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CLOSED-LOOP MODEL VALIDATION USING COPRIME FACTOR UNCERTAINTY MODELS

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Abstract: Model (in)validation techniques are used to bridge the gap between models used in robust control synthesis and uncertainty models obtained from identification experiments. In most applications the aim is to design a robust controller and therefore it is valuable to validate or invalidate an uncertainty model in view of this application by considering a closed-loop model validation technique. In this paper a model validation approach is presented that generalizes the (in)validation of possibly unstable models on the basis of closed-loop experiments with a stabilizing, but possibly unstable, controller. The approach is presented in a robust control framework with an uncertainty model described with coprime factor perturbations. It is shown that this approach yields an affine expression of the uncertainty model in all possible transfer functions that can be measured via a closed-loop experiments, which facilitates the optimization involved with a model invalidation.

Keywords: identification; model invalidation; coprime factors; closed-loop

1. INTRODUCTION

1.1 *Model invalidation*

In trying to model a plant, a distinction can be made between exact and approximate modeling. In case of exact modeling, the attention is focused on trying to model a plant meticulously by trying to capture the dynamical behaviour of plant exactly. Approximate identification is concerned with system identification problems in which the identification technique is used to find approximate models of the plant. Especially if models have to be used for control design, deliberate undermodeling is often required as model-based control design procedures tend to yield controllers that have the same complexity as the model used to compute the controller (Boyd and Barrat, 1991; Zhou *et al.*, 1996). By limiting the order of the model, the order of the controller can be limited during the control design.

In the event of a (deliberate) undermodeling, i.e. models that are too simple to describe the plant completely, a mismatch between the plant and the model is unavoidable. Such a mismatch can be captured in an uncertainty model that consists of a nominal model and an additional model uncertainty or allowable model perturbation. Even if exact modeling is desired on the basis of experimental data, such a model uncertainty

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in inevitable due to incomplete knowledge of the plant or the finite data sequence used during the modeling.

For (robust) control purposes, it is important to verify the validity of the uncertainty model. Without such a verification, the robustness of a controller can not be guaranteed fully. Considering the uncertainty model to be given, the validity of the model can be verified on the basis of an (additional) experiment by a so-called model validation procedure (Smith and Doyle, 1992). The model validation procedure is used to determine whether or not the experimental data could have been produced by the uncertainty model. In general, this is done by determining the level of the smallest model perturbation and the smallest noise signal that could have produced the experimental data. In case these levels exceed the assumed bounds of the uncertainty model, the model is said to be invalidated by the experiment. Although these techniques allow only *invalidation* of uncertainty models, the term model validation is being used extensively.

Several techniques have been reported in the literature that are either based on time domain (Poolla *et al.*, 1994; Rangan and Poolla, 1996) or frequency domain (Smith, 1995; Smith *et al.*, 1997; Token and Chen, 1998) characterizations of the model (in)validation problem. In the time domain approaches, a distinction can be made between a strictly discrete time setting (Poolla *et al.*, 1994) and the more practical framework of continuous time sampled data systems (Rangan and Poolla, 1996; Smith and Dullerud, 1996). In general, these techniques exploit a formulation for the model validation problem that allows an affine optimization to determine the level of the smallest model perturbation and the smallest noise signal that could have produced the experimental data.

Unfortunately, most of these existing techniques focus on an open-loop type of model validation. In the open-loop type of model validation, an uncertainty model is being validated on the basis of open-loop experiments. The open-loop type of model validation techniques exploit the linear appearance of an additive or multiplicative model uncertainty description to solve an affine optimization that addresses the model (in)validation problem (Smith and Doyle, 1992).

1.2 *Closed-loop model invalidation*

Obviously, in the case where the actual plant and/or the uncertainty model are unstable, an open-loop experiment and an open-loop model validation are not desirable. In most situations, open-loop experiments can not be performed as normal operating conditions require a feedback system to comply with safety or performance requirements.

