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Robust Estimation and Automatic Controller Tuning in Vibration Control of Time Varying Harmonic Disturbances

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Abstract: This paper presents theoretical and experimental results of a newly developed automatic controller tuning algorithm called Robust Estimation for Automatic Controller Tuning (REACT) to tune a linear feedback controller to the unknown spectrum of disturbances present in a feedback loop. With model uncertainty and controller perturbations described in (dual) Youla parametrizations, the REACT algorithm allows recursive least squares based tuning of a feedback controller in the presence of model uncertainty to minimize the variance of control performance related signal. It is shown how stability of the feedback can be maintained during adaptive regulation, while simulation and experimental results on a mechanical test bed of an active suspension system illustrate the effectiveness of the algorithm for vibration isolation of periodic disturbances with unknown and varying frequencies.

Keywords: Adaptive Regulation; Model Uncertainty; Vibration Control.

1. INTRODUCTION

Active Vibration Control (AVC) can be powerful tool to suppress undesirable mechanically induced disturbances via feedback control and applications can be found in structural, mechanical and acoustic control, see e.g. R. Fuller et al. (1997); Preumont (2002); Crocker (2007); Du and Xie (2010). AVC relies on the controlled emission of vibrational energy via out-of-phase actuation forces to compensate for external vibrations and is often a combination of feedforward and feedback compensation of acoustic, accelerometer or motion sensor data. The exact out-of-phase and amplitude of the actuation forces as a function of the frequency is crucial for the effectiveness of an active vibration control solution and often relies on the accurate formulation of a dynamic model of either the (mechanical or structural) system or the disturbances.

The use of feedback can alleviate the need to accurately model the exact phase and amplitude of the actuation forces as a function of the frequency. A good example is the internal model principle (Francis and Wonham, 1976) or the more general form of repetitive control (Tomizuka et al., 1989; Steinbuch, 2002) that only requires a (finite memory) resonator to compensate for general periodic disturbances. Feedback compensation of periodic disturbances can be shown to be equivalent to (adaptive) feedforward compensation (Bodson, 2005) under certain conditions. However, the requirements on maintaining closed-loop stability in feedback control of periodic disturbances require model knowledge (Pipeleers et al., 2009) or limitations on the frequency contents of the feedback compensation (Ahn et al., 2007).

A further challenge arises when the spectral contents of the disturbance may change over time. Adaptive solutions for this problem have been proposed using feedforward control (Du and Xie, 2010) that rely on linearly parametrized filters with recursive estimation techniques (de Callafon and Zeng, 2006), but require additional sensors for disturbance monitoring and estimation. Adaptive feedback solutions implementations that use the same estimation principles often separate model estimation from controller tuning (Tokhi and Veres, 2002) to address closed-loop stability. Unfortunately, additional excitation signals during adaptation are needed for accurate model estimation results, while time separation between adaptation and real-time feedback control is required to guarantee closed-loop stability.

Although subsequent model estimation and controller tuning can address adaptive vibration control, often the dynamics of the plant to be controlled is partially known and only the spectral contents of the disturbance changes over time. Such a situation can be characterized by adaptive regulation (Landau et al., 2009) and can be addressed by either estimating and scheduling for the disturbance dynamics (Bohn et al., 2004; Kinney et al., 2007) or tune the feedback controller directly based on time domain observations and the partially known dynamics of the plant to be controlled. The latter approach is the main focus of this paper and it is shown how our REACT algorithm (Kinney and de Callafon, 2009) for Robust Estimation for Automatic

Controller Tuning can be applied to a mechanical test bed of an active suspension system that requires adaptation of a feedback controller for multiple periodic disturbances with unknown and varying frequencies. Using model uncertainty to describe the (partially) know plant dynamics and controller tuning perturbations described in (dual) Youla parametrizations, the REACT algorithm allows minimization of the variance of control performance related signal while maintaining closed-loop stability.

2. AUTOMATIC CONTROLLER TUNING

To set up the REACT algorithm, we first quantify the limited knowledge of the plant in the feedback loop via a controller dependent uncertainty model and then we define the allowable controller perturbations for adaptive regulation. We start out with an initial and fixed feedback controller C that is known to stabilize the (unknown) plant G_o. The initial controller C may be chosen to be zero in case G_o is stable, but the initial controller information is used to describe the uncertainty model of the plant G_o. The initial controller C is a discrete-time transfer function that admits a (right) coprime factorization (Zhou et al., 1996) given by $C = N_c D_c^{-1}$ and a trivial choice for a stable controller C is $N_c = C, D_c = I$. Although the distinction between left and right coprime factorizations is irrelevant for single input single output (SISO) applications, we maintain this distinction throughout the paper as adaptation can also be formulated for multivariable controllers.

