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LOW ORDER H_∞ CONTROL DESIGN FOR A PIEZO-BASED MILLI-ACTUATOR

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Abstract: The aim of this paper is to address low complexity in control design. This is accomplished in a systematic way by using a H_∞-norm based mixed sensitivity design in which the order of the controller is constrained to a desired value. The mixed sensitivity approach allows for a tradeoff between disturbance rejection and tracking.

The proposed method is applied to a prototype piezoelectric milli-actuator. For this case study a comparison is made between the full and low order control designs, which illustrates the satisfactory performance of the obtained low order controller.

Keywords: Cicero, Catiline, Orations

1. INTRODUCTION

In many applications that involve magnetic recording, the aim is to reduce the size or surface on which the magnetic media has to be stored. Especially, in magnetic disk drives there is a need to increase the storage capacity of the disk to reduce the form factor of the disk and to allow faster access to the data recorded on the disk.

In many existing hard disk drive mechanisms, a single Voice Coil Motor (VCM) actuator is used to perform the positioning of a read/write head over the surface of the magnetic disk. Additional design concepts such as advances in head and disk design, interface and channel technologies and the design of so-called micro- and milli-actuators have opened the possibilities for storage capacity improvements (Cheung *et al.*, 1999; Koganezawa *et al.*, 1996; Horsley *et al.*, 1997). However, a systematic approach to develop a (limited complexity) servo controller that is able to control the actuator for positioning of the read/write head is still required.

The aim of this paper is to present a systematic approach for the design of a low order servo controller and to illustrate this design for a specific milli-actuator that can be used in magnetic hard disk drives. The complexity of the servo controller is limited to address costs and implementation issues in a commercial hard disk drive. The systematic approach for such a low order servo controller design is developed in this paper by utilizing a so-called H_∞-optimization with additional constraints on the order of the controller.

The outline of the paper is as follows. First, the hard disk drive milli-actuator, which will serve as a case-study for this paper, is presented in Section 2. Subsequently, in Section 3 the theory on H_∞-norm based optimal control design will be reviewed and the way in which a low order H_∞ controller design can be tackled. The results of Section 3 will be applied to the case-study in Section 4 and the differences between a full order and low order H_∞-norm based controller

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design will be presented. Finally, conclusions are presented in Section 5.

2. PIEZO-BASED MILLI-ACTUATOR

A prototype piezo-based milli-actuator was built to study the properties of track following and reduction of undesired mechanical vibrations with a low order milli-actuator servo controller. The prototype is also used to gather experimental data for modeling purposes (de Callafon *et al.*, 1999). A picture of the prototype being used is given in Figure 1.

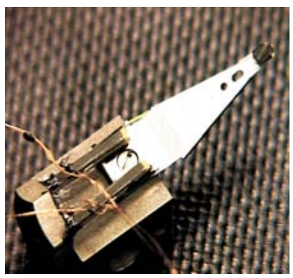

Fig. 1. Bottom view of prototype with special e-block, piezo stacks, wiring and suspension of read/write head. Courtesy of D. Harper, Center for Magnetic Recording Research, Univ. of California, San Diego.

In the prototype design of Figure 1, the connection of the suspension to the e-block is used as a pivoting device. The push/pull configuration of the piezo stacks is used to achieve a radial displacement of the tip of the suspension. The piezo stacks are attached with cyanoacrylate to the bottom of the suspension and a special e-block. The advantage of this proposed design is that it does not modify the shape of the suspension itself, thereby eliminating the need for suspension redesign (de Callafon *et al.*, 1999).

A (nominal) model for the dynamic behavior of the milli-actuator has been developed in () by curve fitting a measured frequency response. For reference purposes, a Bode plot of the measured data and curve fitted 10th order model, denoted by $G(s)$, has been plotted in Figure 2.

The nominal 10th order model $G(s)$ depicted in Figure 2 is used for control design purposes. The aim is to design a controller that is able to perform track following and reduce undesired mechanical vibrations in the flexible suspension. Both full order controllers and low order controllers will be computed via an H_∞-norm based optimal controller design. The performance of both the full and low order controllers are compared.

