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Discussion on: "Robust Design of Predictive Controllers in Presence of Unmodeled Dynamics"

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1. Discussion

In the paper by Canale and Milanese the robustness analysis of a model predictive controller (MPC) algorithm is presented. The MPC algorithm uses an approximate model of the actual plant to both estimate states via an observer and formulate an receding finite horizon regulation control problem. Due to the use of an approximate model, a modeling error is unavoidable and the MPC algorithm is labeled robust by formulating conditions on state contraction and size of the model error to guarantee stability and feasibility of the MPC.

1.1. Main Ideas and Contributions

For the presentation of the robust MPC algorithm, the authors briefly review the main concepts behind a standard MPC regulation problem as also mentioned in [2]. The MPC regulation problem is posed as a constrained (weighted) quadratic optimization that involves states $x_M(k+i\,|\,k)$, $i=1,\ldots,h_\mathrm{p}$ of a model M and inputs $u(k+j\,|\,k)$, $j=0,\ldots,h_\mathrm{c}$ over a prediction h_p and control h_c horizon.

The authors acknowledge that in the absence of full state information $x_M(k)$ or deficiency of the certainty equivalence principle ($M \neq S$), regulation properties of the MPC cannot be guaranteed. A solution is proposed where an approximate model M of the system S is used for modeling purposes. Such an approach is common in the field of (approximate) system identification [1,6,9], especially when a model needs to be used for control design purposes [3]. In this paper, the model M is used to create a standard state observer

$$\hat{x}_S(k) = A\hat{x}_S(k-1) + Bu(k-1) + K_0(y(k-1) - C\hat{x}_S(k-1))$$

to estimate an auxiliary state $\hat{x}_S(k)$ that reflect partial or incomplete information of the dynamics of the system S.

The information of the estimated state $\hat{x}_S(k)$ and the simulated model state $x_M(k)$ is used to formulate a MPC algorithm similar to the standard MPC regulation problem. The subtle differences are found in:

- The constrained (weighted) quadratic optimization that now involves the *estimates states* $\hat{x}_S(k+i\,|\,k)$, $i=1,\ldots,h_\mathrm{p}$ from the observer as an extension to the work by [5] that assumes full state information.
- The state contraction conditions that not only includes a standard nominal state contraction [7]
$$\|x_M(k+1\,|\,k)\|_{2,P_Q} \le \nu \|x_M(k\,|\,k)\|_{2,P_Q}, \quad 0 \le \nu < 1, \tag{1}$$
but also the *robust* state contraction
$$\|\hat{x}_S(k+1\,|\,k)\|_{2,P_Q} \le \mu \|x_M(k\,|\,k)\|_{2,P_Q}, \quad 0 \le \mu < 1, \tag{2}$$
where $\|x(k)\|_{2,W} := \sqrt{x(k)^\mathrm{T} W x(k)}$ and P_Q in $\|\cdot\|_{2,P_Q}$ is the positive definite solution to the Lyapunov equation $A^\mathrm{T} P_Q A - P_Q = -Q$.

To discuss further the effect of partial or incomplete state information, modeling/estimation errors in the estimated state $\hat{x}_S(k)$ are characterized by an additive perturbation $\hat{x}_S(k) = x_M(k) + \delta(k)$. The perturbation $\delta(k)$ is modeled by an unknown, but bounded unstructured additive uncertainty $\delta = \Delta_0 u$ from input u to the model states $x_M = M_0 u$. The bound on the unstructured additive uncertainty Δ_0 is defined via an induced ∞-norm

$$\|\Delta_0\|_Q \leq \gamma_0(Q), \quad \gamma_0(Q) := \sup_u \frac{\|\Delta_0 u\|_{\infty,P_Q}}{\|u\|_\infty},$$

where P_Q is again the positive definite solution to the Lyapunov equation $A^T P_Q A - P_Q = -Q$. Similarly, the size of a nominal model M_0 is characterized by

$$\|M_0\|_Q := \sup_u \frac{\|M_0 u\|_{\infty,P_Q}}{|u\|_\infty},$$

and the ratio of the size $\gamma_0(Q)$ of the unstructured uncertainty and the model size $\|M_0\|_Q$ plays a role in the establishment of the robustness results for the MPC algorithm. The authors indicate that information with respect to the size $\gamma_0(Q)$ of the unstructured uncertainty can be obtained via set membership identification methods [4,8] and details are not discussed further in the paper.

The main technical contributions of the paper are summarized in two propositions that formulate conditions on the size $\gamma_0(Q)$ of the unstructured additive uncertainty Δ_0 and the state contraction of both the estimated states $\hat{x}_S(k)$ and the states $x_M(k)$ of the approximate model M to guarantee feasibility and stability of the MPC. The most important result lists the sufficient feasibility condition

$$\sqrt{1 - \frac{\underline{\sigma}(Q)}{\overline{\sigma}(P_Q)}}\left(1 + \frac{\gamma_0(Q)}{\|M_0\|_Q}\right) < 1, \tag{3}$$

which is a multiplication of the standard (nominal) state contraction and the effect of the worst case error due to the unstructured additive uncertainty. This result is followed by a sufficient condition on the existence of a matrix Q in the Lyapunov equation $A^T P_Q A - P_Q = -Q$ so that (3) can be met. The technical contribution of the paper is ended with a proof of the stability of the proposed robust MPC that heavily relies on the previously derived results. As a final motivation for the use of the proposed MPC algorithm, the results of the robust MPC is illustrated with a small simulation example.