Additionally, in most situations one is specifically interested in validating a model under closed-loop or controlled conditions on the basis of data obtained with a (stabilizing) feedback controller. The reason for this lies in the possible approximations being made during the formulation of the uncertainty model:

- An approximate model with model uncertainty has been developed that specifically aims at modeling the closed-loop behavior of the plant.
- A validation of the uncertainty model under operating conditions (with a feedback controller) is more realistic and desirable.

However, in case of closed-loop experiments, the uncertainty model may appear in a non-affine way in any of the closed-loop transfer functions needed for model validation (Dullerud and Smith, 1999). As a result, the resulting affine optimization used in the open-loop model validation might not be convex when applied to closed-loop systems.

The necessity and the problems associated to closed-loop model validation have been recognized in Dullerud and Smith (1999) and applied to a closed-loop experimental situation in Chen and Smith (1997) and Chen and Smith (1998). It has been observed in these references that the convexity of the model invalidation problem can be preserved by using the knowledge of the controller used in the closed-loop experiments. Furthermore, it has been recognized that a closed-loop configuration has a positive effect on the model validation. This effect is caused by the closed-loop configuration that weighs the effects of the model uncertainties via a specific closed-loop transfer function and addresses the fact that a validation of the uncertainty model under operating conditions (with a feedback controller) is more realistic and desirable.

In the line of the requirements on closed-loop model validation, this paper will generalizes the results on the use of controller information to facilitate closed-loop model validation techniques. The results in this paper focus on the development of model validation techniques that can be applied to data obtained under closed-loop or feedback controlled conditions. Furthermore, in order to present a framework that is applicable to the validation of stable and unstable models using closed-loop experiments, a framework of fractional model representations is being used (de Callafon and Van den Hof, 1997; Van den Hof and de Callafon, 1999). The model validation will be posed as closed-loop validation criterion that involves a closed-loop control objective. In this way, the model is being (in)validated towards its intended model application: robust control design.

2. THE MODEL VALIDATION PROBLEM

2.1 *Problem definition*

In order to provide a formal definition of the model validation problem, it is necessary to provide more details on the uncertainty model, denoted by $\mathcal{P}$, that consists of a nominal model $\hat{P}$ and an additional model uncertainty or allowable model perturbation Δ. A general description of the uncertainty model $\mathcal{P}$ can be given by using an LFT framework (Boyd and Barrat, 1991) that can characterize the uncertainty model $\mathcal{P}$ via an upper fractional transformation $\mathcal{F}_u(Q,\Delta)$ as follows

$$\begin{aligned} \mathcal{P} = \{P \mid P = \mathcal{F}_u(Q,\Delta)\} \text{ with} \\ \mathcal{F}_u(Q,\Delta) := Q_{22} + Q_{21}\Delta(I - Q_{11}\Delta)^{-1}Q_{12} \end{aligned} \qquad (1)$$

where the entries of Q contain the information on the nominal model $\hat{P}$ and how the uncertainty Δ is characterized around the nominal model. Typically, $Q_{22} = \hat{P}$ as for $\Delta = 0$ the nominal model should be obtained. For an (unweighted) output multiplicative uncertainty description it can be verified that $Q_{11} = Q_{21} = 0$ and $Q_{12} = I$.

Given the LFT representation of the uncertainty set $\mathcal{P}$, the entries of Q are assumed to be LTI discrete-time systems. Furthermore, Δ is assumed to be a member of a class of perturbations with $\Delta \in \mathcal{R}H_\infty$ and $\|\Delta\|_\infty < 1$. Given these assumption, the discrete time experimental data $\{u(t), y(t)\}$, coming from the plant, consists of N observations and is described by

$$y(t) = \mathcal{F}_u(Q,\Delta)u(t) + Hd(t) \qquad (2)$$

where d denotes an unknown but bounded noise $d \in l_2$ with $\|d\|_2 < 1$ and H is a stable and stably invertible noise filter.

