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Closed-loop Identification of Hammerstein Systems Using Iterative Instrumental Variables

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Abstract: A feedback connection of a Hammerstein system and a linear controller provides a good approach to modeling systems with actuator nonlinearity or input saturation during closed-loop experiments. This paper discusses closed-loop identification of Hammerstein systems where both input and output signals are correlated with noise, and time domain signals are related via a nonlinear map. The static nonlinearity and the linear dynamic system are parametrized independently by using triangle basis functions and rational functions respectively. The independent parametrization method causes over-parametrization of the Hammerstein system, but parameter separation is conducted using a singular value decomposition. An iterative Instrumental Variables (IV) method minimizing an (filtered) output error (OE) is proposed to identify system parameters. The proposed iterative IV identification method is applied to the experimental closed-loop time domain data from the servo actuator in a Quantum LTO-3 tape drive and the estimation result shows the effectiveness of the proposed algorithm.

Keywords: closed-loop identification, Hammerstein systems, motion control

1. INTRODUCTION

An open-loop Hammerstein system has a block oriented structure where static input nonlinearity and linear dynamics are separated. Usually, Hammerstein systems are parametrized linear in both the input static nonlinearity and the linear dynamic system, so the parameter estimation is reduced to an ordinary least squares (LS) technique or any of its improved versions [Abony et al. (2000)]. Many approaches to identifying open-loop Hammerstein systems have been introduced and some of the recent closed-loop methods were built on these open-loop techniques. The first researchers who studied this problem were Narendra and Gallman (1966) where an iterative identification method was proposed for Hammerstein systems utilizing the alternate adjustment of the parameters of the linear and nonlinear parts of the systems. A non-iterative method to estimate the parameters by minimizing the equation error was proposed in Chang and Luus (1971). An optimal two-stage identification for Hammerstein-Wiener systems was presented in Bai (1998). A simple iterative technique for the estimation of parameters in a Hammerstein model for the case when noise in the output data is correlated was developed in Haist et al. (1973). Hsia (1977) presented modified formulations of the generalized least squares (GLS) estimation algorithm for system parameter identification. Haber (1988) introduced the two-step identification method of the LS parameter estimation based on correlation functions. Greblicki and Pawlak (1986) proposed an algorithm using a nonparametric kernel estimate of regression functions calculated from dependent data.

Even though there has been much research on *open-loop* Hammerstein system identification and *closed-loop* linear system identification [Gilson and Van den Hof (2005)], there are limited results on combining these studies to provide systematic *closed-loop* system identification methods for Hammerstein systems in a closed-loop setting in which both input and output signals are correlated with noise and time domain signals are related via a nonlinear dynamic map. One of the early works on the closed-loop identification of nonlinear systems can be found in Beyer et al. (1979) where a closed-loop identification method is proposed for Hammerstein systems using the LS method, the GLS method and the maximum likelihood method. In addition, Linard et al. (1997) extended closed-loop identification methods (a two-stage method and using right coprime factorizations) for linear dynamic systems to nonlinear dynamic systems and De Bruyne et al. (1998) presented gradient expressions for a closed-loop parametric identification scheme. Unfortunately, these methods are based on linearization of a nonlinear map between time domain signals and the linearization imposes constraints on the actual nonlinear map to be identified. Recently, van Wingerden and Verhaegen (2009) presented an algorithm to identify MIMO Hammerstein systems under open- and closed-loop conditions. Laurain et al. (2009) presented an IV method dedicated to closed-loop Hammerstein systems. Although powerful methods, the parametrization of the static nonlinearity was not discussed in detail in van Wingerden and Verhaegen (2009) and Laurain et al. (2009) .

The objective of this paper is to formulate a procedure that allows unbiased parameter estimation of *closed-loop* Hammerstein systems using the IV technique with a specific parametrization for a piecewise linear approximation of the static nonlinearity. This specific parametrization is chosen due to its efficiency in estimation of non-smooth static nonlinearity, which is very common in feedback control systems. A feedback connection of a Hammerstein system and a linear controller can be used to represent a feedback system with actuator nonlinearity (such as deadzone nonlinearity) or a closed-loop experiment with input saturation. In this study, the closed-loop identification problem for Hammerstein systems within such feedback control systems is expressed as a simple iterative IV method and we illustrate how static nonlinearity and the linear dynamic system can be estimated.