The limited knowledge on the dynamics of the plant G_o is modeled as a nominal model G_x with allowable perturbation or uncertainty Δ_G. The nominal model G_x is a SISO, discrete-time transfer function that is also (internally) stabilized by the initial controller C and has a right coprime factorization given by $G_x = N_x D_x^{-1}$. The right coprime factors (N_c, D_c) of the controller C and (N_x, D_x) of the nominal model G_x and the uncertainty Δ_G are now used to describe the following set of plants

$$\Pi = \{G_\Delta : G_\Delta = (N_x + D_c\Delta_G)(D_x - N_c\Delta_G)^{-1}, \text{ where } \Delta_G \in RH_\infty, \ \|\Delta_G\|_\infty < 1/\gamma\} \tag{1}$$

that models limited knowledge on the dynamics of the plant G_o. The uncertainty Δ_G is chosen such that $G_o \in \Pi$ by overbounding the frequency response of Δ_G. The set in (1) now constitutes the limited knowledge of our plant G_o to be controlled. Knowledge of frequency dependency of Δ_G can easily be incorporated in this set Π by $\|W_G\Delta_G\|_\infty < 1$, where $W_G, W_G^{-1} \in \mathbf{R}H_\infty$, but to simplify notation we simply assume $\|\Delta_G\|_\infty < 1/\gamma$.

From (1) it can be observed that we are considering a model uncertainty that follows dual-Youla parameterization (Douma et al., 2003) of the plant. As the initial controller C stabilizes the unknown plant G_o, it is easy to verify from the dual-Youla parametrization that $\Delta_G \in \mathbf{R}H_\infty$. Notice that if we have no model uncertainty ($\Delta_G = 0$), the knowledge on the plant G_o is equal to the nominal model $G_x = N_x D_x^{-1}$.

We consider a disturbance signal d that is modeled as an additive disturbance on the plant output y via

$$y(t) = G_o(q)u(t) + d(t) \tag{2}$$

For adaptive regulation purposes we consider d as a sum of sinusoids with unknown frequency, magnitude and phase and the frequency may change abruptly (step wise) with time. It should be noted that when $\omega_i, i = 1, \ldots, n_d$ are known constants, this problem is can be solved with the servocompensator theory developed by Francis and Wonham (1976) or the work by de Roover et al. (2000). However, in this paper we are considering the case where the frequencies $\omega_i, i = 1, \ldots, n_d$ are not known a priori and will develop a control algorithm to cancel them automatically via adaptive regulation.

For adaptation of the controller, we consider a perturbed or tuned controller C_Δ given by a standard Youla parametrization

$$C_\Delta = N_{C_\Delta} D_{C_\Delta}^{-1} = (N_c + D_x\Delta_C)(D_c - N_x\Delta_C)^{-1} \tag{3}$$

where N_{C_Δ} and D_{C_Δ} indicate the right coprime factors of the tuned controller. In (3) we use again the right coprime factors (N_x, D_x) of the nominal model G_x and the right coprime factors (N_c, D_c) of the initial controller in C, similarly as in (1). The controller perturbation Δ_C is used to improve controller performance using adaptive regulation while maintaining robust stability in the presence of Δ_G. Notice that if $\Delta_C = 0$, then the perturbed controller is equal to the initial fixed $C = N_c D_c^{-1}$.

The reason we resort to the Youla parametrization in (3) is to guarantee internal stability of the feedback connection of the nominal model G_x and the perturbed controller C_Δ by requiring the Youla parameter $\Delta_C \in \mathbf{R}H_\infty$ (to be stable). In presence of model uncertainty $\Delta_G \in \mathbf{R}H_\infty$ and thus limited knowledge of the plant G_o dynamic with $G_o \in \Pi$, $\Delta_C \in \mathbf{R}H_\infty$ does not suffice to guarantee stability of the feedback connection of the plant G_o and the perturbed controller C_Δ. However, by using the small gain theorem (Van der Schaft, 1996; Zhou et al., 1996) and $\|\Delta_G\|_\infty < 1/\gamma$ it is easily shown that if $\|\Delta_C\|_\infty \leq \gamma$ then the closed loop system of G_o and C_Δ is internally stable (Douma et al., 2003; Kinney, 2009). Using a frequency weighting W_G to bound the model uncertainty $\|W_G\Delta_G\|_\infty < 1$ leads to $\|W_G^{-1}\Delta_C\|_\infty < 1$. Thus, any method that is used to determine the controller perturbation Δ_C should uphold a frequency dependent bound to ensure closed-loop internal stability during adaptive regulation.

