MODEL APPROXIMATION OF PLANT AND NOISE DYNAMICS ON THE BASIS OF CLOSED-LOOP DATA

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Abstract: In this paper, we consider the problem of estimating low order and control relevant models of plant dynamics and additive noise dynamics on the basis of closed-loop experiments. Estimating low order models for both the plant and noise dynamics is important in control design applications that focus on disturbance rejection. Several methods for low order model estimation on the basis of closed-loop data exist in the literature, but fail to address the simultaneous estimation of low order noise models that are relevant in disturbance control problems. In this paper we evaluate and compare some of these methods and propose a new methodology that extends the results to low order noise model estimation. The new methodology is an extended two-stage method where the first stage is used to estimate high order models for filtering purposes. In the second stage, filtered signals are used for low order model approximation. The methodology is illustrated in a realistic simulation study based on the windage disturbance reduction of a flexible hard disk drive suspension.

Keywords: identification; closed-loop; disturbance rejection; hard disk drive

1. INTRODUCTION

For the modeling purposes of a system with unknown or partially known dynamics, system identification techniques can be used to characterize the dynamic behavior of the system (Ljung 1992). Models obtained by system identification techniques can be used for simulation, prediction or control purposes. Models for simulation purposes focus mainly on system dynamics, whereas models for prediction purposes may require open-loop accurate models of both system and noise dynamics to provide reliable prediction of output signals (Ljung 1992). On the other hand, models intended for control purposes may require high quality system dynamic representations of critical closed-loop behavior to design reliable robust servo controllers (Van Den Hof and Schrama 1995).

The need for control oriented modeling has resulted in several methodologies that aim at iteratively im-

proving closed-loop system behavior on the basis of closed-loop experiments (Gevers 2002). In most of the existing methods, the emphasis is placed on the control-relevant approximation of system dynamics only and ignore the approximate modeling of the disturbance dynamics that is relevant in disturbance control. For minimum variance and LQG control, successful modeling and control performance improvements have been shown in (Gevers and Ljung 1986, Hjalmarsson *et al.* 1994), but these results assume consistent estimation of system and disturbance dynamics.

In dealing with closed-loop data, one of the problems in approximate closed-loop identification of plant and noise dynamics is the correlation of the disturbance with any of the signals in the closed-loop. As a result, a so-called direct identification using input and output of the plant will lead to biased approximation results for the system and disturbance dynamics (Van

Den Hof and Schrama 1995, de Callafon 1998). Possible ways to overcome this problem is by assuming low noise correlation condition (Gevers 1993, Zang *et al.* 1995, Åström 1993) that might only be realistic in simulation studies.

A possible way to deal with closed-loop data is a reparametrization of the closed-loop identification problem. Reparametrization can be done by a direct parametrization of the closed-loop transfer function as done in (Donkelaar and Van Den Hof 1996) or in the recursive algorithms for closed-loop identification of (Landau and Karimi 1997, Landau and Karimi 1999). Although powerful for estimating control-relevant plant dynamics, bias approximation results similar to direct identification are obtained in case an approximate noise model is estimated (Karimi and Landau 1998).

The contribution of this paper is to propose an new estimation method that allows for a control-relevant estimation of low order models of both system and noise dynamics. Several other parameter identification methods are reviewed and the bias distribution of the low order plant and noise model estimates of the proposed extended two-stage method are presented for comparison. It is illustrated how control-relevant models for both the system and the disturbance dynamics can be obtained on a case study based on models of a flexible suspension and the windage disturbance found in a conventional hard disk drive.

