectral content of the error microphone signal has been reduced significantly by the generalized FIR filters in the frequency range from 40 till 400Hz.

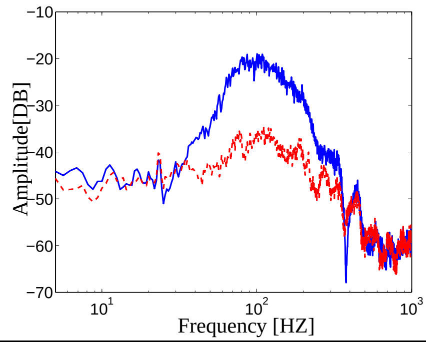

Fig. 6. Estimate of spectral contents of error microphone signal $e(t)$ without ANC (solid) and with ANC using 20th order generalized FIR filter (dotted)

To illustrating the stability and convergence properties of the recursive least square (RLS) estimation, the norm of parameters $\|\theta_i\|$ for $i = 1,\dots,5$ is shown in Figure 7. Since each parameter θ_i in (16) is of dimension $R^{1\times 4}$, only $\|\theta_i\|$ for $i = 1,...,5$ is plotted to provide 5 lines for each multidimensional parameter. From Figure 7, it can be observed that the parameters $\|\theta_i\|$ converge to a steady state very quickly which validates an important property for the recursive least square (RLS) algorithm. Moreover, the parameter values remain constant in the presence of lack of excitation at the middle part of the experiment.

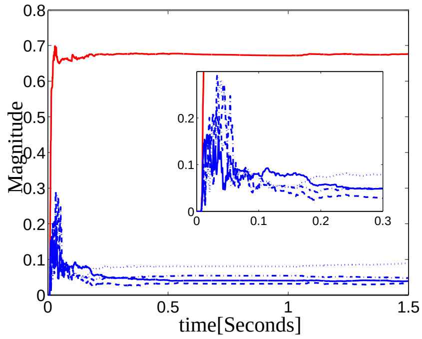

Fig. 7. Norm $\|\theta_i\|$ for $i = 1, ..., 5$ where θ_i is given in (16): θ_1(red); θ_2(solid); θ_3(dotted); θ_4(dashdot); θ_5(dashed)

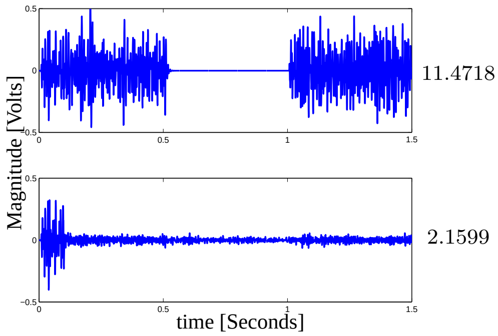

Fig. 8. Evaluation of error microphone signal before ANC (top) and with ANC using 20th order generalized FIR filter (bottom)

The final conformation of the performance of the ANC has been depicted in Figure 8. From Figure 8 it can be observed that, even though the sound disturbance excitation drastically reduces from $t = 0.5$ and $t = 1$ seconds during the experiment, the error microphone signal is not identically zero because of the measurement error of the inlet and outlet microphones. Furthermore, the estimation of the parameters θ_i does not diverge during this time interval due to the robust property of the RLS algorithm. The significant reduction of the error microphone signal observed in the time traces and the norm of the signals displayed on the right part of Figure 8 indicates the effectiveness of the generalized FIR filter for feedforward sound compensation.

6. CONCLUSIONS

In this paper a new methodology has been proposed for the active noise control in an airduct using a feedforward compensation that is parametrized with a generalized FIR filter. The feedforward filter with the linear parametrization is an IIR filter that can be estimated via filtered recursive least squares (RLS) techniques with variable forgetting factor. The design is evaluated on the basis of an experimental active noise cancellation experiment and shows significant sound reduction. The RLS is robust with respect to lack of disturbance excitation by the adaptation of the forgetting factor in the recursive estimation.

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