3. ADAPTIVE ALGORITHM

As mentioned in the previous section, the controller is tuned by finding a Youla parameter Δ_C to improve performance while maintaining closed-loop stability. For simplicity and easy of implementation the perturbation Δ_C is chosen to be a discrete-time Finite Impulse Response (FIR) filter given by

$$\Delta_C(q, \psi) = \sum_{k=1}^{N_\theta} \psi(k) q^{-k}, \ \ \psi(k) \in \mathbf{R} \tag{4}$$

where q is the time-shift operator. The choice for the parametrization (4) ensures $\Delta_C(q, \psi) \in \mathbf{R}H_\infty \ \forall \psi(k) \in \mathbf{R}$. It should be noted that the choice of an FIR filter parametrization using q^{-k} as a basis function can be generalized to a parametrization with rational orthogonal basis functions (Heuberger et al., 2005) while still $\Delta_C(q, \psi) \in \mathbf{R}H_\infty$. For notational brevity we will use $\Delta_C(\psi)$ and drop the time-shift argument q in the following.

To estimate the real-valued parameters ψ in (4) one can use Recursive Least Squares (RLS) or Least Mean Squares (LMS) estimation techniques that also allow the minimization of the variance of control performance related error signal $e(t)$. The equation that relates this error e, the additive output disturbance d and a reference signal r on the output y is given by

$$e(t, \psi) = D_{C_\Delta}(\psi)(D_{C_\Delta}(\psi) + G_\Delta N_{C_\Delta}(\psi))^{-1}(G_\Delta d(t) + r(t))$$

where we used the coprime factors $N_{C_\Delta}(\psi)$ and $D_{C_\Delta}(\psi)$ of $C_\Delta(\psi)$ similar to (3) and G_Δ as defined by the model set Π in (1).

Using a (left) coprime factorization $C = \tilde{D}_c^{-1}\tilde{N}_c$ of the initial controller C and the (right) coprime factorization $G_x = N_x D_x^{-1}$ of the nominal model we can define

$$\Lambda_0 = \tilde{D}_c D_x + \tilde{N}_c N_x$$

and $\Lambda_0, \Lambda_0^{-1} \in \mathbf{R}H_\infty$ as the initial controller C internally stabilizes the nominal model G_x. By defining $\eta(t,\psi)$ as

$$\eta(t,\psi) = \Lambda_o^{-1} D_x e(t,\psi) + \Lambda_o^{-1} N_x y_c(t,\psi), \quad (5)$$

where $y_c(t,\psi) = C_\Delta(q)(r(t) - y(t))$ is the output signal of the adapted feedback controller as defined by (3) and (4), the error signal $e(t,\psi)$ can be rewritten as

$$e(t,\psi) = (D_c - N_x \Delta_C(\psi))\eta(t,\psi) \quad (6)$$

showing the (quasi) affine relation of the error signal $e(t,\psi)$ as a function of the linearly parameterized $\Delta_C(\psi)$ in (4) and the signal $\eta(t,\psi)$. Due to the (quasi) affine relationship, minimizing of the two-norm of $e(t,\psi)$ over ψ would require a nonlinear optimization and would limit real-time implementation due to computational resources.

Instead, we simplify the computation by temporarily fixing the parameter ψ for the signal $\eta(t,\psi)$. We now define $\varepsilon(t,\theta,\psi)$ as

$$\varepsilon(t,\theta,\psi) = (D_c - N_x \Delta_C(\theta))\eta(t,\psi) \quad (7)$$

that separates the *currently* adapted controller $C(\psi)$ implemented on the plant and generating the signal $\eta(t,\psi)$ in (5) from the *newly* adapted controller $C(\theta)$ to be computed. It is clear that $\varepsilon(t,\psi,\psi) = e(t,\psi)$ and small changes between ψ and θ during adaptation maintain this relation. For the actual computation of θ we employ a minimization of the two-norm of $\varepsilon(t,\theta,\psi)$ as function of the linearly paramterized $\Delta_C(\theta)$ similar to (4) and is an affine optimization for which we can formulate recursive solutions as the data $\eta(t,\psi)$ in (5) becomes available.