2. PROBLEM FORMULATION

In order to discuss the problem of estimating low order and control relevant models of plant dynamics and additive noise dynamics on the basis of closed-loop experiments, a feedback connection $T(P_0, C)$ of an unknown plant P_0 and a feedback controller C will be considered here. The feedback connection $T(P_0, C)$ is described in Figure 1 where the output $y(t)$ of the plant is fed back to the input $u(t)$ of the plant. Additionally, an additive noise $v(t)$ acts on the output of the plant which is modeled as a monic stable and stably invertible noise filter H_0 having a white noise input $e(t)$.
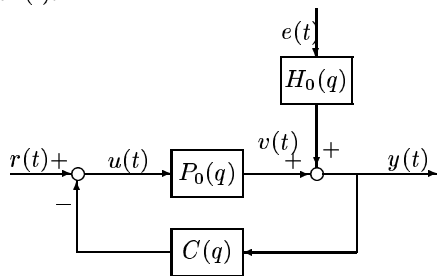

Fig. 1. Closed-loop system

Given Figure 1, the data coming from the plant $P_0(q)$ and subjected to external reference signal $r(t)$ and

additive noise $H_0(q)e(t)$ operating under closed-loop condition can be described as follows:

$$y(t) = P_0(q)S_{in}(q)r(t) + S_{in}(q)H_0(q)e(t) \quad (1)$$
$$u(t) = S_{in}(q)r(t) - C(q)S_{in}H_0(q)e(t) \quad (2)$$

where q is the forward shift operator, and $S_{in}(q)$ is the input sensitivity function defined by

$$S_{in}(q) = \frac{1}{1 + C(q)P_0(q)}$$

Consider the feedback connection $T(P_0, C)$ in Figure 1 and the measurement of the reference $r(t)$ and the output $y(t)$, the objective is to estimate low order models of both plant P_0 and noise H_0 which are important in control design applications that focus on the disturbance rejection.

3. DIRECT METHOD

3.1 Method description

In this method, the input $u(t)$ and output $y(t)$ signals of the plant P_0 are used directly to identify the plant model P_θ and noise model H_θ. In this method the feedback is ignored, so the information of the controller C does not need to be known. Consider a general open loop input/output system with additive disturbances

$$y(t) = P_0 u(t) + H_0 e(t)$$

The one step predictor of the open-loop system is given by (Ljung 1992)

$$y(t|t-1, \theta) = H_\theta^{-1} P_\theta u(t) + (1 - H_\theta^{-1})y(t).$$

To decouple the mutual influence between the plant model and noise model, estimation an independent parametrization of the plant model and noise model can be used. The parameter vector θ is split up in $\theta = [\xi^T \eta^T]^T$, and P_ξ and H_η are the models of P_0 and H_0, respectively. The prediction error is denoted by

$$\epsilon(t, \theta) = y(t) - y(t|t-1, \theta).$$

The resulting prediction error $\epsilon_\theta(t)$ is

$$\epsilon(t, \theta) = H_\theta^{-1}[(P_0 - P_\theta)u(t) + \\ + (H_0 - H_\theta)e(t)] + e(t). \quad (3)$$

and the parameter estimate $\hat\theta$ is found by minimizing the 2-norm of prediction error:

$$\hat\theta = \arg\min_\theta ||\epsilon(t, \theta)||_2 \quad (4)$$

3.2 Biased distribution and conclusion

In the case of open loop identification, $u(t)$ and $e(t)$ are uncorrelated. The minimization (4) can be represented by an integral in the frequency domain, where minimizing argument of (4) is described by

$$\int_{-\pi}^{\pi} \left| H_\theta^{-1}(e^{jw}) \right|^2 \left[\left| P_0(e^{jw}) - P_\theta(e^{jw}) \right|^2 \phi_u(w) \\ + \left| H_0(e^{jw}) - H_\theta(e^{jw}) \right|^2 \phi_e(w) \right] dw. \quad (5)$$

For notational purposes, consider the model set

$$\mathcal{P} := \{P_\theta \mid \theta \in \Theta\} \qquad (6)$$
$$\mathcal{H} := \{H_\theta \mid \theta \in \Theta\} \qquad (7)$$

where Θ is the parameter space that guarantees stability of the prediction error (3). If the true system belongs to the model set, which is defined by $P_0 \in \mathcal{P}$ and $H_0 \in \mathcal{H}$, then a consistent identification of P_0 and H_0 is obtained (Ljung 1992). In case $P_0 \notin \mathcal{P}$ and $H_0 \in \mathcal{H}$, an expression for the approximate identification of the models P_θ and H_θ is derived as follows.