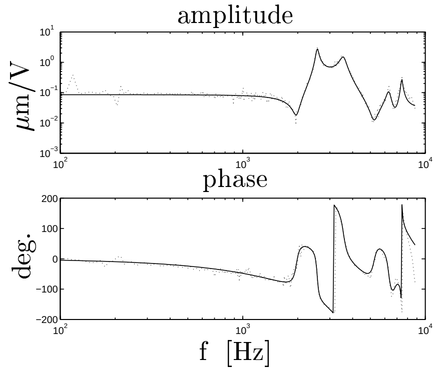

Fig. 2. Bode plot of measured frequency response (dotted) and 10th order linear time invariant model (solid).

3. LOW ORDER H_∞ CONTROLLERS

3.1 *Motivation*

In order to develop a systematic approach for the design of a low order servo controller for an actuator in a magnetic hard disk drives, the theory and results for the design of H_∞-norm based optimal controller design will be used. The reasons to use H_∞-norm based control design is threefold.

First of all, the design of H_∞ (optimal) controllers opens the possibility to include servo control design specifications by formulating frequency dependent bounds on closed-loop transfer functions. This allows for an intuitive and systematic controller design where closed-loop servo design specifications are formulated as frequency dependent weighting functions.

Secondly, the use of an H_∞-norm based control design enables to incorporate modeling uncertainties reasonably easily in the servo control design. In this way, robust performing low order controllers can be designed that take into account inevitable product variabilities or modeling uncertainties that are present in commercial hard disk drives.

Thirdly, the computational routines and results are easily applicable to multivariable systems. In this way, the results applied to the specific test-case of the milli-actuator as presented in this paper, can be extended easily to a low order servo controller design for the combined VCM and milli-actuator.

3.2 *Low order control design*

The H_∞-norm based control design problem under consideration has been analyzed in (Doyle *et al.*, 1989) and can be formulated as follows.

Let a minimal realization of a (generalized) plant $P(s)$ be given by the following state space realization

$$
\begin{aligned}
\dot{x} &= Ax + B_1 w + B_2 u \\
z &= C_1 x + D_{11} w + D_{12} u \\
y &= C_2 x + D_{21} w + D_{22} u
\end{aligned} \quad (1)
$$

where $A \in \mathbf{R}^{n\times n}$, and (A, B_2, C_2) is a stabilizable and detectable and $D_{22} = 0$. The generalized plant $P(s)$ in general will include a (nominal) model of the system to be controlled. For milli-actuator discussed in Section 2, the general plant $P(s)$ will include the 10th order model $G(s)$ given in Figure 2. Additionally, weighting filters that are used to formulate closed-loop servo design specifications are included in the standard plant description of (1).

Now let the servo controller $C(s)$ to be designed be given by the following minimal state space realization

$$
\dot{x}_c = A_c x_c + B_c y \quad (2)
$$

$$
u = C_c x_c + D_c y \quad (3)
$$

where $C(s)$ is of order n_c ($A_c \in \mathbf{R}^{n_c\times n_c}$). Then, by means of the Bounded Real Lemma (Scherer, 1990), the servo controller $C(s)$ satisfies an H_∞-norm bounded performance γ if and only if the matrix inequality

$$
\begin{pmatrix} A_{cl}^T X_{cl} + X_{cl} A_{cl} & X_{cl} B_{cl} & C_{cl}^T \\ B_{cl}^T X_{cl} & -\gamma I & D_{cl}^T \\ C_{cl} & D_{cl} & -\gamma I \end{pmatrix} < 0 \quad (4)
$$

holds for some $X_{cl} > 0$ where

$$
\begin{aligned}
A_{cl} &= \begin{pmatrix} A + B_2 D_c C_2 & B_2 C_c \\ B_c & A_c \end{pmatrix} \\
B_{cl} &= \begin{pmatrix} B_1 + B_2 D_c D_{21} \\ B_c D_{21} \end{pmatrix} \\
C_{cl} &= \begin{pmatrix} C_1 + D_{12} D_c C_2 & D_{12} C_c \end{pmatrix} \\
D_{cl} &= D_{11} + D_{12} D_c D_{21}
\end{aligned}
$$

Finding the lowest value of γ for a given servo controller $C(s)$ for which the above expression still holds, yields an upper bound on the H_∞ performance of the servo controller. By subsequently lowering or minimizing the value γ and characterizing the state space realization of the servo controller $C(s)$, gives rise to a so-called suboptimal H_∞ controller design. The existence of H_∞ controllers of order n_c is fully characterized by the following result (Skelton *et al.*, 1998).