2. STATEMENT OF THE PROBLEM

The Hammerstein system in a closed-loop setting considered in this paper is shown in Figure 1. For the sake of analysis, the reference input $r(t)$ and the controller map $C(\cdot)$ are known. The linearity of the controller is not required as long as the controller information is known. The measurement noise is a colored noise that can be represented by a filtered white noise as $v(t) = H(q)e(t)$, and the filter is assumed to be unknown and will not be estimated. For identification purposes, the input $u(t)$ and output $y(t)$ are measured, whereas the intermediate signal $x(t)$ is unknown. The static nonlinearity $f(\cdot)$ and linear dynamic system $G(q)$ are unknown and need to be estimated.

In a closed-loop setting, the output $y(t)$ and input $u(t)$ are corrupted by measurement noise $v(t)$. Therefore, system parameter estimates obtained from general open-loop identification methods that use the noise corrupted output $y(t)$ and input $u(t)$ will be biased. The purpose of this study is to propose a closed-loop system identification method for Hammerstein systems in a closed-loop setting that yields unbiased system parameter estimates for the linear dynamic system $G(q)$ and static nonlinearity $f(\cdot)$ under a unknown colored disturbance $v(t)$ from the available information (the reference input $r(t)$, the input $u(t)$, the output $y(t)$, and the knowledge of the controller).

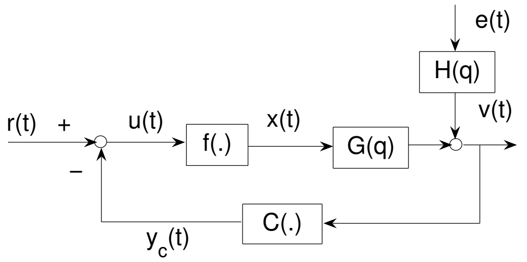


Fig. 1. closed-loop Hammerstein system.

3. SYSTEM DESCRIPTION

3.1 Modeling of static nonlinearity

In feedback control systems, non-smooth static nonlinearity, such as saturation, is common. A piecewise linear approximation is an excellent way to estimate such nonlinearity for feedback control systems since we can achieve good approximation with only a small number of parameters. As a result, a piecewise linear approximation of the static nonlinearity $f(\cdot)$ is used in this study. There are several basis functions that can be used to obtain a piecewise linear approximate of the static nonlinearity. The obvious choice is a piecewise linear function [Van Pelt and Bernstein (2000)]. Another possible basis function is a piecewise triangle function [Gibson (2008)] as shown in Figure 2. Using triangle basis functions $f_m(\cdot)$, the static nonlinearity $f(\cdot)$ is assumed to satisfy the following condition

$$\sup_{u(t)\in[u_{min},u_{max}]} \lim_{M\to\infty} \sum_{m=1}^{M} \mu_m f_m(u(t)) - f(u(t)) = 0 \tag{1}$$

where the center location vector $m = [m_1 \ \cdots \ m_M]^T$, specifying the center locations of triangle basis functions, spans the amplitude of the input vector $u = [u(1) \ \cdots \ u(N)]^T$ and the amplitude vector $\mu = [\mu_1 \ \cdots \ \mu_M]^T$, specifying the amplitudes of triangle basis functions at the center m, is to be estimated. The condition in (1) indicates that the static nonlinearity $f(\cdot)$ can be approximated arbitrary well with a dense grid of triangular basis functions. A sparse grid will provide good approximation for non-smooth nonlinearity if a critical point is well captured. A denser grid is needed to approximate smooth nonlinearity, which will be represented by piecewise linear functions.

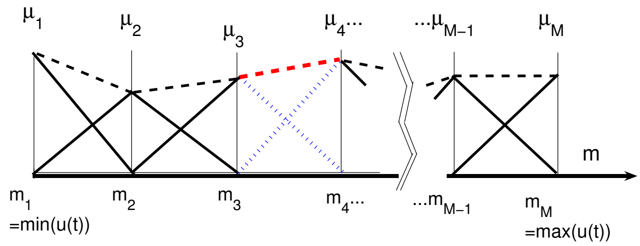


Fig. 2. Triangle basis functions.