3.1 LMS updates

The two-norm of $\varepsilon(t,\theta,\psi)$ in (7) as a function of the linearly parametrized $\Delta_C(\theta)$ is formulated as

$$\min_{\theta \in \mathbf{R}^{N_\theta}} V(\theta,\psi) = \frac{|\varepsilon(t,\theta,\psi)|^2}{2} + \lambda \frac{\theta^T\theta}{2}$$

where the second (regularization) term with weighting λ can be used to limit the change in the parameter θ. The regularization is useful in light of the error term $\varepsilon(t,\theta,\psi)$ in (7) that will resemble $e(t,\psi)$ in (6) for small changes between the currently implemented controller and the newly updated controller. The gradient of $V(\theta,\psi)$ with respect to θ is given by

$$\begin{aligned}\frac{\partial V(\theta)}{\partial\theta} &= \varepsilon(t,\theta,\psi)\frac{\partial\varepsilon(t,\theta,\psi)}{\partial\theta} + \lambda\theta \\ &= -\varepsilon(t,\theta,\psi)X_f(t) + \lambda\theta,\end{aligned}$$

where $X_f(t) = [x_f(t-1) \ldots x_f(t-N_\theta)]^T$ and $x_f(t) = N_x\eta(t,\psi)$.

To formulate a recursive solution for each time step t, consider the update equation for θ given by

$$\begin{aligned}\theta_t &= \theta_{t-1} - \mu\frac{\partial V(\theta)}{\partial\theta} \\ &= \theta_{t-1}(1-\mu\lambda) + \mu\varepsilon(t,\theta,\psi)X_f(t) \ ,\end{aligned}$$

where μ denotes the step size and boils down to a filtered reference leaky Least Mean Squares (LMS) algorithm (Haykin, 2002). Additionally, normalizing the algorithm is possible with

$$\theta_t = \theta_{t-1}(1-\mu\lambda) + \frac{\mu}{\delta + X_f(t)^T X_f(t)}\varepsilon(t,\theta,\psi)X_f(t) \ ,$$

which gives a filtered reference Leaky-NLMS algorithm to update θ (Haykin, 2002).

3.2 RLS updates

Next to the LMS updates, we can recursively minimize the two-norm of $\varepsilon(t,\theta,\psi)$ in (7) as a function of the linearly parametrized $\Delta_C(\theta)$ via Recursive Least Squares minimization. RLS estimation gives in general a faster rate of convergence and the algorithm can be considered as a special case of Kalman filtering where the variance σ of the measurement noise is normalized to $\sigma = 1$ and the state covariance noise matrix $Q = 0$ (Haykin, 2002).

Using the general formulation of the Kalman filter to update the parameter estimate θ at each time step t we find

$$\theta_t = \theta_{t-1} + \varepsilon(t,\theta,\psi)G(t)$$

where $\varepsilon(t,\theta,\psi)$ is updated via

$$\varepsilon(t,\theta,\psi) = D_c\eta(t,\psi) - \varepsilon(t-1,\theta)^T R$$

in which

$$\begin{aligned}G(t) &= \frac{P(t-1)R}{\sigma + R^T P(t-1)R} \\ R &= [R(t-1)\ R(t-2)\ \cdots\ R(t-N_\theta)] \\ R(t) &= N_x\eta(t,\psi)\end{aligned}$$

and $P(t)$ is found via an iterative update of the state covariance matrix

$$P(t) = P(t-1) - \frac{P(t-1)RR^T P(t-1)}{\sigma + R^T P(t-1)R} + qI_{N_\theta \times N_\theta}.$$

The equivalent measurement noise variance σ and equivalent state covariance noise matrix $Q = qI_{N_\theta \times N_\theta}$ in the above algorithm can be adjusted to modify the convergence rate of the RLS algorithm (Haykin, 2002).

3.3 Bounding and Filtering of Parameter Estimate

Once a parameter estimate θ is found by recursive LMS or RLS updates θ_t at each time step, implementation of a newly adapted controller $C(\theta)$ is done by first bounding and filtering the recursive parameter estimate θ_t. Direct implementation of $\Delta_C(\theta_t)$ at each time step might invalidate the small gain condition $\|\Delta_C(\theta_t)\|_\infty \le \gamma$, whereas fast time fluctuations of $C(\theta_t)$ might cause instabilities due to the (fast) time varying nature of the feedback loop. Instead, we will implement $C(\psi_t)$, where ψ_t will be a bounded and filtered version of θ_t.