Theorem 1. Let $\mathcal{N}_x$ and $\mathcal{N}_y$ denote orthonormal bases of the null spaces of (B_2^T, D_{12}^T) and (C_2, D_{21}), respectively. There exists a controller of order n_c which stabilizes the system and yields $\|T_{wz}\|_\infty < \gamma$ if and only if

$$
\mathcal{N}_x^T \begin{pmatrix} AX + XA^T + B_1 B_1^T & XC_1^T + B_1 D_{11}^T \\ C_1 X + D_{11} B_1^T & D_{11} D_{11}^T - \gamma^2 I \end{pmatrix} \mathcal{N}_x < 0 \quad (5)
$$

$$
\mathcal{N}_y^T \begin{pmatrix} A^T Y + YA + B_1 B_1^T & YC_1^T + C_1^T D_{11} \\ C_1 Y + D_{11}^T C_1 & D_{11} D_{11}^T - \gamma^2 I \end{pmatrix} \mathcal{N}_y < 0 \quad (6)
$$

$$
\begin{pmatrix} X & \gamma I \\ \gamma I & Y \end{pmatrix} \geq 0 \quad (7)
$$

together with the rank constraint

$$
\mathbf{Rank} \begin{pmatrix} X & \gamma I \\ \gamma I & Y \end{pmatrix} \leq n + n_c \quad (8)
$$

The constraints (5)–(7) are linear matrix inequalities (LMI) and define a convex set in the variables (X, Y). The suboptimal H_∞ problem with performance γ is solvable if and only if this set is non empty. It should be noted that the rank constraint is satisfied trivially when $n_c = n$, which corresponds to the full order case. In the reduced order case the rank constraint destroys convexity, so that efficient semi–definite programming (SDP) techniques are not applicable and other methods are needed.

Given any solution (X, Y) of (5)–(8), a n_c-th order H_∞ controller can be computed by solving the Bounded Real Lemma inequality (4) for the controller data (A_c, B_c, C_c, D_c). This problem can be solved using SDP.

In general while designing a reduced order controller one has to give up some performance compared to the full order case. This is done by selecting a value of γ higher than the full order case so as to obtain a feasible solution. There is therefore a trade off between controller order and performance that one has to make during the design process.

3.3 *Smooth Optimization Reformulation*

Based on the above theorem the synthesis of H_∞ controllers of order $n_c < n$ consists in finding a pair (X, Y) so that the LMI's (5)–(7) and the rank constraint are satisfied. Since the matrix $RC = \begin{pmatrix} X & \gamma I \\ \gamma I & Y \end{pmatrix}$ is symmetric, the rank constraint (8) is equivalent to requiring that the $k = n - n_c$ smallest eigenvalues of RC be zero. If the eigenvalues of RC are

$$
\begin{aligned}
eig(RC) &= \{\lambda_{2n}, \lambda_{2n-1}, \cdots, \lambda_k, \lambda_{k-1}, \cdots, \lambda_1\} \\
&\quad \text{with } \lambda_{2n} \geq \lambda_{2n-1} \geq \cdots \geq \lambda_k \geq \lambda_{k-1} \geq \cdots \geq \lambda_1
\end{aligned}
$$

Then the constraint (7) is equivalent to $\lambda_1 \geq 0$, and it makes sense to try and minimize λ_k so as to force the k smallest eigenvalues of RC to zero and thereby satisfy the rank constraint. We can therefore formulate the following smooth optimization problem:

$$\begin{aligned} &\min_{X,Y} \lambda_k \begin{pmatrix} X & \gamma I \\ \gamma I & Y \end{pmatrix} \\ \text{subject to } &\lambda_{max}(5) \le 0 \\ &\lambda_{max}(6) \le 0 \\ &\lambda_{min}(7) \ge 0 \end{aligned}$$

where $\lambda_k(M)$ denotes the kth smallest eigenvalue of the matrix M. It is seen that both the objective function and the constraints are just a specific eigenvalue of a symmetric matrix.