3.2 Modeling of closed-loop Hammerstein systems

The noise corrupted output $y(t)$ generated from a Hammerstein system in a closed-loop setting is defined as

$$\begin{aligned} y(t) &= G(q)x(t) + v(t) \\ &= q^{-td}\frac{B(q)}{A(q)}x(t) + H(q)\,\mathrm{e}(t) \end{aligned} \tag{2}$$

where

$$\begin{aligned} &x(t) = f(u(t)) \\ &A(q) = 1 + a_1 q^{-1} + \cdots + a_{n_a} q^{-n_a} \\ &B(q) = b_0 + b_1 q^{-1} + \cdots + b_{n_b} q^{-n_b} \end{aligned} \tag{3}$$

and td indicates the number of steps of time delay of the system. Let $\eta = \left[a_1 \ \cdots \ a_{n_a} \ b_0 \ \cdots \ b_{n_b}\right]^T$ and we assume at least one step time delay in $G(q)$. The noise free output $\hat{y}(t)$ is defined by

$$\hat{y}(t) = \frac{B(q)}{A(q)}\hat{x}(t - td). \tag{4}$$

In order to define $\hat{y}(t)$, one only needs $\hat{x}(t-td), \cdots, \hat{x}(t-n_b-td)$ and $\hat{y}(t-1), \cdots, \hat{y}(t-n_a)$. Subsequently, in order to define $\hat{y}_c(t)$, one only needs information on the feedback control map $\hat{y}_c(t) = C(\hat{y}(t))$. In case of a linear dynamic map, where

$$C(q) = \frac{d_0 + \cdots + d_{n_d} q^{-n_d}}{1 + c_1 + \cdots + c_{n_c} q^{-n_c}},$$

only past values of $\hat{y}_c(t)$ and $\hat{y}(t)$ are needed to compute $\hat{y}_c(t)$. Then

$$\hat{u}(t) = r(t) - \hat{y}_c(t), \ \hat{x}(t) = f(\hat{u}(t)). \tag{5}$$

As a result, noise free input and output signals are generated by the known reference signal only. The linear difference equation between the output $y(t)$ and the intermediate signal $x(t)$ is defined as

$$y(t) = -\sum_{i=1}^{n_a} a_i y(t-i) + \sum_{j=0}^{n_b} b_j x(t - td - j) + w(t)$$

where $w(t) = A(q)v(t)$ and the intermediate signal $x(t)$ is found from the static nonlinearity $f(\cdot)$ as $x(t) = f(u(t))$. As a result, the output $y(t)$ is defined as

$$y(t) = -\sum_{i=1}^{n_a} a_i y(t-i) + \sum_{j=0}^{n_b} b_j f(u(t - td - j)) + w(t). \tag{6}$$

4. PARAMETRIZATION AND ESTIMATION

4.1 Parametrization of static nonlinearity and linear dynamics

In order to define a piecewise linear approximation of the static nonlinearity $f(\cdot)$, a finite value M in (1) can be chosen, whereas the points $m_1, \cdots, m_M$ of a grid over $[u_{min}, u_{max}]$ can be chosen linearly spaced or at strategic locations. Each triangle function $f_m(u(t))$ in (1) has nonzero values through two segments and zeros elsewhere except for the first and the last intervals of the grid.

$$f_m(u(t)) = \begin{cases} \dfrac{u(t) - m_{l-1}}{m_l - m_{l-1}} & \text{for } m_{l-1} \le u(t) < m_l \\ \dfrac{m_{l+1} - u(t)}{m_{l+1} - m_l} & \text{for } m_l \le u(t) < m_{l+1} \\ 0 & \text{Otherwise} \end{cases}$$

$$f_1(u(t)) = \begin{cases} \dfrac{m_2 - u(t)}{m_2 - m_1} & \text{for } m_1 \le u(t) < m_2 \\ 0 & \text{Otherwise} \end{cases}$$

$$f_M(u(t)) = \begin{cases} \dfrac{u(t) - m_{M-1}}{m_M - m_{M-1}} & \text{for } m_{M-1} < u(t) \le m_M \\ 0 & \text{Otherwise} \end{cases}$$