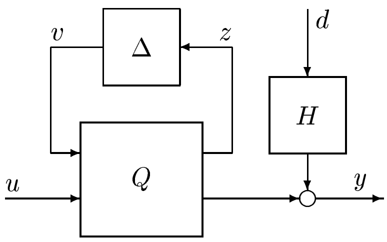

Fig. 1. Uncertainty and noise model

A block diagram of the uncertainty model and the noise model that are subjected to model (in)validation is depicted in Figure 1. The model validation problem can be formalized as follows (Smith and Doyle, 1992).

Problem 1. Given the Q of the uncertainty model $\mathcal{P}$ in (1), the noise model H and observations $\{u(t), y(t)\}$ for $t = 0, 1, \ldots, N-1$, does there exists a discrete time signal d with $\|d\|_2 < 1$ and an uncertainty Δ with $\Delta \in \mathcal{R}H_\infty$ and $\|\Delta\| < 1$ such that (2) holds.

2.2 *Model validation via optimization*

The general approach taken in model validation is an optimization where the value of the smallest model perturbation Δ and the noise signal $d(t)$ is found that could have produced the experimental data in (2). In case the smallest model perturbation or noise signal exceeds the assumed bounds, the model is invalidated by the experimental data.

Posing the condition $\Delta \in \mathcal{R}H_\infty$ and $\|\Delta\| < \alpha$ is a key step in solving the model validation problem by optimizing the value of the smallest model perturbation Δ. This condition has been solved in Poolla *et al.* (1994) for the LTI case and can be summarized as follows.

Lemma 2. Given the signals $v(t)$ and $z(t)$ for $t = 0, 1, \ldots, N-1$ with $v = \Delta z$, as indicated in Figure 1. Then there exists a $\Delta \in \mathcal{R}H_\infty$ with $\|\Delta\|_\infty < \alpha$ if and only if

$$V^T V < \alpha^2 Z^T Z$$

where V and Z are block Toeplitz matrices derived from $v(t)$ and $z(t)$.

With this result, the model validation can be solved via a convex optimization. Crucial in this convex optimization is the fact that the uncertainty Δ appears linearly in (2). With the general LFT form of the uncertainty model in (1), it can be seen that this is the case for $Q_{11} = 0$ and holds for example for an uncertainty model with a multiplicative uncertainty description.

With an the uncertainty Δ appearing linearly in (2), finding the smallest model perturbation Δ and the noise signal $d(t)$ is simply an additional linear constraint added to the convex minimization. With $Q_{11} = 0$ and $z(t) = Q_{12}u(t)$, the following convex optimization needs to be solved.

$$\begin{gathered} \min_\alpha, \text{ subjected to} \\ V^T V < \alpha^2 Z^T Z \\ d^T d < 1 \\ z(t) = Q_{12}u(t) \\ y(t) - Q_{22}u(t) = Q_{21}v(t) + Hd(t) \end{gathered} \qquad (3)$$

In case $\alpha \geq 1$, the model is invalidated by the experimental data $\{u(t), y(t)\}$

Although the optimization mentioned above addresses the model validation problem, the question arises whether or not there are alternative uncertainty descriptions that still allow the use of a convex optimization. Moreover, it is preferable to perform the model validation in closed-loop, as mentioned in the introduction. In case a feedback connection is created around the signals $u(t)$ and $y(t)$, the resulting optimization involved with the model validation may not be convex.

The rest of the paper is devoted to the choice of an uncertainty model that allows for a model validation in closed-loop. The uncertainty structure and the resulting uncertainty model Q will be based on perturbation on coprime factorizations, which generalizes the approach to the validation of stable and unstable models.

3. MODEL STRUCTURE FOR CLOSED-LOOP MODEL INVALIDATION

3.1 *Uncertainty model based on coprime factor perturbations*

For the purpose of the closed-loop model validation using coprime factor uncertainty models, the nominal model $\hat{P}$ is represented in a coprime factor representation

$$\hat{P} = \hat{N}\hat{D}^{-1}$$

where $(\hat{N}, \hat{D})$ denotes a right coprime factorization (*rcf*) of the model $\hat{P}$. The accompanying uncertainty on the model is assumed to be modeled as a perturbations in a dual-Youla parametrization (de Callafon and Van den Hof, 1997). This perturbation uses the knowledge of the feedback controller to characterize the model uncertainty and yields a uncertainty model $\mathcal{P}$ that can be characterized as follows.