In the case the signals u and y are obtained under feedback, substitution (2) into (3) yields the following prediction error

$$\epsilon(t,\theta) = H_\theta^{-1}[(P_0 - P_\theta)\,S_{in}r(t)$$
$$+((P_\theta - P_0)CS_{in}H_0 + (H_0 - H_\theta))e(t)] + e(t) \qquad (8)$$

The last term $e(t)$ can be ignored because it does not contribute to the minimization. Because $r(t)$ and $e(t)$ are uncorrelated, we can get the bias expression of $\hat{\theta}$ as

$$\hat{\theta} = \arg\min_\theta \int_{-\pi}^\pi |H_\theta^{-1}|^2[|P_0 - P_\theta|^2|S_{in}|^2\phi_r$$
$$+|(P_\theta - P_0)CS_{in}H_0 + (H_0 - H_\theta)|^2\phi_e]\,dw \qquad (9)$$

Compare (9) and (5), it shows that the estimation of P_0 and H_0 effect each other even if the plant model P_θ and noise model H_θ are parametrized independently. As a result, a biased estimation of the noise model will leads to a biased estimation of the plant model, and vice versa. This effect can be seen more clearly in case the noise model H_θ is not estimated and fixed to 1, as in an Output Error (OE) model, which yields a parameter estimate

$$\hat{\theta} = \arg\min_\theta \int_{-\pi}^\pi [|P_0 - P_\theta|^2\,|S_{in}|^2\,\phi_r$$
$$+ |(P_\theta C + 1)S_{in}H_0|^2\,\phi_e]\,dw. \qquad (10)$$

When the reference signal $r(t) = 0$ and assuming that $-C^{-1} \in \mathcal{P}$, a biased estimation of $P_\theta = -C^{-1}$ is obtained, even if $P_0 \in \mathcal{P}$. As a result, the estimation of plant model P_θ will be biased, and it depends on the noise present on the closed-loop data.

4. TWO-STAGE IDENTIFICATION

4.1 Method description

In the two-stage method, identification of the plant model and noise model in closed loop is performed in two steps to eliminate the correlation between the input and the noise. The method can be summarized as follows (Van Den Hof and Schrama 1993). In the first step, one identifies a model S_{in}^* of the input sensitivity function S_{in} by considering the map from reference signal $r(t)$ to the plant input $u(t)$ in (2). The estimate S_{in}^* is then used to simulate a noise free input signal $u_r(t)$ via

$$u_r(t) = S_{in}^* r(t).$$

that will be uncorrelated with noise $e(t)$ on the closed-loop data. In case a consistent estimate $S_{in}^* = S_{in}$ is obtained in the first step, (1) rewrites into

$$y(t) = P_0 u_r(t) + S_{in}H_0 e(t)$$

Subsequently, in the second step of this method a plant model P_θ (and possibly a noise model H_θ) can be estimated by minimizing the two-norm of the prediction error

$$\epsilon(t,\theta) = H_\theta^{-1}[P_0 u(t) - P_\theta u_r(t)$$
$$+(H_0 - H_\theta)e(t)] + e(t) \qquad (11)$$

It should be noted that the model S_{in}^* is used only for filtering purposes. No specific restrictions on the order of this models is needed.

In general, the two-stage method is used only to estimate (low order) models P_θ of P_0 in the second step and the estimation of noise filters is omitted. For comparison and analysis purposes, we also consider the estimation of noise models in the standard two-stage method. Rewriting (11) in terms of the reference signal yields

$$\epsilon_\theta(t) = H_\theta^{-1}[(P_0 S_{in} - P_\theta S_{in}^*)\,r(t)$$
$$+ (H_0 S_{in} - H_\theta)\,e(t)] + e(t) \qquad (12)$$

and we will compare the results of noise model estimation with the direct method and the extended two-stage method proposed in this paper.

