

## MODELING PRODUCT VARIABILITIES OF DUAL-STAGE SUSPENSIONS FOR ROBUST CONTROL

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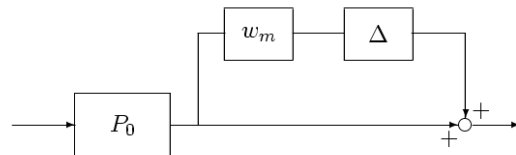
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### Introduction

High-density magnetic recording demonstrations, employing technologies such as dual-stage actuators, are typically achieved under a well-conditioned laboratory environment. Future Hard Disk Drives (HDD) incorporating dual-stage actuators will instead be subjected to all the variability that comes with manufacturing and changing operating conditions. If the same areal densities shown in demonstrations are to be achieved in low cost consumer applications, then the track following servo control system will need to be able to perform adequately in the presence of all possible product variability and uncertainties.

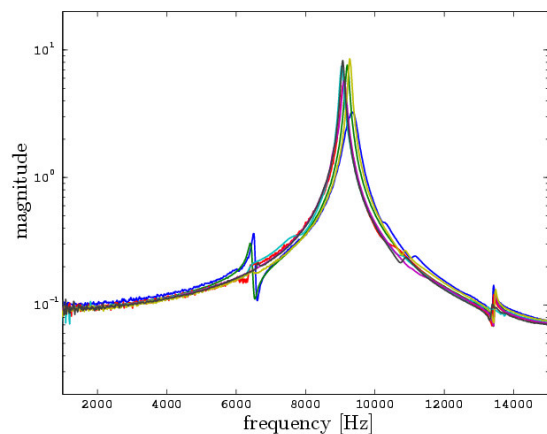
In modern control design approaches for dual-stage HDD which make use of  $H_\infty$  and  $\mu$ -synthesis [1,2], it is possible to incorporate the effects of product variations and various operating conditions in the design process in the form of an uncertainty model. The uncertainty model consists of a nominal model together with a description of the perturbation of the nominal model. This concept is illustrated in Figure 1, in the special case of a multiplicative uncertainty model that will be used in the rest of the paper. In Figure 1,  $P_0$  is the nominal model,  $\Delta$  is the perturbation term and  $w_m$  is a weighting function that describes the uncertainty as a function of frequency. This is expressed in mathematical terms as



**Figure 1** Multiplicative Uncertainty Model

$$P(j\omega) = P_0(j\omega)(1 + w_m(\omega)\Delta(j\omega)) \quad \|\Delta(j\omega)\|_\infty \leq 1 \quad (1.1)$$

In dual-stage suspensions the uncertainties show up as variations in the transfer function from the input of actuators to the measured position output of the slider. This paper focuses on product variability that arise from variations from one suspension to another due to manufacturing as shown in Figure 2, and variations in Z-height. These variations and uncertainties will have a strong effect on the effectiveness of a high-bandwidth and highly accurate dual-stage servo controller design. Experimental data and a systematic modeling and uncertainty characterization are used to capture the variability of a dual stage actuator.



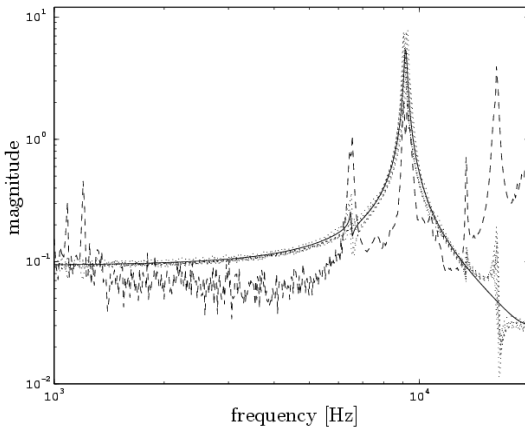
**Figure 2** Magnitude frequency response for 8 different suspensions

## Uncertainty Modeling

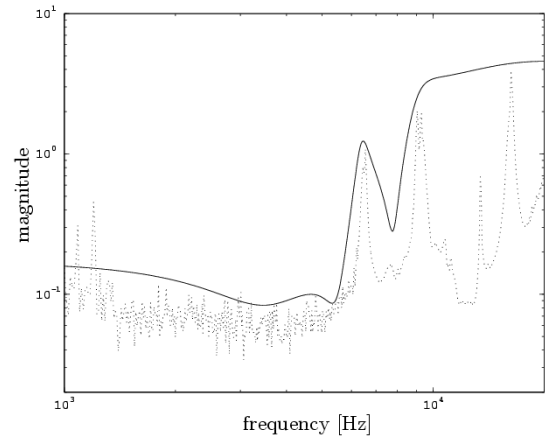
In equation (1.1) both  $P_0$  and  $w_m$  are unknown parametric models and will be determined from frequency response data. An appropriate choice of  $P_0$  can lead to a small uncertainty bound  $w_m$  which will lead to better performance in the control design since less robustness will be needed. To this end let  $F_k(\omega)$  represent the  $k^{\text{th}}$  complex frequency response function in Figure 2, calculated at discrete frequency points. A nominal frequency response function is determined by minimizing the worst-case multiplicative uncertainty bound

$$l_m(\omega) = \min_{\alpha \beta} \max_k \left\| \frac{F_k(\omega) - (\alpha(\omega) + i\beta(\omega))}{(\alpha(\omega) + i\beta(\omega))} \right\| \quad (1.2)$$

where  $F_0(\omega) = \alpha(\omega) + i\beta(\omega)$  is the nominal frequency response function. A nominal parametric model  $P_0(i\omega)$  is obtained by performing a least squares fit [3] of  $F_0(\omega)$ . The results are shown in Figure 3.



**Figure 4** Data  $F_k$ (dotted), nominal model  $P_0$  (solid), and minimum worst case upper bound  $l_f$ (dashed)



**Figure 3** Data  $F_k$ (dotted), nominal model  $P_0$  (solid), and minimum worst case upper bound  $l_f$ (dashed)

To finalize the uncertainty description a model  $w_m$  for the multiplicative model error has to be formulated. The model  $w_m$  is a parametric model that over bounds the unstructured multiplicative uncertainty bound  $l_m(\omega)$  obtained from when the nominal model  $P_0(i\omega)$  is considered. This is expressed in mathematical terms in (1.3).

$$|w_m(\omega)| \geq l_m(\omega) = \max_k \left\| \frac{F_k(\omega) - P_0(i\omega)}{P_0(i\omega)} \right\| \quad (1.3)$$

$w_m$  is obtained by performing a spectral over bounding technique.

## References:

- [1] Rotunno, M. and de Callafon, R., 2000, "Fixed Order  $H_\infty$  Control Design for Dual-Stage Hard Disk Drives" *Proc. IEEE Conf. Decision and Control*.
- [2] Hernandez, D., Park, S., Horowitz R. and Packard, A.K., 1999, "Dual-Stage Track-Following Servo Design for Hard Disk Drives" *Proc. Amer. Contr Conf.*
- [3] Ljung, L., 1998, *System Identification*, Prentice Hall.