1.2. Discussion of Results

The role of the weighting matrix P_Q in (1) and (2) is at first unclear, but the proof of the main results in the paper indicate that the specific choice of P_Q related to the Lyapunov equation

$$A^T P_Q A - P_Q = -Q$$

helps in formulating the state contraction conditions. This is seen by setting $u(k) = 0$, making $x_M(k+1) = Ax_M(k)$ and

$$\begin{aligned}\|x_M(k+1)\|^2_{2,P_Q} &= \|A\, x_M(k)\|^2_{2,P_Q} \\ &= x_M(k)^T A^T P_Q A\, x_M(k)\end{aligned}$$

with P_Q as the positive definite solution to the above mentioned Lyapunov equation it can be seen that $x_M(k)^T A^T P_Q A x_M(k) = x_M(k)^T [P_Q - Q] x_M(k)$ and removes the matrix A. This makes

$$\|x_M(k+1)\|^2_{2,P_Q} = \|x_M(k)\|^2_{2,P_Q} - \|x_M(k)\|^2_{2,Q}$$

and allows a state contraction to be formulated by combining

$$\begin{aligned}\|x_M(k+1)\|^2_{2,P_Q} &= \|x_M(k)\|^2_{2,P_Q} \\ &\quad \times \left(1 - \frac{\|x_M(k)\|^2_{2,Q}}{\|x_M(k)\|^2_{2,P_Q}}\right)\end{aligned}$$

with the result

$$\left(1 - \frac{\|x_M(k)\|^2_{2,Q}}{\|x_M(k)\|^2_{2,P_Q}}\right) \leq \left(1 - \frac{\underline{\sigma}(Q)}{\overline{\sigma}(P_Q)}\right)$$

to guarantee

$$\nu := \sqrt{1 - \frac{\underline{\sigma}(Q)}{\overline{\sigma}(P_Q)}} < 1$$

for the *nominal* state contraction of $x_M(k)$.

The same idea is also applied to prove *robust* state contraction of the estimated state $\hat{x}_S(k)$ by concluding that

$$\begin{aligned}&\hat{x}_S(k+1) = A\hat{x}_S(k), \quad \text{if } u(k) = 0, \\ &\quad k = 0, \ldots, h_c - 1\end{aligned}$$

Although this seems a viable argument, $\hat{x}_S(k)$ is an estimated state, obtained from input $u(k)$ and output measurements $y(k)$ of the system S. Clearly, in case of state estimation errors $\delta(k) \neq 0$ for $u(k) = 0$, $k = 0, \ldots, h_c - 1$ (due to initial conditions) or possible noise on the output measurements $y(t)$, we might find that $\hat{x}_S(k+1) \neq A\hat{x}_S(k)$ making the robust state contraction slightly more complicated.

The use of the estimated state $\hat{x}_S(k)$ in the constrained (weighted) quadratic optimization over a prediction h_p also introduces a small complication in the MPC algorithm. Not only does the state $\hat{x}_S(k\,|\,k)$ have to be estimated, the future states $\hat{x}_S(k+i\,|\,k)$, $i=1,\ldots,h_c$ also have to be predicted in order to be used in the optimization. The authors propose to use the relationship

$$\hat{x}_S(k+1\,|\,k) = A\hat{x}_S(k\,|\,k) + Bu(k\,|\,k)$$

recursively to predict the future state $\hat{x}_S(k+i\,|\,k)$ by

$$\hat{x}_S(k+i\,|\,k) = A^i\hat{x}_S(k\,|\,k) + A^{i-1}Bu(k) + \cdots + Bu(k+i-1)$$

and indicate that $\hat{x}_S(k+i\,|\,k)$ only depends on $\hat{x}_S(k\,|\,k)$ and future inputs. Unfortunately, $\hat{x}_S$ is an estimated state based on an observer structure and $(k+i)$th step-ahead predictions of the state should involve not only involve the current state $\hat{x}(k\,|\,k)$ and future input predictions, but also the $(k+i)$th step-ahead predictions of the output. Unfortunately, in the simulation example these problems are not illustrated, as prediction horizon h_p and control horizon h_c are set to 1.

1.3. Concluding Remarks

The authors have given a detailed analysis that addresses the problem of imperfect model knowledge in MPC regulatory problems. The main idea to use an approximate model of the actual plant to both estimate states via an observer and formulate an receding finite horizon regulation control problem is a promising suggestion. The conditions on state contraction and size of the model error to guarantee stability and feasibility of the MPC are intuitive and understandable to the reader. Special care has to be given to the use of the estimated states in the MPC in case longer prediction horizons are used. Additionally, the unstructured additive uncertainty description to model state estimation errors might be slightly conservative in case state estimation errors are correlated for different states. A structured uncertainty structure would then be better suited to limit conservatism of the uncertainty model.