Proposition 3. Let a nominal model $\hat{P}$ with a *rcf* $(\hat{N}, \hat{D})$ and a controller C with a *rcf* (N_c, D_c) form an internally stable feedback connection. Then an uncertainty model $\mathcal{P}$ is constructed by

$$\begin{aligned} &\mathcal{P}(\hat{N}, \hat{D}, N_c, D_c, \hat{V}, \hat{W}) := \\ &\{P \mid P = (\hat{N} + D_c\Delta_R)(\hat{D} - N_c\Delta_R)^{-1} \text{with} \\ &\Delta_R \in \mathcal{R}H_\infty,\ \Delta := \hat{V}\Delta_R\hat{W} \text{ and } \|\Delta\|_\infty < 1\} \end{aligned} \quad (4)$$

and $\hat{V}$, $\hat{W}$ are stable and stably invertible weighting functions.

As indicated in the proposition, the uncertainty model $\mathcal{P}$ essentially depends on the factorization $(\hat{N}, \hat{D})$ of the nominal model $\hat{P}$, the factorization (N_c, D_c) of the known controller C and the weighting functions $\hat{V}$, $\hat{W}$ that take into account the shape and scaling of the uncertainty. It can be observed that the uncertainty Δ_R acts in a non-trival way on the (coprime factors of the) nominal model $\hat{P}$. It can be seen in Figure 2 that the weighted uncertainty $\Delta_R = \hat{W}^{-1}\Delta\hat{V}^{-1}$ appears in a similar way as a Youla parametrization. To distinguish between the familiar Youla parametrization, the perturbation in Figure 2 is indicated by a dual-Youla parametrization (de Callafon and Van den Hof, 1997).

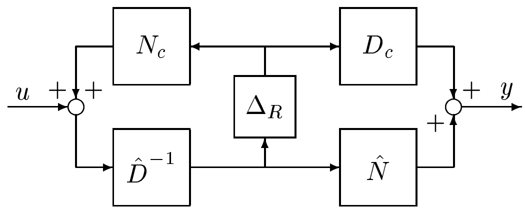

Fig. 2. Representation of $\mathcal{P}$ given in (4) with $\Delta_R = \hat{W}^{-1}\Delta\hat{V}^{-1}$.

With little effort, the nominal model and the perturbation Δ_R in the form of a dual-Youla parametrization can be rewritten in a standard LFT form of (1) with a norm bounded uncertainty $\Delta \in \mathcal{R}H_\infty$. On the basis of Figure 2, a characterization of the coefficient matrix Q in (1) can be given and an alternative representation of the uncertainty model $\mathcal{P}$ in (4) can be obtained.

Corollary 4. The uncertainty model $\mathcal{P}$ given in (4) can be written as

$$\mathcal{P} = \{P \mid P = \mathcal{F}_u(Q, \Delta),\ \|\Delta\|_\infty < 1 \text{ and}$$

$$Q = \left[\begin{array}{c|c} \hat{W}^{-1}\hat{D}^{-1}N_c\hat{V}^{-1} & \hat{W}^{-1}\hat{D}^{-1} \\ \hline (D_c + \hat{P}N_c)\hat{V}^{-1} & \hat{P} \end{array}\right] \quad (5)$$

As mentioned in Section 2.2, it is crucial that $Q_{11} = 0$ in order to exploit a convex optimization for the *open-loop* model validation problem. It can be seen that this model structure does not exhibit this property and is not suitable for an open-loop model validation. However, in the next section it will be illustrated that this model structure is beneficial in a closed-loop model validation.