It should be noted that since RC is positive semidefinite $\sum_{i=1}^{k} \lambda_i(RC) \le k * \lambda_k(RC)$, so that the above minimization problem is very similar to that in (Gahinet and Ignay, 1994) where the sum of the k smallest eigenvalues is minimized. We have found that in practice our method has faster convergence.

The minimization itself is carried out using the code `constr` from the MATLAB Optimization ToolBox. This code requires the gradients of the objective function and of the constraints. These are gradients of the eigenvalues of a symmetric matrix valued function $M(x)$, and are given by

$$\frac{d\lambda_i}{dx} = \left(v_i^T \frac{\partial M}{\partial x_1} v_i, v_i^T \frac{\partial M}{\partial x_2} v_i, \cdots v_i^T \frac{\partial M}{\partial x_n} v_i \right)$$

The code also needs an initial point which is obtained by first solving the following convex optimization problem

$$\begin{aligned} &\min_{X,Y} \operatorname{tr}(X+Y) \\ &\text{subject to} \quad (5)-(7) \end{aligned}$$

This minimization is guaranteed (Ghaoui *et al.*, 1997) to give a controller of order at least $n-1$, and generally does better. The application of this minimization will be applied to the servo control design based on the 10th order model $G(s)$ of the milli-actuator. The aim is to find a low order controller, where the order of the controller is much smaller then the full order of the generalized plant $P(s)$ given in (1).

4. CONTROL DESIGN

In designing the full and reduced order controllers for the piezo-based milli–actuator, an H_∞-norm based mixed sensitivity problem is used. In this H_∞-norm based mixed sensitivity problem a servo controller $C(s)$ is designed in such a way such that H_∞-norm bound

$$\left\| \begin{matrix} w_S S \\ w_T T \end{matrix} \right\|_\infty < \gamma \tag{9}$$

is being satisfied. Adopting the notation $G(s)$ to indicate the 10th order model of the piezo-based milli-actuator, in (9) the $S = 1/(1 + CG)$ denotes the sensitivity and $T = CG/(1 + CG)$ denotes the complimentary sensitivity of the servo loop. Accordingly, w_S and w_T indicate weighting functions that are used to specify the desired shape of respectively the sensitivity and complimentary sensitivity function.

The sensitivity function S plays an important role in the specifying the disturbance rejection. The complimentary sensitivity function T is of importance in tracking. Both functions play a role in the servo control design and they are related by $S + T = I$.

Ideally, S should be small for disturbance rejection. Whereas, T has to be unity (at low frequencies) so as to have good tracking. At high frequencies, T should have a sufficient roll-off so as to attenuate the effect of sensor noise and reduce undesired mechanical resonances at high frequencies. With these design specifications in mind, the weighting functions w_S and w_T in (9) will be characterized in the next section.

4.1 *Weighting Functions*

An important part of the mixed sensitivity approach is selection of the weighting functions w_S and w_T in (9). The weighting functions reflect the servo design specifications. The weighting functions will be incorporated in the standard plant $P(s)$ of (1) used in the suboptimal H_∞-norm based control design. Therefore, the weighting functions should be kept reasonably simple to avoid an unnecessary model order increase of the standard plant $P(s)$. With the servo design specifications in mind, the following weighting filters have been chosen.

- For the sensitivity the following weighting function was chosen (Skogestad and Postlethwaite, 1996).

$$w_S(s) = \frac{s/M + \omega_B}{s + \omega_B A} \tag{10}$$

  This weighting function gives an upper bound on $|S|$ equal to $A \le 1$ at low frequencies, and equal to $M \ge 1$ at high frequencies. The zero crossing is at the frequency ω_B which is approximately the bandwidth requirement. The coefficients were chosen as $M = 2$, $A = 1e-3$ and $\omega_B = 800 * 2\pi$. The chosen value of M guarantees a Gain Margin(GM) of 2 and a Phase Margin(PM) of 29°.
- For the complimentary sensitivity the following weighting function was chosen.

$$w_T(s) = \frac{s}{\omega_T} \tag{11}$$

  where $\omega_T = 4500 * 2\pi$. Although this weighting function is an improper transfer function (i.e., has more zeros than poles), the product of $w_T(s)$ and $G(s)$ is proper and thus has a state–space realization. This above choice of w_T is such as to guarantee a -20dB/decade roll off above $4500Hz$ so as to synthesize the closed-loop to measurement noise and unmodeled dynamics.

