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MULTIVARIABLE FEEDBACK RELEVANT SYSTEM IDENTIFICATION OF A WAFER STEPPER SYSTEM

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ABSTRACT

This paper discusses the approximate and feedback relevant parametric identification of a positioning mechanism present in a wafer stepper. The positioning mechanism in a wafer stepper is used in chip manufacturing processes for accurate positioning of the silicon wafer on which the chips are to be produced. The accurate positioning requires a robust and high performance feedback controller that enables a fast through put of silicon wafers. A set of multivariable finite dimensional linear time invariant discrete time models will be estimated, that is suitable for model-based robust control design of the positioning mechanism.

INTRODUCTION

Wafer steppers combine a high accuracy positioning and a sophisticated lithographic process to manufacture integrated circuits (chips) via a fully automated process. By means of a photolithographic process, the chip architecture is exposed on the surface of a wafer, a silicon disk covered with photo resist. In the application discussed in this paper, the wafer is supposed to carry approximately 80 chips. In order to expose the surface of the wafer, each chip is processed sequentially. Such a sequential process is needed as only one mask of the chip layout is available during the exposure fase of the photolithographic process. For that purpose, the wafer is placed on a moving table that needs to be moved (stepped) in 3 Degrees Of Freedom (3DOF) accurately for the sequential processing of the chips on the wafer.

Clearly, both the accuracy and the speed of the servo mechanism during the subsequent steps of the wafer will influence the success and throughput of the production process of the chips on the wafer. Sophisticated control of this (multivariable) servo mechanism can help in achieving a required throughput by designing a multivariable feedback controller that is able to satisfy high performance requirements (de Roover *et al.* 1996). A model that describes the dynamical behaviour of the servo mechanism is needed to design such a controller thoughtfully.

A dynamical model can be obtained by first principle modelling, see e.g. de Roover and van Marrewijk (1995). Although such a model provides valuable knowledge of the dynamical behaviour, either the numerical completion of specific elements in the servo system is undiscoverable or deliberate assumptions are posed to simplify the modelling. This causes the model to deviate from the actual dynamical behaviour of the system. Alternatively, a system identification procedure can be exploited in which experimental data is used directly. In this way, a model describing the dynamical behaviour is evaluated directly on the basis of the data coming from the actual system (Ljung 1987).

Although both modelling procedures provide insight in the dynamical behaviour of the positioning mechanism present in a wafer stepper, it is impossible to exactly characterize all phenomena describing the dynamics. On the one hand exact modelling can be impossible or too costly, on the other hand control design methods can get unmanageable if they are applied to models of high complexity. As a result, the model obtained is only an approximation of the

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system to be controlled. As the validity of any approximate model hinges on its intended use, the modelling procedure being applied should take into account the intended use of the model; control design.

MODELLING FOR CONTROL

In this paper the attention is focused on deriving Finite Dimensional Linear Time Invariant (FDLTI) models via system identification techniques that approximates the dynamical behaviour of the positioning mechanism in a wafer stepper. For an existing servo mechanism present in a wafer stepper, time domain observations are gathered to estimate models that can be used for subsequent controller design. The aim of this paper is to outline the system identification procedure being used and the performance improvement obtained when designing a multivariable controller.

In order to estimate models suitable for control design, the following requirements should be satisfied. Preferably, the models should be a linear description of actual system to be controlled. In this way, standard tools for linear model-based control design can be used. Furthermore, control design methods become unmanageable if they are applied to models of high complexity. Hence, linear models should have a reasonable model order in order to formulate a manageable control design problem. As the models will be necessarily approximative, it should contain those dynamical aspects that are important for control design (Schrama 1992*b*). Finally, the identification procedure being used should be able to deal with data that is obtained under closed-loop (controlled) conditions. This is due to the fact that many engineering systems are unable to operate without additional control, including the position servo mechanism of the wafer stepper.

Estimating a linear model can be done by existing system identification techniques reported in the literature (Ljung 1987, Söderström and Stoica 1989) and available in the corresponding commercial software packages (Ljung 1995). However, application of these techniques to find models on the basis of closed-loop experiments that capture the dominant dynamical aspects relevant for feedback, is by far trivial. Estimating such models boils down to the fact that models, suitable for control design, can only be found by taking the closed loop operation of the model into account (Schrama 1992*a*). In general, this leads to identification problem in which the criterion used for designing the subsequent controller should also be used to deduct the model. See for example the work by Zang *et al.* (1995) for LQG-based controller design.

As the resulting model is just an approximation of the system to be identified, the controller based on the model has to be robust against any dissimilarities between the model and the system. This has been a motivation for the development of identification techniques that estimate an upper bound on the model error, see for example the contributions by Goodwin *et al.* (1992), Helmicki *et al.* (1993), Partington and Mäkilä (1995) Mäkilä and Partington (1995) and the references therein. The resulting model error constitutes an allowable model perturbation around a nominal model being estimated and defines a set of models where the actual system is assumed to be an element of. Subsequently, a robust controller can be designed on the basis of this set of models (Doyle *et al.* 1992). In this approach stability and performance requirements are guaranteed for the complete set of models, that includes the actual system to be controlled. The estimation of such a set of models for the design of a robust controller for the positioning mechanism of the wafer stepper is the main item in this paper.

In order to estimate such a set of models by the estimation of a (low complexity) nominal model along with its allowable model perturbation, the identification procedure discussed in this paper uses the algebraic framework of stable fractional model representations, similarly as in de Callafon *et al.* (1994) or Van den Hof *et al.* (1995). The reasoning to use such a fractional model representations is due to the ability to deal with both stable, unstable or marginally unstable systems, such as the positioning mechanism discussed in this paper. As such, this approach enables one to find a set of feedback relevant models by estimating stable factorizations of a nominal model along with a stable perturbation on the allowable model perturbations. Furthermore, the fractional approach can deal with observations obtained under closed-loop (controlled) conditions relatively easily.

WAFER STEPPER SERVO MECHANISM

Description of servo mechanism

The servo mechanism discussed in this paper is an integral part of the Silicon Repeater 3rd generation (SIRE3) wafer stepper. The moving table, called the wafer chuck, that needs to position the wafer, is equipped with a air bearing and placed on a large suspended granite block to reduce the effect of external vibrations. The position of the wafer chuck on the horizontal surface of the granite block is measured by means of laser interferometry. A schematic overview of this servo mechanism is depicted in Figure 1.

Relative movements of the wafer chuck are measured by determining the phase shift of the laser beams reflected on the mirror block depicted in Figure 1. As the horizontal plane allows three degrees of freedom, three laser measurements uniquely determine the horizontal position of the

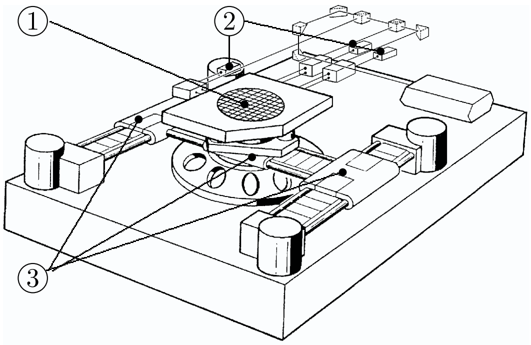

Figure 1. SCHEMATIC VIEW OF A WAFER STAGE; 1:WAFER CHUCK, 2:LASER INTERFEROMETERS, 3:LINEAR MOTORS.

wafer, whereas three linear motors are used to position the wafer chuck in 3DOF. This makes the servo mechanism of the wafer stepper a multivariable system, having three inputs and three outputs. The inputs reflect the currants to the three linear motors, whereas the outputs are constructed by measuring the position of the wafer chuck both in x-, y-direction (translation) and the ϕ-direction (rotation).