Although the coefficient matrix Q in (5) looks complicated, it can be written as as simple multiplication

$$Q = \begin{bmatrix} \hat{W}^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} \\ \bar{Q}_{21} & \bar{Q}_{22} \end{bmatrix} \begin{bmatrix} \hat{V}^{-1} & 0 \\ 0 & I \end{bmatrix}$$

where the entries of the (unweighted) coefficient matrix $\bar{Q}$ are given by

$$\begin{bmatrix} \hat{D}^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} I & 0 \\ \hat{P} & I \end{bmatrix} \begin{bmatrix} C & I \\ I & 0 \end{bmatrix} \begin{bmatrix} D_c & 0 \\ 0 & I \end{bmatrix}. \quad (6)$$

The multiplication of the transfer functions in (6) indicates the construction of the (unweighted) coefficient matrix $\bar{Q}$. Furthermore, it can be observed that $\bar{Q}$ is invertible, as all the matrices in (6) are known to be invertible, whereas

$$\begin{bmatrix} I & 0 \\ \hat{P} & I \end{bmatrix} \text{ and } \begin{bmatrix} C & I \\ I & 0 \end{bmatrix}$$

are unimodular. From the multiplication in (6) it can be observed that Q will have a McMillan degree equal to the sum of the McMillan degree of the (nominal) model $\hat{P}$ and the controller C.

3.2 *Favorable properties of coprime factor based uncertainty model*

The appearance of the (unstructured) model uncertainty Δ_R in the uncertainty model $\mathcal{P}$ in (5) is more complicated than a standard additive or multiplicative uncertainty. However, the only knowledge needed to construct the uncertainty model $\mathcal{P}$ is a coprime factorization of the nominal model $\hat{P}$ and the controller C. With this information, the construction of the model uncertainty Δ_R is not more complicated than a standard uncertainty description. Due to the specific appearance of the model uncertainty in the uncertainty model of (5), the following favorable properties can be summarized.

- All models in the uncertainty model $\mathcal{P}$ are stabilized by the know feedback controller C, irrespective of the size or shape of Δ_R.
- The uncertainty Δ_R in (5) will appear in an affine way in any closed-loop transfer function.

The first property indicates the usefulness of the uncertainty set $\mathcal{P}$ of (5). Basically, only those models P are captured in the set $\mathcal{P}$ that are stabilized by the controller C. This is an appealing and intuitive result as the plant, currently operating in a feedback connection, is know to be stabilized by the feedback controller C. Hence, only those models can be validated that are known to be stabilized by the controller C.

The second property will enable the application of an affine optimization, similar to (3) to address the closed-loop model validation problem. Clearly, this generalizes the application of model validation techniques to closed-loop systems. Furthermore, the use of stable coprime factorizations in the uncertainty model $\mathcal{P}$ of (5) generalizes the approach to stable and unstable plants and controllers.

For an explanation of these properties, consider the transfer function matrix

$$T(P,C) := \begin{bmatrix} P \\ I \end{bmatrix} (I+CP)^{-1} \begin{bmatrix} C & I \end{bmatrix} \qquad (7)$$

which captures all possible closed-loop transfer functions. With the general closed-loop transfer function matrix $T(P,C)$ given in (7) and the uncertainty model given in (5), the following result can be given.

Lemma 5. Consider the uncertainty model $\mathcal{P}$ of (5) and the controller C used in the construction of $\mathcal{P}$. With the definition of the weighted closed-loop transfer function matrix $T(P,C)$ given in (7), the uncertainty model $\mathcal{P}$ satisfies

$$\mathcal{P} = \{P \mid T(P,C) = \mathcal{F}_u(M,\Delta) \text{ with } \Delta \in \mathcal{R}H_\infty,\ \|\Delta\|_\infty < 1\}$$

where the entries of M are given by

$$\begin{aligned} M_{11} &= 0 \\ M_{12} &= \hat{W}^{-1}(\hat{D}+C\hat{N})^{-1}\begin{bmatrix} C & I \end{bmatrix} \\ M_{21} &= -\begin{bmatrix} I \\ -C \end{bmatrix} D_c\hat{V}^{-1} \\ M_{22} &= T(\hat{P},C) \end{aligned} \qquad (8)$$

The entries of M in (8) are all known quantities. It can be verified that all these entries are stable *if and only if* the controller C internally stabilizes the nominal model $\hat{P}$, as mentioned in the construction in the set $\mathcal{P}$ in Proposition 3. With $\hat{V}$, $\hat{W}$ stable and stably invertible and $\Delta \in \mathcal{R}H_\infty$, all models $P \in \mathcal{P}$ are stabilized by the know feedback controller C, irrespective of the size or shape of Δ_R.