4.2 Biased distribution and conclusion

By minimizing the 2-norm of the prediction error (12) during the second step of the two-stage method, the parameter estimate $\hat{\theta}$ can be represented by the following integral expression (Van Den Hof and Schrama 1993)

$$\hat{\theta} = \arg\min_\theta \int_{-\pi}^\pi \left|H_\theta^{-1}\right|^2 [|\,(P_0 - P_\theta)\,S_{in}$$
$$+P_\theta\,(S_{in} - S_{in}^*)\,|^2\phi_r + |H_0 S_{in} - H_\theta|^2\,\phi_e]\,dw \qquad (13)$$

From the above expression, the following remarks can be made with respect to the bias distribution of this method.

- In the case that the plant and noise model are identified independently, the bias of the noise model does not effect the estimation of the plant model.

- The estimation of the noise model H_θ is always biased, and it tends to $H_0 S_{in}$, which is the closed-loop noise model.

- The estimation of the plant model P_θ depends on the estimation of the input sensitivity function S_{in} obtained from the first step. In case $S_{in}^* \neq S_{in}$, it can be observed from equation (13) that the term $P_\theta\,(S_{in} - S_{in}^*)$ effects the fitting of $P_\theta \to P_0$. As a result, no explicit tunable expression for the misfit between P_0 and P_θ is obtained. However, this term can be made small by obtaining a consistent estimate of the sensitivity function in the first step of the method.

5. EXTENDED TWO-STAGE METHOD

5.1 Method description

The extended two-stage method, just as its name implies, is similar to the previously mentioned two-stage method. But the main difference lies in the use of a noise model estimate in the two steps of this method. To explain the extended two-stage method in more details, define $\bar{P} = P_0 S_{in}$ and $\bar{H} = H_0 S_{in}$ for notational convenience. $\bar{P}$ and $\bar{H}$ indicate the closed-loop transfer functions in (1). Using the knowledge of the controller C, (1) can be rewritten into the following two expressions:

$$y(t) = \bar{P}r(t) + \bar{H}e(t) \tag{14}$$

$$y(t) = P_0(1 - C\bar{P})r(t) + H_0(1 - C\bar{P})e(t) \tag{15}$$

From (15) it can be observed that with knowledge of $\bar{P}$, the controller C and a time realization of $e(t)$, the estimation of P_0 and H_0 becomes a standard open-loop identification problem. Furthermore, $\bar{P}$, $\bar{H}$ and a time realization of $e(t)$ are accessible from (14) by performing a consistent identification. From these observations, the extended two-stage method can be summarized as follows:

(1) Using the reference signal $r(t)$ and the output signal $y(t)$ according to (14) to perform an standard open-loop identification of $\bar{P}$ and $\bar{H}$. Using the estimated models $\bar{P}_*$ and $\bar{H}_*$, compute the closed-loop prediction error

$$\varepsilon_{cl}(t) = \bar{H}_*^{-1}\left(y(t) - \bar{P}_* r(t)\right). \tag{16}$$

(2) The estimated models $\bar{P}_*$ and $\bar{H}_*$ are used to create a filtered input $u_f(t)$ and a filtered prediction error $\varepsilon_f(t)$:

$$u_f(t) = (1 - C\bar{P}_*)r(t) \tag{17}$$

$$\varepsilon_f(t) = (1 - C\bar{P}_*)\varepsilon_{cl}(t). \tag{18}$$

Use the signals $u_f(t)$ and $\varepsilon_f(t)$ according to (15) to estimate low order models P_θ and H_θ by the output error minimization

$$\hat{\theta} = \arg\min_\theta \left\| y(t) - [P_\theta(q)\ H_\theta(q)]\begin{bmatrix} u_f(t) \\ \varepsilon_f(t) \end{bmatrix} \right\|_2 \tag{19}$$

During the open-loop identification of $\bar{P}$ and $\bar{H}$ in the first step of this method, a stable plant model $\bar{P}$ and a stable and stably-invertible noise model $\bar{H}$ are estimated. The reason for the construction of the closed loop residuals in (16) in the first step of the method is to allow control over the order of the estimated noise model in the second step. An alternative would be to compute a noise model from the estimate $\bar{H}_*$ using knowledge of C and a model P_θ, but this would lead to a higher order estimate of the noise model.