Before implementing the controller parameters ψ_t, a bound on $\Delta_C(\psi)$ must be enforced to maintain stability robustness. It was shown in (Kinney and de Callafon, 2009; Kinney, 2009) that if

$$\|\psi\|_2 \le \frac{\gamma}{\sqrt{N_\theta}} \quad (8)$$

then

$$\|\Delta_C(\psi)\|_\infty \le \gamma \quad (9)$$

for any fixed value $\psi = \psi_t$. Based on this result, we will constrain the parameters ψ to the set

$$\mathbf{S} = \left\{\psi : \|\psi\|_2 \le \frac{\gamma}{\sqrt{N_\theta}}\right\} .$$

It was also shown in (Kinney and de Callafon, 2009; Kinney, 2009) that low-pass filtering will improve the convergence of the adaptive algorithm. Specifically, if we use the bound on $\|\psi\|_2$ in (8) then the fluctuations in ψ_t may be fast, while still maintaining robust stability. If we use the bound on $\|\Delta_C\|_\infty$ in (9), then the fluctuations in ψ_t must be slow to ensure stability. This is because the bound (8) is more conservative than (9).

However, in both cases low-pass filtering is needed to ensure that θ_t converges to the correct value, whereas low-pass filtering is not needed when $\Delta_G = 0$. With

$$\bar{\theta}_t = \frac{\gamma}{\sqrt{N_\theta}} \cdot \frac{\theta_t}{\|\theta_t\|_2}$$

and using simple first order filtering on the parameter estimates θ_t, the algorithm for updating the actual controller parameter ψ_t via the Youla parametrization

$$\begin{aligned} C_\Delta(\psi_t) &= (N_c + D_x \Delta_C(\psi_t))(D_c - N_x \Delta_C(\psi_t))^{-1} \\ \Delta_C(\psi_t) &= \sum_{k=1}^{N_\theta} \psi_t(k) q^{-k}, \;\; \psi_t(k) \in \mathbf{R} \end{aligned} \quad (10)$$

can now be written as

$$\psi_t = \begin{cases} \psi_{t-1} - \varepsilon(\psi_{t-1} - \theta_t) & \text{if } \|\theta_t\|_2 < \dfrac{\gamma}{\sqrt{N_\theta}} \\ \psi_{k-1} - \varepsilon(\psi_{k-1} - \bar{\theta}) & \text{otherwise} \end{cases}$$

in which ε is a pole location for parameter filtering.

4. APPLICATION TO VIBRATION ISOLATION

To demonstrate the REACT algorithm, it will applied to a mechanical test bed of an active suspension system located at the GIPSA-lab in Grenoble, France.

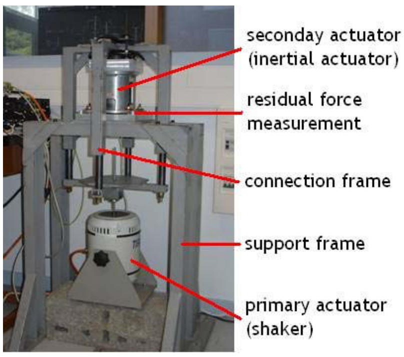

Fig. 1. Overview of mechanical test bed for vibration isolation

The test bed has two actuators that can generate forces and vibrations that are measured by a residual force sensor, as indicated in Fig. 1. The primary actuator is a conventional shaker that allows the creation of the (artificial) vibration disturbances $d(t)$ in (11), whereas the secondary actuator is an inertial actuator that allows the creation of controlled forces via a Voice Coil Motor to actively suppress vibration disturbances. The requirement would be to suppress unknown harmonic disturbances in the frequency range between 50 and 100Hz at different levels of complexity (multiple simultaneous harmonic disturbances). Frequency response data from the mechanical test bed is used to create a nominal model G_x of the secondary path (secondary actuator to residual force sensor), an initial stabilizing controller C and a quantification of the model uncertainty Δ_G to formulate the set of model Π in (1) to quantify the incomplete knowledge of the 'true' plant G_o to be controlled.