Experimental set up

In order to perform an identification and test the control of the servo mechanism, an experimental set up has been provided by the Philips Research Laboratories and has been depicted in Figure 2

Figure 2. PHOTO OF EXPERIMENTAL SET UP

The experimental set up is equipped with a computer interface to measure the position in x-, y- and ϕ-direction of the wafer chuck on discrete time samples via a digital signal processor. Due to safety requirements and operating conditions of the laser interferometers, the signals can be measured only if a (digital) controller is used to control the positioning of the wafer chuck. Such a digital controller can be implemented using the same digital signal processor.

Consequently, only (discrete time) measurements obtained under feedback can be gathered for identification purposes. Additional external reference signals can be applied to the feedback connection of the positioning mechanism to provide sufficient excitation (Ljung 1987) while gathering data for identification. A schematic overview of the signals that can be accessed in the feedback connection is depicted in the block diagram of Figure 3.

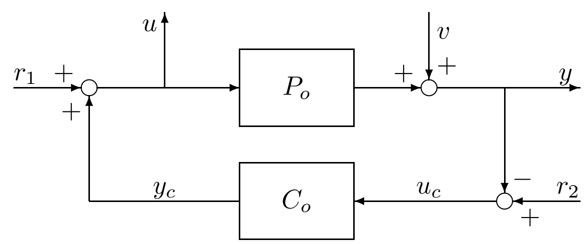

Figure 3. BLOCK DIAGRAM OF EXPERIMENTAL SET UP OF FEEDBACK CONTROLLED POSITIONING MECHANISM

As indicated in Figure 3, the positioning mechanism of the wafer chuck is denoted by P_o, while the feedback controller currently used to control P_o is denoted by C_o. In the current experimental set up, the controller C_o consists of 3 parallel PID controllers controlling the positioning in x- y- and ϕ-direction separately. The feedback connection of P_o and the controller C_o is denoted by $\mathcal{T}(P_o, C_o)$.

Control of the positioning mechanism

Next to the purpose of providing sufficient excitation of $\mathcal{T}(P_o, C_o)$, the reference signals in Figure 3 can be used to move or step the wafer chuck in a desired direction. As such, the signals r_1 and r_2 can be used to evaluate the performance of the feedback controlled positioning mechanism by applying a reference signal r_2 and a feed forward signal r_1 in order to track a certain desired position signal y of the wafer chuck. In this way, the input signal u_c to the controller C_o reflects the servo error between a desired reference r_2 and the actual desired position y.

Controlling the positioning mechanism of the wafer chuck aims at minimizing the servo error, while moving the

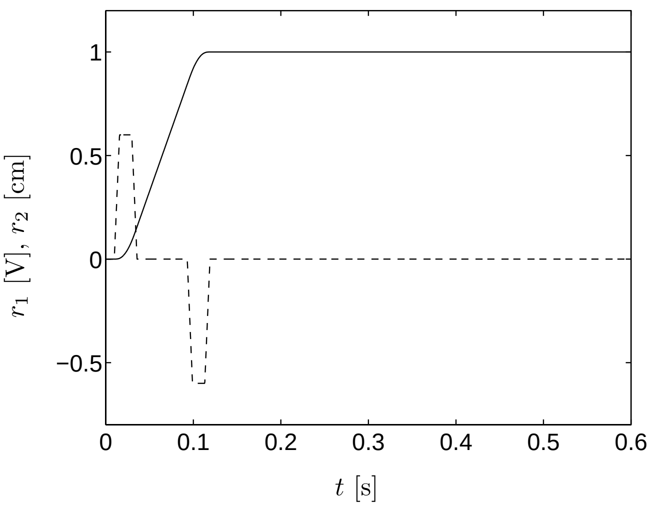

Figure 4. SHAPE OF REFERENCE SIGNAL r_2 (—) AND FEEDFORWARD SIGNAL r_1 (- -)

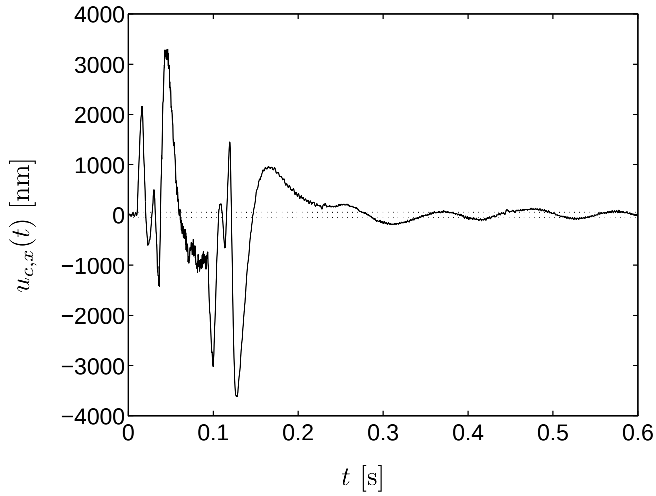

Figure 5. SERVO ERROR RESPONSE TO A STEP IN X-DIRECTION

chuck as fast as possible. The design specification for the SIRE3 wafer stepper is to bring the servo error within a bound of 52nm (4 times the measurement resolution) as soon as possible after a step has been performed. This is due to the fact that the chuck must be kept in a constant position before a chip can be exposed on the surface of the wafer.

Henceforth, controlling the positioning of the wafer chuck requires the combined design of both a feedback controller and the appropriate reference r_2 and feed forward signal r_1 (de Roover *et al.* 1996). In this paper however, the attention is focused on the identification of a set of models, denoted by $\mathcal{P}$, to improve the design of the feedback controller only.

In order to compare feedback controllers designed on the basis of the set of models $\mathcal{P}$ being estimated, the signals r_2 and r_1 are fixed to some prespecified desired trajectory. This prespecified trajectory is based on the dominating open loop dynamical behaviour of P_o that is given by a double integrator, relating the force generated by the linear motors to the position of the wafer chuck. Based on this relatively simple model, r_2 will denote a desired position profile, whereas r_1 denotes (a scaled) acceleration profile obtained by computing the second derivative of r_2. A typical shape of the reference signal r_2 and the feed forward signal r_1 to position the wafer chuck in either the x- or y-direction over 1cm is depicted in Figure 4.

In Figure 4, the position profile r_2 is obtained by allowing a maximum jerk (derivative of acceleration) and a maximum speed of the wafer chuck. The resulting acceleration profile r_1 is the second derivative of r_2. Application of both reference signals in either an x- or y-direction is labelled as a step respectively in x- or y-direction. Using these specified reference signals r_1 and r_2 for the current experimental set up in which 3 parallel PID controllers are used to control the positioning in x- y- and ϕ-direction separately, the servo error $u_{c,x}$ depicted in Figure 5 for a step in the x-direction is obtained.