Compared to the two-stage method it can be observed that the extended two-stage method also uses the knowledge of $\bar{H}_*$ to estimate lower order approximations of H_0 in the second step. Only the signals $r(t)$ and $y(t)$ are used in this method, but it should be observed that the knowledge of the controller C can be replaced by an additional measurement of $u(t)$.

Similar to the two-stage method in (Karimi and Landau 1998), the models $\bar{P}_*$ and $\bar{H}_*$ are only used for filtering purposes. No specific restrictions on the order of these models is needed, as they are only used for filtering purposes. Moreover, the computation of the prediction error $\varepsilon_{cl}(t)$ can be used for model assessment purposes to validate the models $\bar{P}_*$ and $\bar{H}_*$ being estimated (Ljung 1992).

5.2 Biased distribution and conclusion

A result for the bias distribution of the estimation of plant model P_θ and noise model H_θ is given in the following theorem.

Theorem 1. Consider the first step in the extended two-stage method with estimates $\bar{P}_*$ and $\bar{H}_*$ with

$$\bar{P}_* \neq P_0 S_{in}, \quad \bar{H}_* \neq \bar{H} \tag{20}$$

then the minimization of (19) is equivalent to

$$\min_\theta \int_{-\pi}^{\pi} [|(P_0 - P_\theta)S_{in} + (\bar{P}_* - \bar{P})(P_\theta C +$$
$$H_\theta(1 - C\bar{P}_*)\bar{H}_*^{-1})|^2 \phi_r(w) + |(H_0 - H_\theta)S_{in} + \tag{21}$$
$$H_\theta(S_{in} - (1 - C\bar{P}_*)\bar{H}\bar{H}_*^{-1})|^2 \phi_e(w)]\, dw$$

where P_θ and H_θ denote the models estimated in the second step of the extended two-stage method.

Proof: With (20), (16) rewrites to

$$\varepsilon_{cl}(t) = \bar{H}_*^{-1}(\bar{P} - \bar{P}_*)r(t) + \bar{H}_*^{-1}\bar{H}e(t) \tag{22}$$

The prediction error can be computed as follows:

$$\epsilon_\theta(t) = y(t) - P_\theta u_f - H_\theta \varepsilon_f$$
$$= P_0 S_{in} r(t) + H_0 S_{in} e(t) - P_\theta u_f - H_\theta \varepsilon_f \tag{23}$$

Using (17), (18) and (22), (23) can be written as

$$\epsilon_\theta(t) = [(P_0 - P_\theta)S_{in} + (\bar{P}_* - \bar{P})(P_\theta C$$
$$+ H_\theta(1 - C\bar{P}_*)\bar{H}_*^{-1})]r(t) + [(H_0 - H_\theta)S_{in} \tag{24}$$
$$+ H_\theta(S_{in} - (1 - C\bar{P}_*)\bar{H}\bar{H}_*^{-1})]e(t)$$

which leads to the bias distribution (21). □

This results gives the bias distribution for the general case. Useful insight in the bias distribution of P_θ and H_θ in the special cases are described in the following corollaries.

Corollary 1. Let $\bar{P}_* = P_0 S_{in}$, $\bar{H}_* = \bar{H}$ in the first step in the extended two-stage method, then minimization of (19) is equivalent to

$$\min_\theta \int_{-\pi}^{\pi} [|\ P_0(e^{jw}) - P_\theta(e^{jw})|^2 |S_{in}|^2 \phi_r(w)$$
$$+ |H_0(e^{jw}) - H_\theta(e^{jw})|^2 |S_{in}|^2 \phi_e(w)]\, dw \tag{25}$$

Proof: Substitute $\bar{P}_* = P_0 S_{in}$, $\bar{H}_* = \bar{H}$ into (21), then (25) is obtained. □

It is easily observed that in the case $\bar{P}_* = P_0 S_{in}$, $\bar{H}_* = \bar{H}$, the difference $|P_0 - P_\theta|^2$ is weighted by the reference spectrum ϕ_r and the difference $|H_0 - H_\theta|^2$ is weighted by noise spectrum ϕ_e. Both are weighted by the input sensitivity function S_{in}, which is advantageous for the closed-loop approximation of P_0 by P_θ and H_0 by H_θ.