4.1 Nominal model estimation

Based on the requirement of being able to suppress unknown harmonic disturbances in the frequency range between 50 and 100Hz at different levels of complexity (multiple simultaneous harmonic disturbances), we anticipate controller perturbations $\Delta_C(\psi_t)$ in (10) during tuning in the same frequency range. Furthermore, the requirement of stability robustness based on the model uncertainty Δ_G in (1) bounded by $\|W_G \Delta_G\|_\infty < 1$ requires $\|W_G^{-1} \Delta_C\|_\infty \leq 1$ putting immediate restrictions on the size of the model uncertainty Δ_G in the frequency range between 50 and 100Hz.

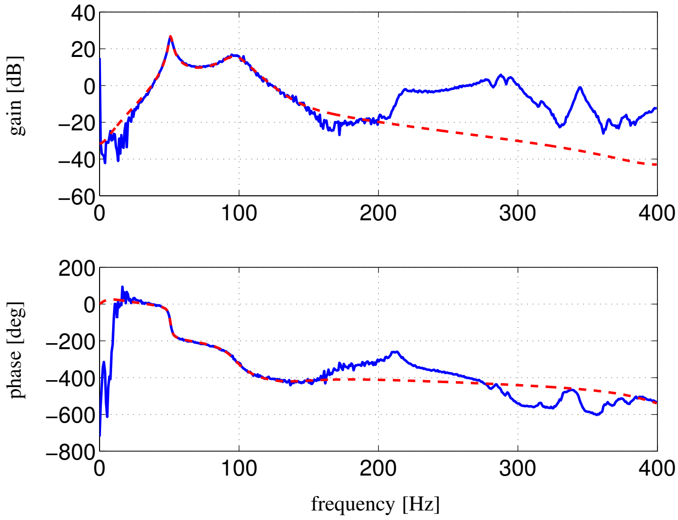

Fig. 2. Amplitude (top) and phase (bottom) Bode response of frequency domain data of secondary path (solid) and estimated sixth order discrete-time model G_x.

Given a frequency response $G_o(\omega)$ of the plant G_o, a nominal model G_x and an initial controller C that stabilizes both G_o and G_x, the model perturbation $\Delta_G(\omega)$ in (1) can be computed frequency point wise via

$$\Delta_G(\omega) = D_c^{-1}(\omega)(1 + G_o(\omega)C(\omega))^{-1}(G_o(\omega) - G_x(\omega))D_x(\omega)$$

where D_c and D_x are found from the right coprime factorization of C and G_x respectively. From the above expression it can be observed that the model uncertainty is shaped by the additive difference between G_o and G_x, weighted by the closed-loop sensitivity function $(1 + G_o C)^{-1}$ and the (inverse) of the coprime factors D_c and D_x. Since the dynamics G_o of the mechanical test bed and the model G_x are both inherently stable, the initial controller is simply chosen as $C = 0$ and $N_c = 0$, $D_c = 1$. Furthermore, with G_x a stable model, we can simply choose $N_x = G_x$ and $D_x = 1$ reducing the closed-loop weighted uncertainty Δ_G to a straightforward additive uncertainty $\Delta_G(\omega) = (G_o(\omega) - G_x(\omega))$. It should be noted that a more carefully chosen initial controller C can reduce the effect of the uncertainty $\Delta_G(\omega)$ in the frequency range for adaptive regulation.

To obtain a nominal model G_x, we use standard (iterative) least-squares curve fitting on the frequency response data $G_o(\omega)$, stressing the need to find a low complexity model G_x with a small additive error in the frequency range between 50 and 100Hz. The Bode response of the resulting model has been depicted in Fig. 2 and in the frequency range of interested we find a model error bound $1/\gamma \approx 0.2232$, requiring us choose $\gamma \approx 4.5$ to bound the controller parameter estimates to guarantee stability robustness during adaptive regulation.