It can be observed from Figure 5 that the servo error $u_{c,x}$ is hardly within the bounds of 52nm indicated by the dotted lines. Furthermore, $u_{c,x}$ exhibits a low frequent vibration after the step has ended. As a result, the settling time of the step is strongly influenced and both an improvement of the speed of decay and a reduction of the low frequent vibration of the servo error is desired to improve the behaviour of the servo mechanism.

PRELIMINARIES

Data obtained from experimental set up

For analysis purposes, P_o is considered to be a discrete time linear time invariant map that is characterized by the difference equation

$$y(t) = P_o(q)u(t) + v(t)$$

where $t = k\Delta T,\ k = 0, 1, 2, \ldots$ denotes the discrete time character of the signals being processed by the digital processor and $qu(t) = u(t + 1)$ denotes the forward shift. The signals u and y respectively denote the input (currants to the linear motors) and a disturbed output (measured position in x-, y- and $\phi-$direction) of the positioning mechanism. The signal v is used to model to disturbance that may present on the output y. The signals u and y are measurable and sampled with a sampling time $\Delta T = 3 \cdot 10^{-4}$, while known reference signals r_1 and r_2 are applied to provide sufficient excitation of $\mathcal{T}(P_o, C_o)$.

It is assumed that the feedback connection $\mathcal{T}(P_o, C_o)$ is well posed, that is $\det(I + C_o P_o) \not\equiv 0$ (Boyd and Barrat 1991) and the mapping from the signals $col(r_2, r_1)$ onto $col(y, u)$ is given by the transfer function matrix $T(P_o, C_o)$ with

$$T(P_o, C_o) := \begin{bmatrix} P_o \\ I \end{bmatrix} (I + C_o P_o)^{-1} \begin{bmatrix} C_o & I \end{bmatrix}, \qquad (1)$$

As a result, the data obtained from the feedback connection $\mathcal{T}(P_o, C_o)$ of Figure 3 can be described by

$$\begin{bmatrix} y \\ u \end{bmatrix} = T(P_o, C_o) \begin{bmatrix} r_2 \\ r_1 \end{bmatrix} + \begin{bmatrix} I \\ -C_o \end{bmatrix} (I + P_o C_o)^{-1} v \qquad (2)$$

For identification purposes, it is presumed that the noise v is uncorrelated with the external reference signals r_1, r_2 and that it can be modelled as the output of a monic stable and stably invertible noise filter H_0 having a white noise input e (Ljung 1987).

Norm-based control design

As indicated in Figure 5 the behaviour of the servo mechanism needs to be improved in order to reduce the settling time of the wafer chuck. For that purpose, a multivariable feedback controller is (re)designed on the basis of the set of models $\mathcal{P}$ found by system identification.

In order to design the feedback controller, a norm-based control design will be used. In this way, the design specifications are translated in a control objective function, whereas a norm of the function is used to indicate the performance of the resulting feedback connection. For notational convenience a control objective function is denoted by $J(P, C) \in R\mathcal{H}_\infty$, where P and C are FDLTI (possibly unstable) mappings and used to denote respectively a system and a feedback controller. The notion of performance will be characterized by the value of the norm $\|J(P, C)\|_\infty$: a smaller value of $\|J(P, C)\|_\infty$ indicates better performance (Van den Hof and Schrama 1995).

The mapping from the reference signals (r_2, r_1) to the output and input signals (y, u) of the plant P_o is given by the matrix $T(P_o, C_o)$ in (1). In a similar way, a feedback connection of a system P and a controller C can be studied by inspecting the matrix $T(P, C)$ with

$$T(P, C) := \begin{bmatrix} P \\ I \end{bmatrix} (I + CP)^{-1} \begin{bmatrix} C & I \end{bmatrix}, \qquad (3)$$

Note that a feedback connection $\mathcal{T}(P, C)$ is internally stable if and only if $T(P, C) \in R\mathcal{H}_\infty$ (Schrama and Bosgra 1993). In order to incorporate control design specification for the map $T(P, C)$, the control objective function $J(P, C)$ is taken to be a weighted form of the matrix $T(P, C)$ given in (3) and is defined as follows

$$\|J(P, C)\|_\infty := \|U_2 T(P, C) U_1\|_\infty \qquad (4)$$

where U_2 and U_1 are (square) weighting functions. The weighting functions U_1 and U_2 are chosen in such a way that the bandwidth of the resulting feedback connection can be adjusted, which will increase the speed of decay of the resulting servo error depicted in Figure 5. Furthermore, the weighting functions can be used to design a controller C that allows for an additional suppression of the low frequent vibration of the servo error.

The performance characterization (4) is fairly general and will be used for analysis purposes in this paper. In this perspective, the performance objective function $J(P, C)$ as given in (4) will be used to evaluate both the identification of a set of models $\mathcal{P}$ and the additional reduction of a robust controller designed based on the set $\mathcal{P}$. For that purpose, the set of models $\mathcal{P}$ as used in this paper is discussed below.

Characterization of the set of models

In order to design a robust controller for the positioning mechanism of the wafer stepper, the estimation of a single approximate (nominal) model does not suffices. To be robust against any dissimilarities between a model and the actual system P_o, a set of models $\mathcal{P}$ needs to be estimated. Such a set of models allows one to capture the actual system P_o in the robust controller design, provided that $P_o \in \mathcal{P}$. An (upper) LFT

$$\mathcal{F}_u(Q, \Delta) := Q_{22} + Q_{21}\Delta(I - Q_{11}\Delta)^{-1} Q_{12} \qquad (5)$$

provides a general notation to represent all models $P \in \mathcal{P}$ as follows

$$\mathcal{P} = \{P \mid P = \mathcal{F}_u(Q, \Delta) \\ \text{with } \Delta \in R\mathcal{H}_\infty \text{ and } \|\Delta\|_\infty < 1\}$$

where Δ indicates an unknown (but bounded) uncertainty. The entries of the coefficient matrix Q in (5) dictate the way in which the set of models $\mathcal{P}$ is being structured. As a special entry one can recognize the nominal model, denoted by $\hat{P}$, for which $\Delta = 0$

$$\hat{P} := \mathcal{F}(Q, 0) = Q_{22}$$

Employing the knowledge of the controller C_o implemented on the system P_o for experimental considerations, the set of models $\mathcal{P}$ will be characterized by using the algebraic theory of fractional model representations (Vidyasagar 1985). In this way, the coefficient matrix Q in (5) is formed by considering a model perturbation that is structured according to a Youla-Kucera parametrization. Following this parametrization, the set of models used in this paper is structured as follows

$$\begin{aligned} \mathcal{P} = \{P \mid P = (\hat{N} + D_c\bar{\Delta})(\hat{D} - N_c\bar{\Delta})^{-1} \\ \text{with } \bar{\Delta} \in R\mathcal{H}_\infty \text{ and } \|\hat{V}\bar{\Delta}\hat{W}\|_\infty < 1\} \end{aligned} \quad (6)$$

where (N_c, D_c) and $(\hat{N}, \hat{D})$ respectively denote a right coprime factorization (*rcf*) of the controller C_o implemented on the system P_o and a nominal model $\hat{P}$, that satisfies $T(\hat{P}, C_o) \in R\mathcal{H}_\infty$. The (stable and stably invertible) weighting functions $\hat{V}$, $\hat{W}$ are used to normalize the upper bound on $\hat{V}\bar{\Delta}\hat{W}$.