The Bode plots of the inverse of the weighting functions w_S and w_T are shown in Figure 3.

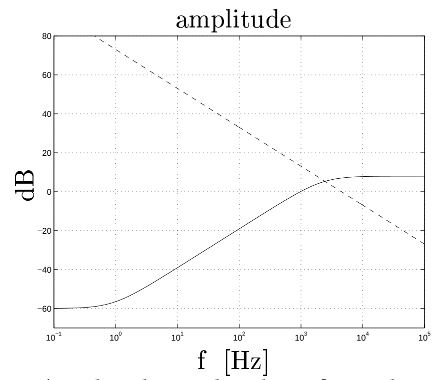

Fig. 3. Amplitude Bode plot of weighting functions $w_S(s)$ (solid) and $w_T(s)$ (dashed).

4.2 *Full Order Controller*

With the weighting functions selected as above the augmented standard plant $P(s)$ has 11 states. The value of gamma for the full order case is $\gamma = 1$. The fullorder H_∞ control design ($n_c = n$) yields controllers of the same order as the generalized plant $P(s)$. Hence, the full order controller $C(s)$ has 11 states.

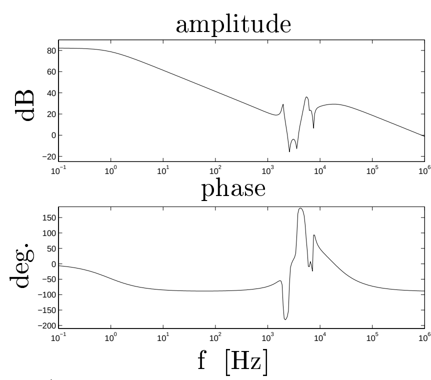

Fig. 4. Amplitude and phase Bode plot of full order (11th order) controller design.

In Figure 4 the Bode plot of the full order is shown. It is known for the H_∞-norm based mixed sensitivity problem that the designed (full order) controller will cancel the stable poles of the model (Doyle *et al.*, 1989). It can be seen from the Bode plot of the model $G(s)$ in Figure 2 and the Bode plot of the full order controller $C(s)$ in Figure 4 that this is indeed the case.

4.3 *Low Order Controller*

For the low order controller design, the minimization discussed in Section 3.3 will be used. Obviously, due to the lower order of the controller being designed, the H_∞-norm performance bound $\gamma = 1$ of the full order controller cannot be reached. As less design freedom is used in a lower order controller, the performance level must be increased in order to find an H_∞ optimal controller.

As a result, for an H_∞-norm performance bound of $\gamma = 1.5$, a 3rd order controller can be obtained. Setting γ to the higher level of $\gamma = 2.5$, a 2nd order controller can be computed. To show the results for the low order control design, the attention will be focused on the low (2nd) order controller and the Bode plot of the 2nd order controller computed for H_∞-norm performance bound of $\gamma = 2.5$ is shown in Figure 5. For comparison, the Bode plot of the full order controller is also plotted in Figure 5.

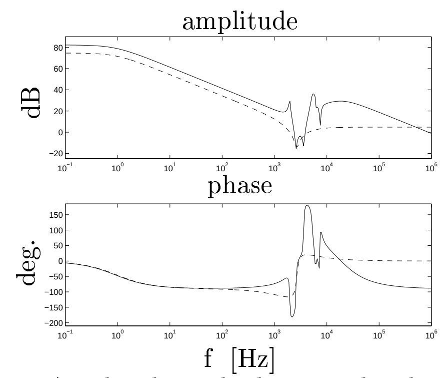

Fig. 5. Amplitude and phase Bode plot of full order (11th order) controller design (solid) and low order (2nd order) controller design (dashed).

It can be observed from Figure 5 that the 2nd order controller has an integrating action and a lead action around 2kHz. However, the lower order controller has sacrificed performance in terms of a lower gain at low frequencies due to the limited design freedom in the controller to control and stabilize resonance modes in the piezo-electric milli-actuator around the 2kHz.