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Final Comments by the Authors

M. Canale and M. Milanese

The paper discussion is based upon the following main points:

(a) robustness aspects with respect to measurements noise;
(b) effects of the model uncertainty on the computation of the control law; and
(c) conservativeness of the performances results due to the unstructured nature of the model uncertainty.

1. Point (a)

In the recent years important results have been achieved in the field of Model Predictive Control (MPC). In particular, starting from nominal stability and performance properties of MPC, the researchers' attention has been focused on robustness issues. In such context, the stability characteristics of predictive control laws have been investigated under two different settings (see [1] and the references therein):

1. robustness against certain classes of non decaying disturbances;
2. robustness in presence of model uncertainty.

The present paper deals with the second of the above settings and, in particular, on the design of a robust predictive control law in face of an unstructured model set. Indeed, the problem of dealing robust stability properties considering both model uncertainty and non-decaying output disturbances is an important issue and it is at present under investigation.

2. Point (b)

The proposed predictive control design relies on the constrained minimization of the following functional cost:

$$\min_{U[u(k\,|\,k)\,\dots\,u(k+h_c-1\,|\,k)]} \left\{ \sum_{i=1}^{h_p} \|\hat{x}_S(k+i\,|\,k)\|^2_{2W_p} + \sum_{i=0}^{h_c} \|u(k+i\,|\,k)\|^2_{2W_c} \right\}, \quad (1)$$

which depends on the ahead predictions of the "estimated state" $\hat{x}_S$. As the state $\hat{x}_S$ can not be computed exactly due to the model uncertainty, two different approaches may be followed to evaluate its predictions inside the functional cost:

1. use the nominal model equations to propagate the state evolution (i.e. $\hat{x}_S(k+i\,|\,k) \approx x_M(k+i\,|\,k)$);
2. evaluate, on the basis of the model uncertainty, an upper bound of the functional cost as done in [2].

In the first case the predictions of $\hat{x}_S$ are made by means of the nominal model M_O:

$$x_M(k+1) = Ax_M(k) + Bu(k)$$

and starting from the value $\hat{x}_S(k\,|\,k)$:

$$\begin{aligned}\hat{x}_S(k+i\,|\,k) &\approx x_M(k+i\,|\,k) \\ &= A^i\hat{x}_S(k\,|\,k) + A^{i-1}Bu(k) \\ &\quad + \cdots + Bu(k+i-1).\end{aligned} \quad (2)$$

This approach is analogue to what is usually done in standard synthesis procedures in the robust control literature (see e.g. the robust H_∞ methodologies) and it is performed via the optimization of a nominal cost under robust stability constraints. On the other hand, it has also to be noticed that such "nominal" predictions of $\hat{x}_S$ do not take into account, according to the observer equation, the effects of the future output predicted values. In order to take into account the influence of the modeling errors via the output predictions on $\hat{x}_S$ inside the functional cost, the following min–max problem can be considered:

$$\min_{U[u(k\,|\,k)\,\dots\,u(k+h_c-1\,|\,k)]} \sup_{\|\Delta_O\|_Q \le \gamma_O(Q)} \times \left\{ \sum_{i=1}^{h_p} \|\hat{x}_S(k+i\,|\,k)\|^2_{2W_p} + \sum_{i=0}^{h_c} \|u(k+i\,|\,k)\|^2_{2W_c} \right\}. \quad (3)$$

Using the same arguments as in the proof of Proposition 1 it can be proved that:

$$\sup_{\|\Delta_O\|_Q \le \gamma_O(Q)} \|\hat{x}_S(k\,|\,k)\|_{2P_Q} = \left(1 + \frac{\gamma_O(Q)}{\|M_O\|_Q}\right) \|x_M(k)\|_{2P_Q}, \quad (4)$$

so that the optimization problem (5a) in the paper can be replaced as

$$\min_{U[u(k|k)\dots u(k+h_c-1|k)]} \left\{ \sum_{i=1}^{h_p} \|x_M(k+i|k)\|^2_{2W_p^{rob}} + \sum_{i=0}^{h_c} \|u(k+i|k)\|^2_{2W_c} \right\}, \quad (5)$$

where W_p^{rob} is a suitable weighting matrix which depends on W_p, γ_O and $|M_O|_Q$. This way, the design can be again casted to a nominal optimization problem as stated in the paper.

3. Point (c)

As a final comment, it has to be remarked that the purpose of the present paper was to investigate the possibility of designing a predictive controller by using approximated models expressed as unstructured additive model set and to evaluate its achievable performances. Anyway we agree to the fact that the unstructured nature of the uncertainty may lead to more conservative results both for the more general description and for the correlation errors present in state estimation. Indeed, the identification procedure may be worked out in order to derive different uncertainty models from the system input to each of the estimated state. The design procedure performed on the basis of this structured uncertainty model could lead to less conservative results. On the other hand, the derivation of such model appears, at the moment, a quite difficult non-trivial task.

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