In order to design a robust controller for the system P_o on the basis of the set of models $\mathcal{P}$, $P_o \in \mathcal{P}$ must be guaranteed. In order to $P_o \in \mathcal{P}$, additional prior information on the plant P_o must be introduced. This is due to the fact that $P_o \in \mathcal{P}$ cannot be validated solely on the basis of finite time, possibly disturbed, observations coming from the plant P_o (Mäkilä *et al.* 1995, Ninness and Goodwin 1995). Such information is in accordance with the uncertainty modelling procedure of Hakvoort (1994), that is used in this paper to bound the uncertainty $\bar{\Delta}$ in (6).

The LFT characterization of the models P within the set of models of (5) can be represented by the block diagram given in Figure 6. It can be verified from the map $col(d, u)$ to $col(z, y)$ in Figure 6 that the coefficient matrix Q in the LFT of (5) is given by

$$Q = \left[\begin{array}{c|c} \hat{W}^{-1}\hat{D}^{-1}N_c\hat{V}^{-1} & \hat{W}^{-1}\hat{D}^{-1} \\ \hline (D_c + \hat{P}N_c)\hat{V}^{-1} & \hat{P} \end{array}\right] \quad (7)$$

Consequently, the matrix Q contains all the relevant information in order to characterize the set of models $\mathcal{P}$. In

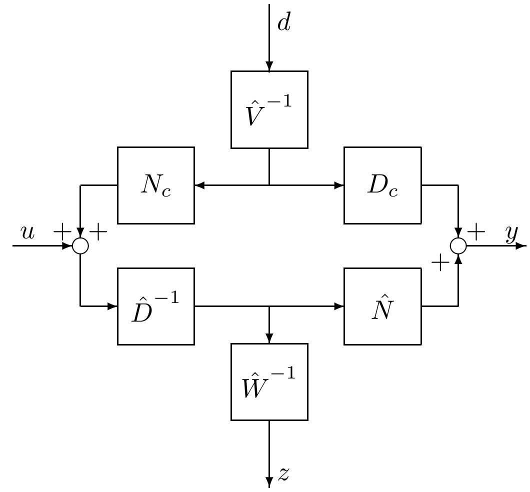

Figure 6. BLOCK DIAGRAM OF LFT REPRESENTATION

(7), the nominal model $\hat{P}$, or its *rcf* $(\hat{N}, \hat{D})$, and the stable and stably weighting filters $\hat{V}$ and $\hat{W}$ are the unknown quantities to be estimated.

Feedback relevant identification

In order to find a set of models that take into the intended application of the design of a controller, knowledge of the controller C_o that is implemented on the system P_o can be exploited to estimate a set of models $\mathcal{P}$. In order to estimate the set of models $\mathcal{P}$ given in (6), a factorization of a nominal model and frequency dependent stable and stably invertible weighting filters must be estimated.

To control the complexity of the controller being designed, it is required to bound the complexity of the nominal model $(\hat{N}, \hat{D})$ and the weighting filters $(\hat{V}, \hat{W})$. By again exploiting the knowledge of the controller C_o, an approximate identification of both a nominal model and the weighting filters can be tuned towards the intended control application. In other words, a set of models $\mathcal{P}$, subjected to the condition $P_o \in \mathcal{P}$, should be estimated such that

$$\sup_{P \in \mathcal{P}} \|J(P, C_o)\|_\infty \quad (8)$$

is minimized. In this way, a set of models is found for which the worst case performance for the controller C_o is minimized.

Minimizing (8) using the limited complexity *rcf* $(\hat{N}, \hat{D})$ and weighting filters $(\hat{V}, \hat{W})$ simultaneously is intractable. Therefore, minimization of (8) is tackled by estimating the *rcf* $(\hat{N}, \hat{D})$ and the pair $(\hat{V}, \hat{W})$ separately. Clearly, by the separate identification of the *rcf* $(\hat{N},\hat{D})$ of a nominal model $\hat{P}$ and the weighting filters $(\hat{V},\hat{W})$ only an upper bound on (8) can be minimized. However, available tools for the identification of a nominal factorization and an uncertainty bound can be exploited to complete the estimation of the set of models.

ESTIMATION OF A NOMINAL MODEL

Access to coprime factorizations

The first step in the characterization of the set of models $\mathcal{P}$, is the (approximate) identification of a stable nominal factorization $(\hat{N},\hat{D})$ of a (possibly unstable) nominal model $\hat{P}$. Access to a *rcf* of the system P_o for identification purposes can be obtained by a simple filtering of the signals present in the feedback connection $\mathcal{T}(P_o,C_o)$.

Inspecting (2), the transfer functions (P_oS_{in},S_{in}), with $S_{in}=(I+C_oP_o)^{-1}$, can be considered to be a stable (right) factorization of the system P_o with $P_o=[P_oS_{in}][S_{in}]^{-1}$. Denoting $r:=r_1+C_or_2=u+C_oy$ it can be observed that (P_oS_{in},S_{in}) is accessible from data as u and y are measured. To avoid the presence and estimation of common unstable zeros in the stable right factorization of P_o, the factorization needs to be a *rcf*. Furthermore, a *rcf* is not unique and access to different factorizations would be preferable.

As indicated in Van den Hof *et al.* (1995) or de Callafon and Van den Hof (1995*b*), an additional filtering of the reference signal r via $x:=Fr$ can be introduced to fulfil these requirements. With (2) this yields

$$x=F\begin{bmatrix}C_o & I\end{bmatrix}\begin{bmatrix}r_2\\ r_1\end{bmatrix}=F\begin{bmatrix}C_o & I\end{bmatrix}\begin{bmatrix}y\\ u\end{bmatrix} \tag{9}$$

and (2) reduces to

$$\begin{bmatrix}y\\ u\end{bmatrix}=\begin{bmatrix}P_oS_{in}F^{-1}\\ S_{in}F^{-1}\end{bmatrix}x+\begin{bmatrix}(I+P_oC_o)^{-1}\\ -C_o(I+P_oC_o)^{-1}\end{bmatrix}v \tag{10}$$

where $(P_oS_{in}F^{-1},S_{in}F^{-1})$ can be considered to be a (right) factorization of the system P_o.

In order to let $(P_oS_{in}F^{-1},S_{in}F^{-1})$ be a *rcf* of the system P_o, the form of the filter F in (9) is restricted and the result is summarized below.

Lemma 1. *Let P_o and C_o form a stable feedback connection $\mathcal{T}(P_o,C_o)$ then the following statements are equivalent.*

(i) $(P_oS_{in}F^{-1},S_{in}F^{-1})$ *is a* rcf.