Corollary 2. Let $\bar{P}_* \neq P_0 S_{in}$, $\bar{H}_* = \bar{H}$ in the first step in the extended two-stage method, then the minimization of (19) is equivalent to

$$
\min_\theta \int_{-\pi}^{\pi} [|(P_0 - P_\theta)S_{in} + (\bar{P}_* - \bar{P})(P_\theta C +
$$
$$
H_\theta (1 - C\bar{P}_*)\bar{H}_*^{-1})|^2 \phi_r(w) + |(H_0 - H_\theta)S_{in} \quad (26)
$$
$$
+ (\bar{P}_* - \bar{P})C H_\theta|^2 \phi_e(w)]\, dw
$$

Proof: Substitute $\bar{P}_* \neq P_0 S_{in}$, $\bar{H}_* = \bar{H}$ into (21) and the result in (26) is obtained. $\square$

In the case $\bar{P}_* \neq P_0 S_{in}$, $\bar{H}_* = \bar{H}$, two terms including $(\bar{P}_* - \bar{P})$ are introduced that will effect the fitting of P_θ to P_0, H_θ to H_0, respectively. The biased estimation of $\bar{P}$ will effect the estimation of the noise model. No explicit tunable expressions of the misfit between P_0 and P_θ, and between H_0 and H_θ are obtained. This situation is similar to the two-stage method.

Corollary 3. Let $\bar{P}_* = P_0 S_{in}$, $\bar{H}_* \neq \bar{H}$ in the first step in the extended two-stage method, then the minimization of (19) is equivalent to

$$
\min_\theta \int_{-\pi}^{\pi} [|(P_0 - P_\theta)|^2 |S_{in}|^2 \phi_r(w) + |(H_0 - H_\theta)S_{in}
$$
$$
+ H_\theta (1 - \bar{H}\bar{H}_*^{-1})S_{in}|^2 \phi_e(w)]\, dw
$$

Proof: Substitution of $\bar{P}_* = P_0 S_{in}$, $\bar{H}_* \neq \bar{H}$ into (21) yields the result. $\square$

In the case $\bar{P}_* = P_0 S_{in}$, $\bar{H}_* \neq \bar{H}$, a term $H_\theta (1 - \bar{H}\bar{H}_*^{-1})$ is created to effect the fitting of H_θ to H_0, weighted by S_{in}. No explicit tunable expressions of the misfit between H_0 and H_θ is obtained.

Summarizing, the following result can be mentioned for the extended two-stage method.

• This method gives an unbiased estimation of the plant and noise model when they are in the model set.

• In case (high order) consistent estimates are obtained in the first step, a tunable bias expression for both the plant model P_θ and the noise model H_θ is obtained in the second step.

• Even though we use the higher order models to fit $\bar{P}$ and $\bar{H}$ in the first step, the orders of the models P_θ and H_θ can be reduced significantly in the second step.

• The bias of the noise model will effect the estimation of the plant model in some special cases, and vice versa.

6. APPLICATION TO CASE STUDY

The case study in this paper is a simulation study based on a model of a flexible mechanical suspension and the windage disturbance found in a conventional hard disk drive (HDD) (Crowder and de Callafon 2003). A schematic representation of the system under consideration is illustrated in Figure 2. Using the notation P_0 and H_0 to respectively represent the dynamics of the flexible suspension and the dynamics of the windage disturbances, a block diagram similar to Figure 1 is obtained.
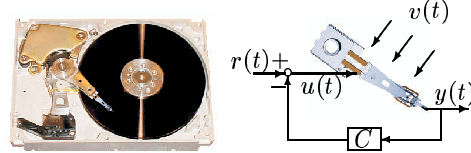

Fig. 2. Configuration of HDD (left) and schematic representation of flexible suspension, windage disturbance and servo controller C (right).