In each segment of the m-axis, the resulting linear function as indicated by the (red/shaded) dashed line in Figure 2 is defined by two overlapping triangle functions in the segment, as indicated by the two (blue) dotted lines. This piecewise linear parametrization using triangle basis functions can approximate not only a smooth but also a non-smooth static nonlinearity function $f(\cdot)$. With the amplitude vector $\mu = [\,\mu_1 \;\cdots\; \mu_M\,]^T$, $x(t)$ can be rewritten as

$$x(t) = f(u(t)) \approx \rho^T(u(t))\mu \tag{7}$$

where $\rho^T(u(t))$ is defined as

$$\rho^T(u(t)) = \left[\cdots\; 0 \;\; \frac{m_{k+1} - u(t)}{m_{k+1} - m_k} \;\; \frac{u(t) - m_k}{m_{k+1} - m_k} \;\; 0 \;\cdots \right] \quad \text{for } m_k \le u(t) < m_{k+1} \tag{8}$$

where m_k and m_{k+1} are the center locations of the triangle basis functions.

Approximating the non-smooth static nonlinearity function $f(\cdot)$ by the finite dimensional approximation in (7), (8), the output vector $Y = [y(1) \;\cdots\; y(N)]^T$, with the output $y(t)$ in (6), can be written in matrix form as

$$Y = \Phi\theta_0 + W$$

where θ_0 denotes the parameter to be estimated. Following the parametrization of $G(q)$ in (2) and (3) we see that the real valued $(n_a + M \cdot (n_b + 1)) \times 1$ parameter θ_0 has a structure characterized by

$$\theta_0 = [\, a_1 \;\cdots\; a_{n_a} \;\; b_0\mu_1 \;\cdots\; b_0\mu_M \;\cdots\; b_{n_b}\mu_1 \;\cdots\; b_{n_b}\mu_M \,]^T .$$

In this parametrization, an arbitrary gain may be distributed between the static nonlinearity and the linear dynamic system [Chou and Verhaegen (1999); Wigren (1993)]. In order to avoid an ambiguous gain, the scaling of either the linear dynamic system or the static nonlinearity can be normalized. Here, we choose to normalize the scaling of the linear dynamic system by setting $b_0 = 1$. The modified parameter vector θ to be estimated is now defined as

$$\theta = \left[\theta_1 \;\cdots\; \theta_{n_a} \; \theta_{n_a+1} \;\cdots\; \theta_{n_a+M} \;\cdots\; \theta_{n_a+M\cdot(n_b+1)}\right]^T \tag{9}$$

where $\theta_1, \;\cdots,\; \theta_{n_a}$ are used to capture $a_1, \;\cdots,\; a_{n_a}$ and $\theta_{n_a+1}, \cdots, \theta_{n_a+M\cdot(n_b+1)}$ are used to capture $b_1, \cdots, b_{n_b}$, and $\mu_1, \cdots, \mu_M$. This parametrization method over-parametrizes the Hammerstein system, but the separation of θ into $b_1, \cdots, b_{n_b}$ and $\mu_1, \cdots, \mu_M$ is solved using a singular value decomposition outlined in Section 4.3. The regressor data Φ is defined as

$$\begin{aligned} \Phi &= [\, \Phi_a \;\; \Phi_{b\mu} \,] \\ &= [\, \phi_1(t) \;\; \phi_2(t) \;\cdots\; \phi_{n_a+M\cdot n_b}(t) \;\; \phi_{n_a+M\cdot(n_b+1)}(t) \,] \end{aligned} \tag{10}$$

where

$$\Phi_a = \begin{bmatrix} -y(1-1) & -y(1-2) & \cdots & -y(1-n_a) \\ -y(2-1) & -y(2-2) & \cdots & -y(2-n_a) \\ -y(3-1) & -y(3-2) & \cdots & -y(3-n_a) \\ \vdots & \vdots & \ddots & \vdots \\ -y(N-1) & -y(N-2) & \ldots & -y(N-a_{n_a}) \end{bmatrix}$$

$$\Phi_{b\mu} = \begin{bmatrix} \rho^T(u(1-td)) & \cdots & \rho^T(u(1-td-n_b)) \\ \rho^T(u(2-td)) & \cdots & \rho^T(u(2-td-n_b)) \\ \vdots & \vdots & \vdots \\ \rho^T(u(N-td)) & \cdots & \rho^T(u(N-td-n_b)) \end{bmatrix}$$

with $\rho^T(u(t))$ in (8) and the equation error vector $W = [\, w(1) \;\cdots\; w(N)\,]^T$.