(ii) *there exists a* rcf (N_x,D_x) *of an auxiliary model* P_x *with* $T(P_x,C)\in R\mathcal{H}_\infty$ *such that*

$$F=[D_x+C_oN_x]^{-1} \tag{11}$$

Both conditions on F imply $F\begin{bmatrix}C_o & I\end{bmatrix}\in R\mathcal{H}_\infty$.

Proof. See Van den Hof *et al.* (1995).

Consequently, a simple filtering (9) of the signals present in the feedback connection $\mathcal{T}(P_o,C_o)$ allows the access to a *rcf* of the system P_o. As a result, the following proposition to access a *rcf* of the system P_o on the basis of closed loop signals can be given.

Proposition 2. *Let the plant P_o and a controller C_o create an stable feedback connection $\mathcal{T}(P_o,C_o)$, then* (2) *can be rewritten as*

$$\begin{bmatrix}y\\ u\end{bmatrix}=\begin{bmatrix}N_{o,F}\\ D_{o,F}\end{bmatrix}x+\begin{bmatrix}I\\ -C_o\end{bmatrix}[I+P_oC_o]^{-1}v$$

where x is given in (9)*, F is given in* (11) *and $(N_{o,F},D_{o,F})$ is the* rcf *of the plant P_o given by*

$$\begin{bmatrix}N_{o,F}\\ D_{o,F}\end{bmatrix}=\begin{bmatrix}P_o\\ I\end{bmatrix}[I+CP_o]^{-1}[I+CP_x]D_x \tag{12}$$

Since x in (9) is uncorrelated with v, Proposition 2 gives rise to an equivalent open loop identification problem of the *rcf* $(N_{o,F},D_{o,F})$ of the system P_o.

Feedback relevant estimation of coprime factorizations

In the estimation of the *rcf* $(\hat{N},\hat{D})$, minimization of (8) must be taken into account when estimating a nominal factorization $(\hat{N},\hat{D})$. Furthermore, $\hat{P}=\hat{N}\hat{D}^{-1}$ is subjected to internal stability of the feedback connection $\mathcal{T}(\hat{P},C_o)$ in order to characterize the set of models $\mathcal{P}$ given in (6).

Clearly, at this stage the set of models $\mathcal{P}$ is unknown and (8) cannot be computed. In fact, the set of models $\mathcal{P}$ is arbitrarily large as the norm bounded uncertainty $\bar{\Delta}$ is (6) has not been characterized. Consequently, for any nominal model $\hat{P}$ there exist a norm bounded uncertainty $\bar{\Delta}$ that forms a set of models $\mathcal{P}$ for which $P_o\in\mathcal{P}$. As $P_o\in\mathcal{P}$, for any nominal model $\hat{P}\in\mathcal{P}$ the following upper bound for $\|J(\hat{P},C_o)\|_\infty$ can be given.

$$\|J(P_o,C_o)\|_\infty\leq\|J(P_o,C_o)\|_\infty+\|J(\hat{P},C_o)-J(P_o,C_o)\|_\infty$$

As $\|J(P_o,C_o)\|_\infty$ in the above expression does not depend on the nominal model $\hat{P}$, a *rcf* $(\hat{N},\hat{D})$ of a nominal model can be found by minimizing

$$\begin{aligned} &\|J(P,C_o) - J(P_o,C_o)\|_\infty = \\ &= U_2[T(P_o,C_o) - T(\hat{P},C_o)]U_1 \end{aligned} \tag{13}$$

and constitutes a control relevant criterion for the estimation of a nominal model.

Estimating a *rcf* $(\hat{N},\hat{D})$ of a nominal model by minimizing (13) can be done by minimizing an additive weighted difference between the *rcf* $(N_{o,F},D_{o,F})$ of the system P_o given in (12) and the *rcf* $(\hat{N},\hat{D})$ of the nominal model. This additive difference can be characterized as follows.

Lemma 3. *Let P_o and C_o create a stable feedback connection $\mathcal{T}(P_o,C_o)$ and let $(N_{o,F},D_{o,F})$ be the* rcf *of P_o given by* (12) *where F is any filter satisfying* (11). *Consider any model $\hat{P}$, then*

(i) *there exists a* rcf $(\hat{N},\hat{D})$ *of the model $\hat{P}$ such that $\hat{D} + C_o\hat{N} = F^{-1}$.*
(ii) *$U_2[T(P_o,C_o) - T(\hat{P},C_o)]U_1$ equals*

$$U_2\left(\begin{bmatrix} N_{o,F} \\ D_{o,F}\end{bmatrix} - \begin{bmatrix}\hat{N} \\ \hat{D}\end{bmatrix}\right) F \begin{bmatrix} C_o & I\end{bmatrix} U_1 \tag{14}$$

where $(\hat{N},\hat{D})$ is a rcf *of $\hat{P}$ that satisfies* (i).

For a proof of this lemma and a discussion of the minimization of (14) one is referred to de Callafon and Van den Hof (1995*b*). The estimation of a nominal factorization for the positioning mechanism of the wafer stepper will be illustrated in the next section.

Estimation of nominal factorizations

To estimate a nominal factorization $(\hat{N},\hat{D})$, frequency domain measurements of the factorization $N_{o,F}(\omega)$, $D_{o,F}(\omega)$ along a prespecified frequency grid are used. Subsequently, the curve fitting procedure described in de Callafon and Van den Hof (1995*a*) is used to tackle the weighted minimization of (14) frequency wise. As the curve fitting procedure is a non-linear optimization, an initial estimate is required to start the optimization. For that purpose, a multivariable least squares curve fitting procedure is used de Callafon *et al.* (1996).

An amplitude Bode plot of the *rcf* $(\hat{N},\hat{D})$ being estimated can be found in Figure 7. The resulting estimate of $col(\hat{N},\hat{D})$ is a 30th order discrete time multivariable model having 6 inputs and 3 outputs. Computing $\hat{P} = \hat{N}\hat{D}^{-1}$ yields a 30th order nominal model, having 3 inputs and 3 outputs. The Amplitude Bode plot of the model $\hat{P}$, along with the available frequency domain data computed via $N_{o,F}(\omega)D_{o,F}(\omega)^{-1}$ is depicted in Figure 8.

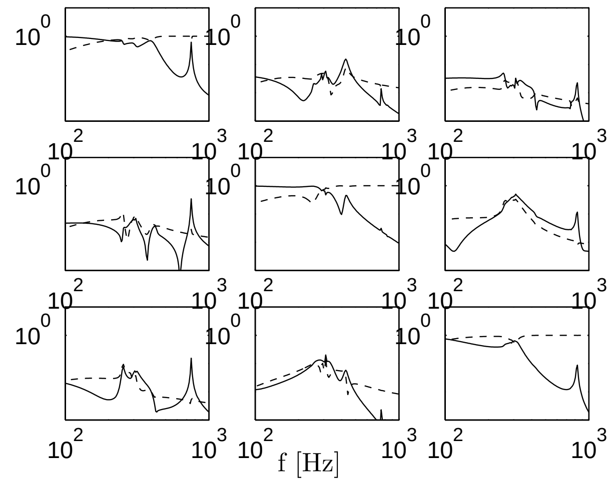

Figure 7. AMPLITUDE BODE PLOT OF ESTIMATED COPRIME FACTORS $\hat{N}$ (—) AND $\hat{D}$ (- -)