4.4 *Comparison*

For comparison of the low (2nd) order and full (11th) order controller, in Figure 6 a Bode plot of the loop gain $L = GC$ of both controllers 2nd order. As mentioned before, the lower order controller has sacrificed performance in terms of a lower gain at low frequencies. This can be seen in Figure 6 by the lower value of the cross-over frequency or bandwidth of 800Hz instead of 1kHz in case of the full order controller.

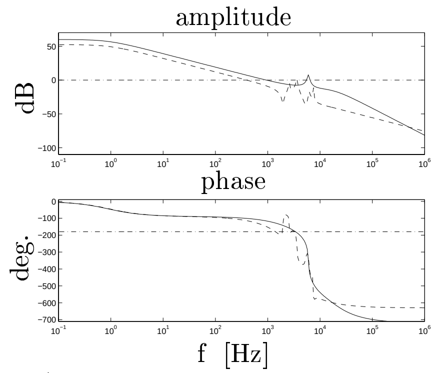

Fig. 6. Amplitude and phase Bode plot of loop gain based on full order controller (solid) and low order controller (dashed).

Moreover it can be seen that the full order controller cancels most of the stable poles in $G(s)$ as the loop gain appears to extremely smooth around the cross-over frequency. Although this allows for a higher cross-over frequency, the resulting high-order controller may not be very robust against variations in the model $G(s)$. Due to the limited design freedom in the low order controller, these phenomena do not occur.

A final comparison between the full and low order controller is depicted in Figure 7. In this figure the results of a closed-loop reference step on the servo control of the milli-actuator has been displayed for both the low and full order controller. It can be seen that the full order controller controls yields a faster step response with more control over the flexibilities in the suspension of the milli-actuator. However, this is at the price of sudden changes and fluctuations in the control signal (the input signal to the piezo-electric element). The control signal of the low order controller is much smoother as the low order servo controller does not attempts to cancel the resonance modes of the milli-actuator.

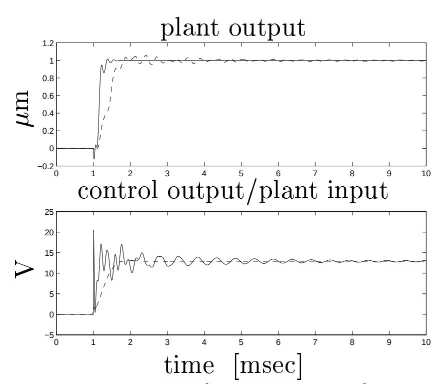

Fig. 7. Response to reference step of output (top) and control output (bottom) using full order controller (solid) and low order controller (dashed).

The results of the comparison between the full and reduced order controller have been summarized in Table 4.4. From this table it can be seen that although 9 orders has been sacrificed in the low order controller, the controller still has a very acceptable servo performance in terms of bandwidth and settling time.

Performance	Full Order	2nd Order
Gain Margin [dB]	7.5	6.2
Phase Margin [deg]	64	52
Bandwidth [Hz]	1000	800
Rise Time [ms]	0.2	0.6
Settling Time [ms]	0.33	0.72

5. CONCLUSIONS

In this paper a procedure to design low order H_∞-norm based servo controllers has been reviewed and applied to the servo control design of a piezo-based milli-actuator used in a hard disk drive. A previously identified model of the milli-actuator has been used to design a servo controller with a limited complexity that can be used to control the piezo-based milli-actuator.

The design of low order H_∞ controllers can be cast as a set of linear matrix inequalities and a rank constraint on a positive semi–definite matrix. There are various ways in which the rank constraint can be imposed. In this paper, a minimization of the smallest eigenvalues of the symmetric matrix has been chosen to address the rank constraint. The computational procedure for computing the low order controllers involves a (non-linear smooth) optimization.

On the basis of a 11th order augmented plant model, both a full (11th) and a low (2nd) order servo controller were computed for the piezo-based milli-actuator. In a comparison study it was found that the servo performance of the low order controller is comparable to the full (11th) order case. The systematic design of low order controllers presented in this paper is promising for similar hard disk drive applications as low order controllers are required to address low cost implementations issues.

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