4.2 IV estimation

If a linear LS method is used to estimate the system parameters, the estimated $\hat{\theta}$ will be biased in cases where the equation error $w(t)$ is not white noise. In order to overcome this problem, an instrumental variable (IV) method can be used [Söderstöm and Stoica (2002)]. The main idea of the IV method is to modify the LS estimate by using general correlation vectors called instruments so that the estimate of the system parameter vector $\hat{\theta}$ becomes consistent for an arbitrary disturbance. An instrument ξ is chosen to be uncorrelated with the equation error vector W and correlated with the regressor Φ such that

$$\begin{cases} E[\xi^T W] = 0 \\ E[\xi^T \Phi] \text{ is a nonsingular matrix} \end{cases} \tag{11}$$

where 0 represents a zero vector and E represents the expectation operator. With the instrument ξ defined under the condition given in (11), an asymptotically unbiased estimate is obtained by using the IV estimate

$$\hat{\theta}^N_{IV} = (\xi^T\Phi)^{-1}\xi^T Y \tag{12}$$

that has the same dimension and structure as the modified parameter θ in (9).

The instrument ξ can be defined by using a noise free output $\hat{y}(t)$ and a noise free input $\hat{u}(t)$. First, auxiliary models $\hat{G}$ and $\hat{f}$ are identified from closed-loop signals $u(t)$ and $y(t)$ using the least squares method, and noise free signals $\hat{y}(t)$ and $\hat{u}(t)$ are generated with these auxiliary models using the known reference signal $r(t)$ as indicated in (4) and (5) respectively. As a result, ξ is defined as

$$\xi = \Phi_{nf} = \left[\Phi_{a,nf} \;\; \Phi_{b\mu,nf}\right] \tag{13}$$

where

$$\Phi_{a,nf} = \begin{bmatrix} -\hat{y}(1-1) & -\hat{y}(1-2) & \cdots & -\hat{y}(1-a_{n_a}) \\ -\hat{y}(2-1) & -\hat{y}(2-2) & \cdots & -\hat{y}(2-a_{n_a}) \\ -\hat{y}(3-1) & -\hat{y}(3-2) & \cdots & -\hat{y}(3-a_{n_a}) \\ \vdots & \vdots & \ddots & \vdots \\ -\hat{y}(N-1) & -\hat{y}(N-2) & \ldots & -\hat{y}(N-a_{n_a}) \end{bmatrix}$$

$$\Phi_{b\mu,nf} = \begin{bmatrix} \rho^T(\hat{u}(1-td)) & \cdots & \rho^T(\hat{u}(1-td-n_b+1)) \\ \rho^T(\hat{u}(2-td)) & \cdots & \rho^T(\hat{u}(2-td-n_b+1)) \\ \vdots & \vdots & \vdots \\ \rho^T(\hat{u}(N-td)) & \cdots & \rho^T(\hat{u}(N-td-n_b+1)) \end{bmatrix}$$

It should be noted that the IV estimate as formulated in (12) does not necessarily exhibit any optimality properties other than the guarantee of giving a consistent estimate on average.

4.3 Parameter separation

As explained in Section 4.1, since the static nonlinearity and the dynamic linear system are parameterized independently, the system is over-parametrized by the modified parameter vector given in (9). As the parameter estimate $\hat{\theta}_{IV}^N$ in (12) has the same structure, we need to separate the parameters of the linear dynamic system in

$$\eta = [\, a_1 \cdots a_{n_a} \;\; b_1 \cdots b_{n_b} \,]^T$$

and the parameters for the piecewise linear approximation of $f(\cdot)$ in

$$\mu = [\, \mu_1 \;\cdots\; \mu_M \,]^T .$$

Once $\hat{\theta}_{IV}^N$ in (12) is estimated, $\hat{a} = [\hat{a}_1 \cdots \hat{a}_{n_a}]^T$ is easily obtained. The parameter vectors $\hat{b} = [\hat{b}_1 \cdots \hat{b}_{n_b}]^T$ and $\hat{\mu}$ can be separated using the Singular Value Decomposition (SVD) [Bai (1998)]. First the parameter vector $\hat{\theta}_{IV}^N$ is reorganized into $\Gamma_{b\mu}$ given by

$$\Gamma_{b\mu} = \begin{bmatrix} \hat{\mu}_1 & \cdots & \hat{\mu}_M \\ \vdots & \vdots & \vdots \\ \hat{b}_{n_b}\hat{\mu}_1 & \cdots & \hat{b}_{n_b}\hat{\mu}_M \end{bmatrix}.$$