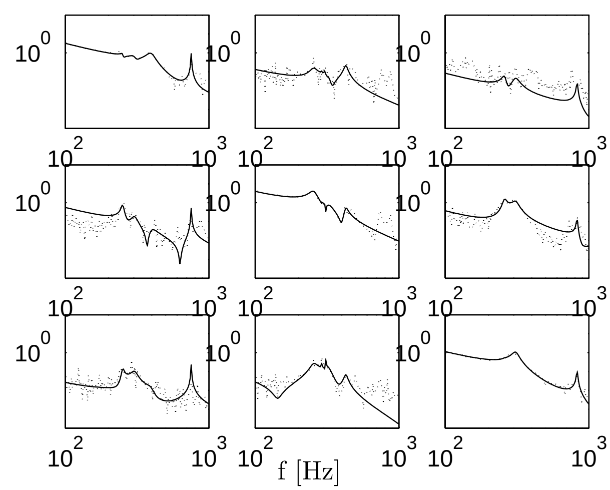

Figure 8. AMPLITUDE BODE PLOT OF COMPUTED $\hat{P}$ (—) AND FREQUENCY DOMAIN DATA (···)

Although stability of $\mathcal{T}(\hat{P},C_o)$ is not guaranteed by the estimation of the coprime factorization $(\hat{N},\hat{D})$ discussed here, the model $\hat{P}$ is stabilized by C_o. This mainly due to the fact that a good fit of the frequency domain data is obtained in closed-loop relevant frequency area around 200Hz.

ESTIMATION OF MODEL UNCERTAINTY BOUNDS

Access to model uncertainty

Once a *rcf* of a nominal model is obtained, an estimation of the allowable model perturbation $\bar{\Delta}$ in (6) can be performed. Estimation of an allowable model perturbation involves the characterization of an upper bound on $\bar{\Delta}$ in (6) via the stable and stably invertible filters $(\hat{V}, \hat{W})$ such that (8) is being minimized and $P_o \in \mathcal{P}$. For that purpose, first (an upper bound on) the allowable model perturbation $\bar{\Delta}$ is determined by applying a model error bounding estimation technique. An uncertainty estimation routine such as the procedure described by Hakvoort (1994) can be used to obtain a frequency dependent upper bound for $\bar{\Delta}$

$$\|\bar{\Delta}(\omega)\| \leq \delta(\omega) \text{ with probability} \geq \alpha \tag{15}$$

where α is a prechosen probability. In the multivariable case, the upper bound (15) can be obtained for each transfer function. Subsequently, stable and stably invertible weightings $\hat{V}$ and $\hat{W}$ can be determined that overbound the estimated upper bound $\delta(\omega)$.

Clearly, in order to estimate a frequency dependent upper bound on $\bar{\Delta}$, the map $\bar{\Delta}$ must be accessible from data. The following proposition provides the access to $\bar{\Delta}$ simply by an appropriate filtering of the signal present in the feedback connection $\mathcal{T}(P_o, C_o)$.

Proposition 4. *Consider C_o with* rcf (N_c, D_c) *and $\hat{P}$ with* rcf $(\hat{N}, \hat{D})$. *Let $\mathcal{T}(P_o, C_o)$ and $\mathcal{T}(\hat{P}, C_o)$ be stable and define*

$$z := (D_c + \hat{P}N_c)^{-1} \begin{bmatrix} I & -\hat{P} \end{bmatrix} \begin{bmatrix} y \\ u \end{bmatrix} \tag{16}$$

then $\bar{\Delta}$ in

$$z = \bar{\Delta}x + D_{c,i}(I + P_o C_o)^{-1} v \tag{17}$$

satisfies $\bar{\Delta} \in RH$, while x is given in (9) *and is uncorrelated with v.*

Consequently, Proposition 4 constitutes an open loop bounded error identification problem to find an upper bound for a stable $\bar{\Delta}$. The estimated upper bound of $\bar{\Delta}$ in (15) can then be used to complete the characterization of the set of models $\mathcal{P}$.

Feedback relevant estimation of model uncertainty

Limiting the complexity of a controller designed on the basis of the set of models $\mathcal{P}$ being identified also requires the complexity of the weighting filters $(\hat{V}, \hat{W})$ in (7) to be bounded. As a consequence, the estimated upper bound $\delta(\omega)$ in (15) needs to be approximated and over bounded by low complexity weighting filters $(\hat{V}, \hat{W})$. Using the LFT representation of the set of models $\mathcal{P}$ given in (7), the performance of a controller C applied to any model $P \in \mathcal{P}$ can be rewritten in terms of an LFT. The result has been summarized in the following lemma and will be used to address the estimation of limited complexity weighting filters $(\hat{V}, \hat{W})$.

Lemma 5. *Consider the set $\mathcal{P}$ defined in* (6) *that uses the knowledge of the controller C_o and let C be any controller such that the map $J(P, C) = U_2 T(P, C) U_1$ is well-posed for all $P \in \mathcal{P}$. Then*

$$J(P, C) = \mathcal{F}_u(M, \Delta) \; \forall P \in \mathcal{P}$$

where the entries of M are given by

$$\begin{aligned} M_{11} &= -\hat{W}^{-1}(\hat{D} + C\hat{N})^{-1}(C - C_o)D_c\hat{V}^{-1} \\ M_{12} &= \hat{W}^{-1}(\hat{D} + C\hat{N})^{-1} \begin{bmatrix} C & I \end{bmatrix} U_1 \\ M_{21} &= -U_2 \begin{bmatrix} -I \\ C \end{bmatrix} (I + \hat{P}C)^{-1}(I + \hat{P}C_o)D_c\hat{V}^{-1} \\ M_{22} &= U_2 \begin{bmatrix} \hat{N} \\ \hat{D} \end{bmatrix} (\hat{D} + C\hat{N})^{-1} \begin{bmatrix} C & I \end{bmatrix} U_1 \end{aligned} \tag{18}$$

Proof. By algebraic manipulation, see de Callafon and Van den Hof (1997).

It can be observed from (18) that substitution of $C = C_o$ yields $M_{11} = 0$. This implies that when the controller C_o is applied to the estimated set of models $\mathcal{P}$, the upper LFT $\mathcal{F}_u(M, \Delta)$ modifies into

$$M_{22} + M_{21}\Delta M_{12} \tag{19}$$

which is an affine expression in Δ. Substituting M_{21} and M_{12} in (19) with $\Delta = \hat{V}\bar{\Delta}\hat{W}$ yields the following expression

$$M_{22} + M_{21}\Delta M_{12} = M_{22} + W_2\bar{\Delta}W_1$$

where

$$\begin{aligned} W_2 &= -U_2 \begin{bmatrix} -D_c \\ N_c \end{bmatrix} D_c \\ W_1 &= \hat{D}^{-1}(I + C_o\hat{P})^{-1} \begin{bmatrix} C_o & I \end{bmatrix} U_1 \end{aligned} \tag{20}$$

Consequently, the effect of replacing an accurate (and high order estimate) of the upper bound $\bar{\Delta}$ by a low order upper bound approximation $\tilde{\Delta}$ on the (robust) performance $\|J(P_o,C_o)\| = \|M_{22}+W_2\bar{\Delta}W_1\|$ can be bounded by the following triangular inequality

$$\begin{array}{c}\|M_{22}+W_2\bar{\Delta}W_1\| \leq \\ \|M_{22}+W_2\tilde{\Delta}W_1\| + \|W_2(\bar{\Delta}-\tilde{\Delta})W_1\|\end{array} \quad (21)$$

From (21) it can be observed that, similar to identification of a low complexity factorization of a nominal model, a weighted difference between the actual and highly complex uncertainty $\bar{\Delta}$ and the low complexity approximation $\tilde{\Delta}$ must be taken into account. The weightings W_2 and W_1 are given in (20) and are known, once a nominal factorization $(\hat{N},\hat{D})$ has been estimated.