It can also be observed that the uncertainty Δ appears affinely in all possible (weighted) closed-loop transfer functions of $T(P,C)$. The affine representation of the dual-Youla based model uncertainty Δ can be exploited to formulate a framework to (in)validate uncertainty models on the basis of closed-loop data.

4. APPLICATION TO CLOSED-LOOP MODEL VALIDATION

With the LFT representation of the uncertainty model $\mathcal{P}$ in (5) and the application of the feedback controller C to the models in $\mathcal{P}$, a closed-loop uncertainty model depicted in is obtained.

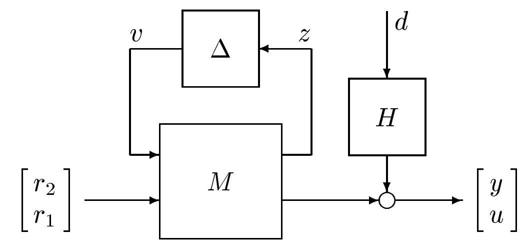

Fig. 3. Closed-loop uncertainty model

In Figure 3 the signals r_2 and r_1 indicate external closed-loop reference signals, whereas u and y denote the input and output of the plant. On the basis of this closed-loop representation, a closed-loop model validation problem can be formulated where the affine appearance of the model uncertainty Δ in (8) is used. The discrete time experimental data $\{u(t), y(t)\}$, coming from the closed-loop plant, consists of N observations and is described by

$$\begin{bmatrix} y(t) \\ u(t) \end{bmatrix} = \mathcal{F}_u(M, \Delta) \begin{bmatrix} r_2(t) \\ r_1(t) \end{bmatrix} + Hd(t) \qquad (9)$$

where d denotes an unknown but bounded noise $d \in l_2$ with $\|d\|_2 < 1$ and H is a stable and stably invertible noise filter. The closed-loop model validation problem can now be formalized as follows.

Problem 6. Given the M of the uncertainty model $\mathcal{P}$ in (8), the noise model H and observations $\{r_2(t), r_1(t), y(t), u(t)\}$ for $t = 0, 1, \ldots, N-1$, does there exists a discrete time signal d with $\|d\|_2 < 1$ and an uncertainty Δ with $\Delta \in \mathcal{R}H_\infty$ and $\|\Delta\| < 1$ such that (9) holds.

Along the lines of Section 2.2, the closed-loop model validation problem can be solved via a convex optimization, as $M_{11} = 0$. With the uncertainty Δ appearing linearly in (8), finding the smallest model perturbation Δ and the noise signal $d(t)$ is simply an additional linear constraint added to the convex minimization:

$$\begin{aligned} &\min_{\alpha}, \text{ subjected to} \\ &V^T V < \alpha^2 Z^T Z \\ &d^T d < 1 \\ &z(t) = M_{12} \begin{bmatrix} r_2(t) \\ r_1(t) \end{bmatrix} \\ &\begin{bmatrix} y(t) \\ u(t) \end{bmatrix} - M_{22} \begin{bmatrix} r_2(t) \\ r_1(t) \end{bmatrix} = M_{21} v(t) + Hd(t) \end{aligned}$$

In case $\alpha \geq 1$, the model is closed-loop invalidated by the close-loop experimental data $\{r_2(t), r_1(t), y(t), u(t)\}$.

5. CONCLUSIONS

In this paper a model validation approach is presented that generalizes the (in)validation of possibly unstable models on the basis of closed-loop experiments with a stabilizing, but possibly unstable, controller. The approach is presented in a robust control framework with an uncertainty model described with coprime factor perturbations. It is shown that this approach yields an affine expression of the uncertainty in all possible transfer functions that can be measured via a closed-loop experiments. This allows the model validation problem to be solved via standard convex optimization techniques.

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