The singular value decomposition of $\Gamma_{b\mu}$ is given as

$$\Gamma_{b\mu} = U\Sigma V^T$$

where $U_{(n_b+1)\times(n_b+1)}$ and $V_{M\times M}$ are orthogonal matrices, and $\Sigma_{(n_b+1)\times M}$ is a rectangular diagonal matrix. The positive diagonal entries of Σ are called singular values. With the constraint, $b_0 = 1$, the parameter vectors $\hat{\eta}$ and $\hat{\mu}$ can be calculated by

$$\begin{aligned} \hat{\eta} &= [\, \hat{a}^T \;\; \hat{b}^T \,] \\ &= [\, \hat{\theta}_{IV}^N(1:n_a)^T \;\; U(:,1)/U(1,1) \,] \\ \hat{\mu}^T &= \sigma_1 V^T(1,:)\cdot U(1,1) \end{aligned} \tag{14}$$

where σ_1 is the largest singular value and $U(1,1)$ denotes the first nonzero element of $U(:,1)$, where the notation (1,:) and (:,1) are used to denote the first row and the first column in a matrix respectively. In this way, the optimal system parameter vectors $\hat{b}$ and $\hat{\mu}$ are obtained by minimizing the matrix Frobenius norm given by

$$[\hat{\mu}, \hat{b}] = \arg \min_{\mu\in R^M,\; b\in R^{n_b+1}} ||\Gamma_{b\mu} - \hat{b}\hat{\mu}^T||_F^2$$

with the constraint, $b_0 = 1$.

4.4 Iterative IV estimation

Although the IV estimate described in Section 4.2 is not derived explicitly to exhibit any optimality properties, the choice of an instrument ξ can be refined to include some optimality properties for the IV estimate. A general class of IV estimators is given by

$$\hat{\theta}_{IV}^N = (\xi^T\Phi^L)^{-1}\xi^T Y^L$$

where

$$Y^L = \left[y^L(1) \cdots y^L(N)\right]^T, \;\; y^L(t) = L(q)y(t) \tag{15}$$

and

$$\begin{aligned} \Phi^L &= \left[\phi_1^L(t) \cdots \phi_{n_a+M\cdot(n_b+1)}^L(t)\right], \\ \phi_i^L(t) &= L(q)\phi_i(t) \end{aligned} \tag{16}$$

that allows an additional filtering of the output $y(t)$ and the regressor Φ with a filter $L(q)$ [Söderstöm and Stoica (2002); Ljung (1999)]. One optimality principle commonly used in IV estimate is minimum variance optimality criterion. Since the choice of the instrument ξ affects the parameter variance considerably, the minimum variance optimality property depends on the choice of the instrument ξ. This minimum variance IV estimate can be obtained by using the following choice of instruments

$$\begin{aligned} L(q) &= H(q)^{-1}A(q)^{-1} \\ \xi &= \Phi_{nf}^L \text{ (filtered noise free regressor)} \end{aligned} \tag{17}$$

where $H(q)^{-1}$ is the inverse of the noise filter in (2) and $A(q)^{-1}$ is the inverse of $A(q)$ in (3). The optimal choice of instruments in (17) is motivated by minimizing a prediction error. In this study, we are particularly interested in minimizing an (filtered) Output Error (OE), motivating the choice of a fixed noise filter $H(q) = H^*(q)$. For an OE model, $H^*(q) = 1$ is chosen. As a result, $L(q) = A(q)^{-1}$ will be used in this study. This choice of instrument requires prior knowledge of the pole locations of $G(q)$ in (2) and a noise free regressor Φ_{nf} in (13). Although impractical at first due to the required prior information, this problem can be solved by formulating an optimization scheme for the optimal IV estimate allowing an iterative procedure to update the necessary prior information. With an initial (IV) parameter estimate $\hat{\theta}_{IV}^N = \hat{\theta}_k^N$ to model the static nonlinearity $\hat{f}$ and the linear dynamic system $\hat{G}$, one could employ an iterative solution that consists of the following computational steps:

Step 1 : Separate $\hat{\theta}_k^N$ into $\hat{\eta}$ and $\hat{\mu}$ in (14) and generate noise free input $\hat{u}(t)$ and noise free output $\hat{y}(t)$ using (4) and (5) respectively to define Φ_{nf} in (13).