Estimation of model uncertainty

Given the nominal factorization $(\hat{N},\hat{D})$ and a normalized *rcf* (N_c,D_c) of the controller C_o, an estimation of the allowable model perturbation $\bar{\Delta}$ in (6) is performed. For that purpose, the uncertainty estimation as presented in Hakvoort (1994) has been applied to estimate a frequency dependent upper bound on $\bar{\Delta}$. As a complete discussion of the uncertainty estimation procedure of Hakvoort (1994) is beyond the scope of this paper, only the result is presented in Figure 9.

It can be observed from Figure 9 that the upper bound of the frequency domain estimation of $\bar{\Delta}$ is crossing the upper bound $\delta(\omega)$. Partly, this is due to the fact the upper bound only holds within a prespecified probability of 95%.

USING THE IDENTIFIED SET FOR CONTROL DESIGN

On the basis of the identified set of models, a robust controller was designed via a μ-synthesis (Zhou *et al.* 1996). As $\delta(\omega)$ is only a frequency dependent upper bound for $\bar{\Delta}$, low frequent weighting filters $(\hat{V},\hat{W})$ are used to parametrize the upper bound on the estimated uncertainty bound $\delta(\omega)$ depicted in Figure 9. In this way, the estimated upper bound can be taken into account during a robust controller design.

In the construction of $(\hat{V},\hat{W})$ the weightings W_1 and W_1 given in (20) are used to emphasize the frequency range for the upper bounding of $\delta(\omega)$ by the parametric stable and stablly invertible weightings $(\hat{V},\hat{W})$ is most critical. It can be observed from (20) that the input sensitivity $(I+C_o\hat{P})^{-1}$, based on the nominal model $\hat{P}$, is incorporated in the weightings given in (20). As a consequence, the weightings emphasize (again) the closed-loop relevant frequency area around 200Hz.

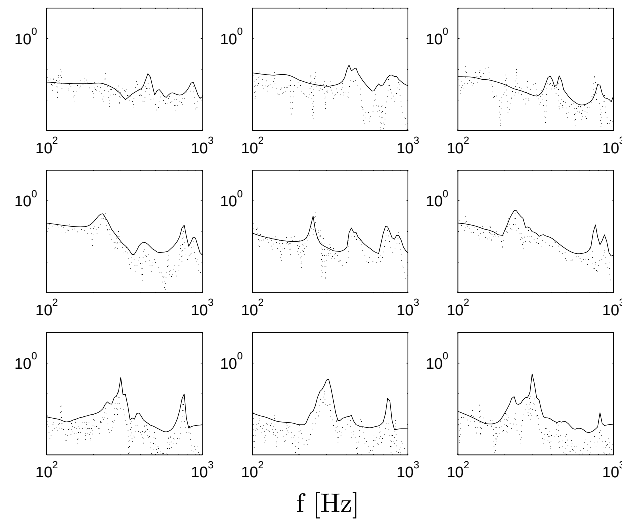

Figure 9. AMPLITUDE BODE PLOT OF ESTIMATED UNCERTAINTY BOUND $\delta(\omega)$ (—) OF $\bar{\Delta}$ AND FREQUENCY DOMAIN ESTIMATE OF $\bar{\Delta}$ $(\cdots)$

Extracting the controller C from the LFT given in (18), the following *lower* LFT $\mathcal{F}_l(G,C)$ can be obtained for the synthesis of a robust controller.

Proposition 6. *Consider the map M given in* (18). *Then $M=\mathcal{F}_l(G,C)$ where G is given by*

$$\begin{bmatrix}\hat{W}^{-1} & 0 & 0\\ 0 & U_2 & 0\\ 0 & 0 & I\end{bmatrix}\left[\begin{array}{c|cc|c}\hat{D}^{-1}N_c & 0 & \hat{D}^{-1} & \hat{D}^{-1}\\ \hline (D_c+\hat{P}N_c) & 0 & \hat{P} & \hat{P}_i\\ 0 & 0 & I & I\\ \hline -(D_c+\hat{P}N_c) & I & -\hat{P} & -\hat{P}\end{array}\right]\begin{bmatrix}-\hat{V}^{-1} & 0 & 0\\ 0 & U_1 & 0\\ 0 & 0 & I\end{bmatrix}$$

Invoking the μ-design, a high order multivariable feedback controller is obtained. In order to implement the controller being designed, an additional closed-loop controller reduction Wortelboer (1993) was used to reduce the controller to a 32nd order state space realization. A comparison between the controller C_o previously implemented on the system P_o and the newly designed controller C is given in terms of the amplitude Bode plot depicted in Figure 10.

In order to show the improvement of the positioning control of the servo mechanism in the wafer stepper, the reference signals r_1 and r_2 depicted in Figure 4 are put on the newly designed feedback connection $\mathcal{T}(P_o,C)$. A comparison with the servo error of Figure 5 obtained with the previous controller C_o is depicted in Figure 11. It can be seen from Figure 11 that both the speed and the accuracy of positioning have been improved successfully.

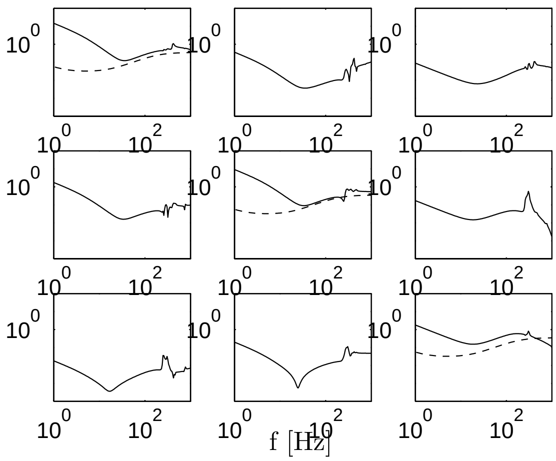

Figure 10. AMPLITUDE BODE PLOT OF OLD CONTROLLER C_o (- -) AND NEWLY DESIGNED CONTROLLER C (—)