4.2 Simulation Results

The adaptive regulation capabilities of REACT for harmonic disturbances is first demonstrated via a simulation study in which different levels of complexity (multiple simultaneous harmonic disturbances) are used. The different levels of complexity used in our simulation study are summarized in Table 1 and involves the application of a harmonic disturbance

$$d_n(t) = \sum_{k=1}^{n} A_k \sin(2\pi f_k t + \phi_k) \qquad (11)$$

where the number n of harmonics is equivalent to the complexity level n. In our simulation results, the disturbance $d_n(t)$ in (11) starts at $t = 5$ sec and the frequencies f_k in Herz undergo *step wise changes* every 3 seconds by cyclically rotating through the numerical values of f_k, $k = 1, \ldots, n$ listed in the columns of Table 1.

level	frequencies [Hz]		
$n=1$	$f_1 = 75$	$f_1 = 85$	$f_1 = 65$
$n=2$	$f_1 = 75$ $f_2 = 105$	$f_1 = 85$ $f_2 = 100$	$f_1 = 65$ $f_2 = 90$
$n=3$	$f_1 = 75$ $f_2 = 105$ $f_3 = 85$	$f_1 = 85$ $f_2 = 100$ $f_3 = 80$	$f_1 = 65$ $f_2 = 90$ $f_3 = 70$

Table 1. Frequency of harmonic disturbances for different level n of complexity. Sequence of step-wise changes in the frequency follow the numerical values listed in column 1, 2, 1, 3 and 1.

The harmonic disturbance $d_n(t)$ in (11) is passed through the primary path of the active vibration control system. To limit the real-time computations, a model of the primary path is not used in our algorithm and the amplitude A_k, frequency f_k and phase ϕ_k are unknown to the adaptation algorithm. Obviously, using a model of the primary path could further improve the results summarized here. For the recursive estimation of the parameters θ_t of the Youla parameter $\Delta_C(\theta_t)$ we use $N_\theta = 26$ tapped delays in the FIR filter and the Kalman filter-based RLS algorithm explained in Section 3.2 using an equivalent measurement noise variance $\sigma = 1000$ and an equivalent state covariance noise matrix $Q = 0.1 \cdot I_{26\times 26}$ for relative fast convergence results. The number $N_\theta = 26$ of FIR filter coefficients is chosen as a trade-off between handling multiple harmonics in adaptive regulation, computational complexity and variance of the parameter estimates. For the bounding and filtering of the recursive parameter estimate θ_t as explained in Section 3.3 we choose a norm $\gamma = 4$ to satisfy the robust stability requirement and a first order filter coefficient $\varepsilon = 0.4$ to smoothen the recursive parameter estimate θ_t.

An overview of the REACT simulation results based on the above mentioned numerical values is plotted in Fig. 3. For each level n indicated in Table 1, the output of the vibration control system (residual force) without control is plotted in a dotted (green) line for comparison purposes. It can be observed that the REACT algorithm based on RLS estimation of the controller parameters converges relatively fast whenever step changes in the frequency of the disturbance $d_n(t)$ occur. Convergence depends on the size of the step in the frequency, the actual frequency and number of frequencies ($n = 1, 2$ or 3) in the disturbance, but in most cases the output of the vibration control system is brought back down to the noise level of the force sensor.

4.3 *Experimental Results*

For the experimental verification of the REACT algorithm on the actual mechanical test bed of an active suspension, only the

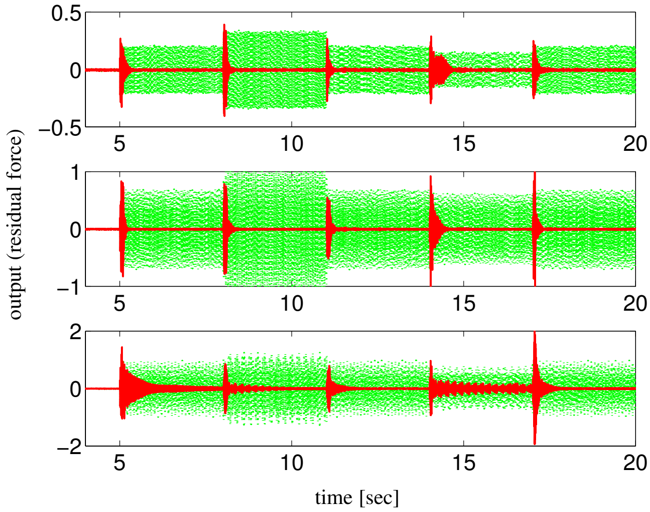

Fig. 3. Overview of level 1 (top), level 2 (middle) and level 3 (bottom) REACT simulation results for the mechanical test bed of an active suspension system using step-wise changes in multi-harmonic disturbance with frequencies listed in Table 1. The light (green) dotted lines indicate *without* control, and the (red) solid line indicate the situation with the REACT controller for adaptive regulation.

level 1 (single harmonic disturbance) was tested. Experimental results consisted of step-wise application of single harmonic disturbances with a frequency ranging from 45 Hz to 105 Hz in steps of 5 Hz. The REACT adaptation algorithm was able to control all harmonic disturbances in this frequency range with a typical converge ranging from 0.1 second to 1 second. Due to the space limitations of this paper, some of the typical results at the extremes of 50 Hz and 100 Hz and an interesting (temporary) burst phenomena for an 85 Hz has been depicted in Fig. 4. The temporary burst phenomena is most likely caused by the limited resolution of the small residual force sensor after adaptation and/or stick and friction effects of the active suspension system.