Step 2 : Define the filter $L(q, \hat{\theta}_{k-1}^N) = A(q, \hat{\theta}_{k-1}^N)^{-1}$. If the filter $L(q, \hat{\theta}_{k-1}^N)$ is unstable, project the poles outside the unit circle inside the unit circle.

Step 3 : Define the instrument (filtered noise free regressor) $\xi_{k-1} = \Phi_{nf}^L$ by filtering Φ_{nf} from **Step 1**, the filtered output vector Y_{k-1}^L in (15), and the filtered regressor Φ_{k-1}^L in (16).

Step 4 : Compute the IV estimate. $\hat{\theta}_k^N = (\xi_{k-1}^T\Phi_{k-1}^L)^{-1}\xi_{k-1}^T Y_{k-1}^L$

Step 5 : Stopping criterion of the algorithm. If $\|\hat{\theta}_k^N - \hat{\theta}_{k-1}^N\|/\|\hat{\theta}_{k-1}^N\| > \varepsilon$, go to **Step 1**

In the above steps, the stable filter $L(q)$, the filtered output vector Y^L, the filtered regressor Φ^L and the instrument ξ are updated using $\hat{\theta}_{k-1}^N$ during the iterations over k. **Step 1** creates a noise free regressor Φ using the noise free input $u(t)$ and the noise free output $y(t)$ generated from closed-loop simulations. In **Step 2** the filter $L(q)$ is updated to provide the correct filtering for the instrument ξ, the output vector Y, and the regressor Φ in **Step 3**. **Step 4** is the actual computation of the IV estimate and **Step 5** formulates a stopping criterion for the algorithm by looking at the relative parameter error. With $\hat{\theta}_k^N$, where N is large enough to capture the real parameter θ_0, we have

$$\lim_{N\to\infty} E\{\hat{\theta}_k^N\} = \theta_0$$

Fig. 3. Quantum LTO-3 tape drive.

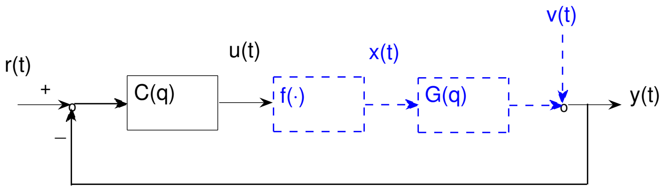


Fig. 4. Closed-loop experimental setup of a Quantum LTO-3 tape drive. The excitation signal $r(t)$ and the linear controller $C(q)$ are known. The input to the static nonlinearity $u(t)$ and the output $y(t)$ are measured. The static nonlinearity $f(\cdot)$ and linear dynamic system $G(q)$ are unknown and need to be estimated under a unknown colored disturbance $v(t)$ (the dotted line indicates unknown parts and the solid line indicates the known parts).

for each iteration step k. If the iteration converges, $\hat{\theta}^N_{IV} = \hat{\theta}^N_k$ and with the above asymptotic property we have found the optimal solution that satisfies

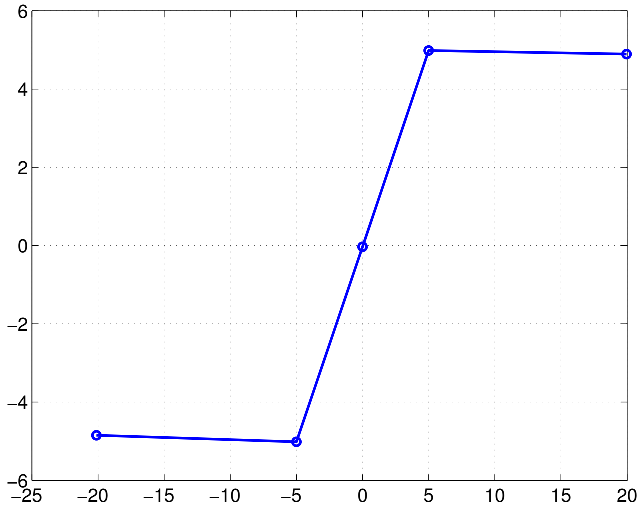
$$\hat{\theta}^N_{IV} = \text{sol}_\theta E[\xi^T(Y^L - \Phi^L\theta)] = 0 \tag{18}$$

where E represents the expectation operator and sol_θ means solve the following equation for θ.