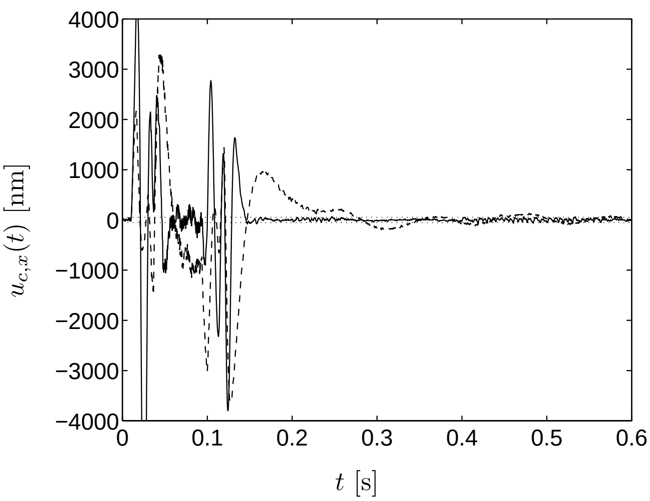

Figure 11. SERVO ERROR RESPONSE TO A STEP IN X-DIRECTION WITH OLD CONTROLLER C_o (- -) AND NEW CONTROLLER C (—)

CONCLUSIONS

This paper discusses the approximate and feedback relevant parametric identification of a servo mechanism present in a wafer stepper. Via the identification of a set of models, built up from a nominal model along with an allowable model perturbation, the dynamical behaviour of the servo mechanism has been modelled.

The feedback relevant identification in this paper is based on the algebraic theory of stable fractional representations. This framework leads to an equivalent open loop identification of a stable factorization of a nominal model and an allowable model perturbation written in terms of a (dual) Youla parametrization. Both the estimation of nominal factorization and the uncertainty estimation can be performed in a feedback relevant way, taking the intended control application of the estimated set of model into account.

The estimated set of models is used for the design of a robust controller for which significant improvement of the positioning mechanism has been illustrated.

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REFERENCES

Boyd, S.P. and C.H. Barrat (1991). *Linear Controller design – Limits of Performance*. Prentice–Hall, Englewood Cliffs, New Jersey, USA.

de Callafon, R.A., D. Roover and P.M.J. Van den Hof (1996). Multivariable least squares frequency domain identification using polynomial matrix fraction descriptions. In *Proc. 35th IEEE Conference on Decision and Control*. Kobe, Japan. pp. 2030–2035.

de Callafon, R.A. and P.M.J. Van den Hof (1995*a*). Control relevant identification for H_∞–norm based performance specifications. In *Proc. 34th IEEE Conference on Decision and Control*. New Orleans, USA. pp. 3498–3503.

de Callafon, R.A. and P.M.J. Van den Hof (1995*b*). Filtering and parametrization issues in feedback relevant identification based on fractional model representations. In *Proc. 3rd European Control Conference,* Vol. *1*. Rome, Italy. pp. 441–446.

de Callafon, R.A. and P.M.J. Van den Hof (1997). Suboptimal feedback control by a scheme of iterative identification and control design. *Mathematical Modelling of Systems,* Vol. *3*. pp. 77–101.

de Callafon, R.A., P.M.J. Van den Hof and D.K. de Vries (1994). Identification and control of a compact disc mechanism using fractional representations. In *Prepr. 10th IFAC Symp. on System Identification,* Vol. *2*. Copenhagen, Denmark. pp. 121–126.

Doyle, J.C., B.A. Francis and A.R. Tannenbaum (1992). *Feedback Control Theory*. MacMillan, New York, USA.

Goodwin, G.C., M. Gevers and B. Ninness (1992). Quantifying the error in estimated transfer functions with application to model order selection. *IEEE Trans. on Automatic Control, AC–37*. pp. 913–928.

Hakvoort, R.G. (1994). System Identification for Robust Process Control: Nominal Models and Error Bounds. PhD thesis. Delft University of Technology, Delft, the Netherlands.

Helmicki, A.J., C.A. Jacobson and C.N. Nett (1993). Least squares methods for H_∞ control–oriented system identification. *IEEE Trans. on Automatic Control, AC–38*. pp. 819–826.

Ljung, L. (1987). *System Identification: Theory for the User*. Prentice–Hall, Englewood Cliffs, New Jersey, USA.

Ljung, L. (1995). *System identification toolbox user's guide*. The Mathworks, Inc.

Mäkilä, P.M. and J.R. Partington (1995). On boundend–error identification of feedback systems. *Int. Journal of Adaptive Control and Signal Processing,* Vol. *9,* No. *1*. pp. 47–61.

Mäkilä, P.M., J.R. Partington and T.K. Gustafson (1995). Worst–case control–relevant identification. *Automatica,* Vol. *31*. pp. 1799–1819.

Ninness, B. and G.C. Goodwin (1995). Estimation of model quality. *Automatica,* Vol. *31*. pp. 1771–1797.

Partington, J.R. and P.M. Mäkilä (1995). Analysis of linear methods for robust identification in l_1. *Automatica,* Vol. *31*. pp. 755–758.

de Roover, D., O.H. Bosgra, F.B. Sperling and M. Steinbuch (1996). High–performance motion control of a flexible mechanical servomechanism. *Selected Topics in Identification, Modelling and Control,* Vol. *9*. pp. 69–77. O.H. Bosgra, P.M.J. Van den Hof and C.W. Scherer (Eds.).

de Roover, D. and R.N. van Marrewijk (1995). Systematic analysis of the dynamic behaviour of an electro–mechanical servosystem. *Selected Topics in Identification, Modelling and Control,* Vol. *8*. pp. 75–83. O.H. Bosgra, P.M.J. Van den Hof and C.W. Scherer (Eds.).

Schrama, R.J.P. (1992*a*). Accurate identification for control design: the necessity of an iterative scheme. *IEEE Trans. on Automatic Control, AC–37*. pp. 991–994.

Schrama, R.J.P. (1992*b*). Approximate Identification and Control Design with Application to a Mechanical System. PhD thesis. Delft University of Technology, Delft, the Netherlands.

Schrama, R.J.P. and O.H. Bosgra (1993). Adaptive performance enhancement by iterative identification and control design. *Int. Journal of Adaptive Control and Signal Processing,* Vol. *7,* No. *5*. pp. 475–487.

Söderström, T. and P. Stoica (1989). *System identification*. Prentice–Hall, Englewood Cliffs, New Jersey, USA.

Van den Hof, P.M.J. and R.J.P. Schrama (1995). Identification and control – closed loop issues. *Automatica,* Vol. *31,* No. *12*. pp. 1751–1770.

Van den Hof, P.M.J., R.J.P. Schrama, R.A. de Callafon and O.H. Bosgra (1995). Identification of normalised coprime plant factors from closed–loop experimental data. *European Journal of Control,* Vol. *1,* No. *1*. pp. 62–74.

Vidyasagar, M. (1985). *Control System Synthesis: A Factorization Approach*. MIT Press, Cambridge, Massachusetts, USA.

Wortelboer, P.M.R. (1993). Frequency Weighted Balanced Reduction of Closed–Loop Mechanical Servo–Systems: Theory and Tools. PhD thesis. Delft University of Technology, Delft, the Netherlands.

Zang, Z., R.R. Bitmead and M.R. Gevers (1995). Iterative weighted least–squares identification and weighted LQG control design. *Automatica,* Vol. *31*. pp. 1577–1594.

Zhou, K., J.C. Doyle and K. Glover (1996). *Robust and Optimal Control*. Prentice–Hall, Upper Saddle River, New Jersey, USA.