The consistent estimation of (relatively high 12th order) discrete time models for $P_0(q)$ and $H_0(q)$ on the basis of experimental data has been illustrated in (Crowder and de Callafon 2003). For illustrative purposes, an amplitude Bode plot of the 12th order models of P_0 and H_0 found in (Crowder and de Callafon 2003) is given in Figure 3.
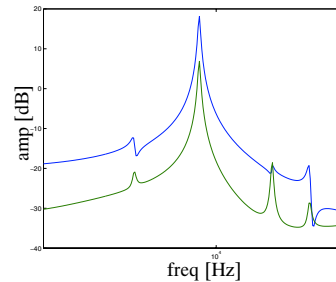

Fig. 3. Amplitude Bode plot of system dynamics P_0 (top line) and noise dynamics H_0 (bottom line)

The servo controller $C(q)$ used during the closed-loop experiments in our case study is a 3rd order lead/lag compensator given by

$$
C(q) = \frac{5.33q^2 - 0.125q - 4.954}{q^3 - 0.2697q^2 - 0.5338q + 0.09558}
$$

and closed-loop data of 4096 data points, sampled at 51.2kHz is obtained with r and e as independent white noise sequences with unit variance. The objective of the case study is to find low (4th) order models P_θ and H_θ of P_0 and H_0 depicted in Figure 3 on the basis of closed-loop experiments.

Figure 4, Figure 5, Figure 6 are the bode plots of the identified plant and noise models, compared with real system using direct method, two stage method and extended two-stage method respectively. By analyzing and comparing the characteristics of these methods, the following results can be summarized.

- Low order approximation with the direct method gives biased estimation results for both the plant and the disturbance dynamics.

- A low order approximation of plant dynamics is successful for the two-stage method. However, the method yields biased results for the noise filter.

- The extended two-stage method can be used to obtain low order approximations of both the plant model and the noise dynamics.
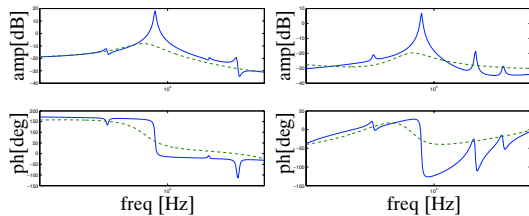
Fig. 4. Application of direct identification. Left: Bode plot of plant P_0 (solid) and the 4th order model P_θ (dashed). Right: Bode plot of H_0 (solid) and the 4th order noise model H_θ (dashed).
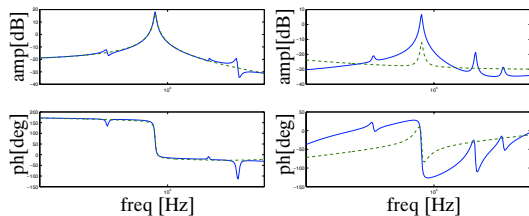
Fig. 5. Application of two-stage method. Left: Bode plot of plant P_0 (solid) and the 4th order model P_θ (dashed). Right: Bode plot of H_0 (solid) and the 4th order noise model H_θ (dashed).
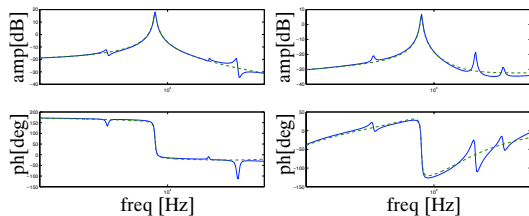
Fig. 6. Application of extended two-stage method. Left: Bode plot of plant P_0 (solid) and the 4th order model P_θ (dashed). Right: Bode plot of H_0 (solid) and the 4th order noise model H_θ (dashed).

7. CONCLUSIONS

In this paper, several methods for low order plant model and noise model identification are discussed and compared in terms of the bias distribution of the approximate estimation. A new extended two-stage estimation method is proposed to improve the approximate estimation of both plant and noise model dynamics. The method is evaluated on the basis of simulated closed-loop data from a hard disk drive experiment and shows improvements with respect to low order approximation of plant and noise models.

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