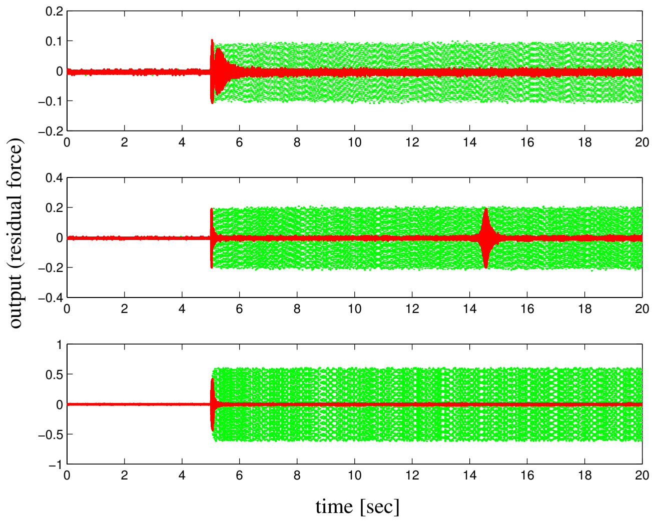

Fig. 4. Experimental results for 50 Hz (top), 85 Hz (middle) and 100 Hz disturbance (bottom). The light (green) dotted lines indicate *without* control, and the (red) solid lines indicate the situation with the REACT controller.

Adaptation and convergence for consecutive step changes in the frequencies of the single harmonic disturbances proved to be harder for the REACT algorithm than originally anticipated from the simulation results in Fig. 3. Nevertheless, the algorithm was able to maintain stability robustness and adaptively regulate all step-wise changes in the frequencies of the disturbance to an acceptable level, as indicated by the results in Fig. 5.

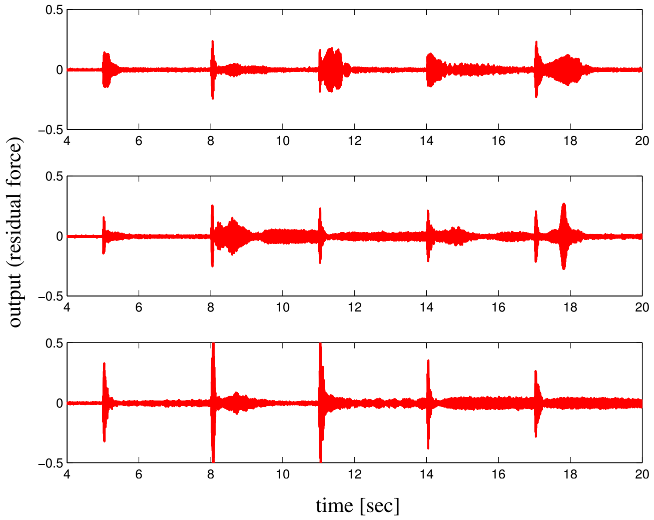

Fig. 5. Experimental results for cyclical step wise changes in the frequency of the single harmonic disturbance. Top: 55, 65 and 45 Hz, middle: 75, 85 and 65 Hz and bottom: 95, 105 and 85 Hz.

5. CONCLUSIONS

The REACT algorithm for Robust Estimation for Automatic Controller Tuning is successfully demonstrated for a vibration control problem in a mechanical test bed of an active suspension system. Periodic disturbances with unknown and varying frequencies have been regulated adaptively by REACT. With model uncertainty and controller perturbations described in (dual) Youla parametrizations and a Recursive Least Squares estimation algorithm with a bound on the parameter size determined by the model uncertainty, the REACT algorithm allows tuning of a feedback controller in the presence of model uncertainty to minimize the variance of control performance related signal. With simulations and experimental results it is shown how stability of the feedback is maintained during adaptive regulation and how REACT is able to reduce the harmonic vibrations for different levels of complexity.

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