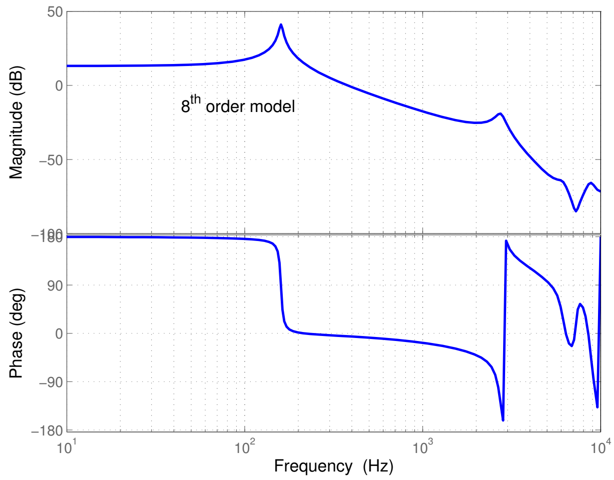
5. EXPERIMENTAL RESULTS

In this section, the proposed iterative IV method is applied to the experimental closed-loop time domain data from the servo actuator in a Quantum LTO-3 tape drive in order to identify the actuator dynamics and static nonlinearity existing in the closed-loop experiment. In this experiment, the tape drive was running at $4m/s$ causing periodic disturbances due to Lateral Tape Motion (LTM). An excitation signal r was added to the output signal (the only change is from $u(t) = r(t) - y_c(t)$ in Figure 1 to $u(t) = C(q)(r(t) - y(t))$ in Figure 4) and the excitation level was chosen such that the control signal $u(t)$ to the plant was being saturated during the experiment. A total of $1,406,251$ actuator output measurements, in the form of a Position Error Signal (PES) at $16bit$ resolution, was measured for $70.3126sec$ sampled at $20kHz$. The controller $C(q)$ implemented during experiments is known. Only $N = 10,000$ (for $0.05sec$) data was used for the system identification. $M = 5$ (the total number of grid points) with $m = [min(u)\ \ -5\ 0\ 5\ max(5)]$ is used to model static nonlinearity (we can start with $M > 5$ and remove unnecessary grid points as we go) and an 8^{th} order model with 1 step time delay is used to model the linear dynamic system. The configuration of the experiment is shown in Figure 4. The results of applying the proposed iterative IV identification method to the closed-loop time domain data from the servo actuator in a Quantum LTO-3 tape drive is shown in Figure 5 and Figure 6.

Knowing that the LTO-3 drive has a saturation of $\pm 5V$ on the control input, it can be observed from Figure 5-(a) that the input



(a)



(b)

Fig. 5. (a) The plot of the identified static nonlinearity function $\hat{f}(\cdot)$. (b) The Bode plot of the identified linear dynamic system $\hat{G}(q)$.

saturation is properly estimated. In addition, several resonance modes have been estimated in the linear dynamic response of the actuator as indicated in Figure 5-(b). The resulting closed-loop Hammerstein system represents the LTO-3 actuator under control with input saturation and the simulation of this closed-loop Hammerstein system shows excellent agreement with the experiment data, as shown in Figure 6.

6. CONCLUSIONS

An iterative IV method minimizing an (filtered) OE is proposed to identify a Hammerstein system in a closed-loop setting. Piecewise linear approximation is used for modeling static nonlinearity due to its efficiency in dealing with non-smooth static nonlinearities. The properties of an IV estimation depend on the choice of the instrument. In this paper, an instrument is chosen as a filtered noise free regressor where the filter is derived by a priori knowledge of the pole locations of the linear dynamic system and where the noise free regressor is computed from simulated closed-loop input and output signals generated by the known reference signal. For accurate computation of the

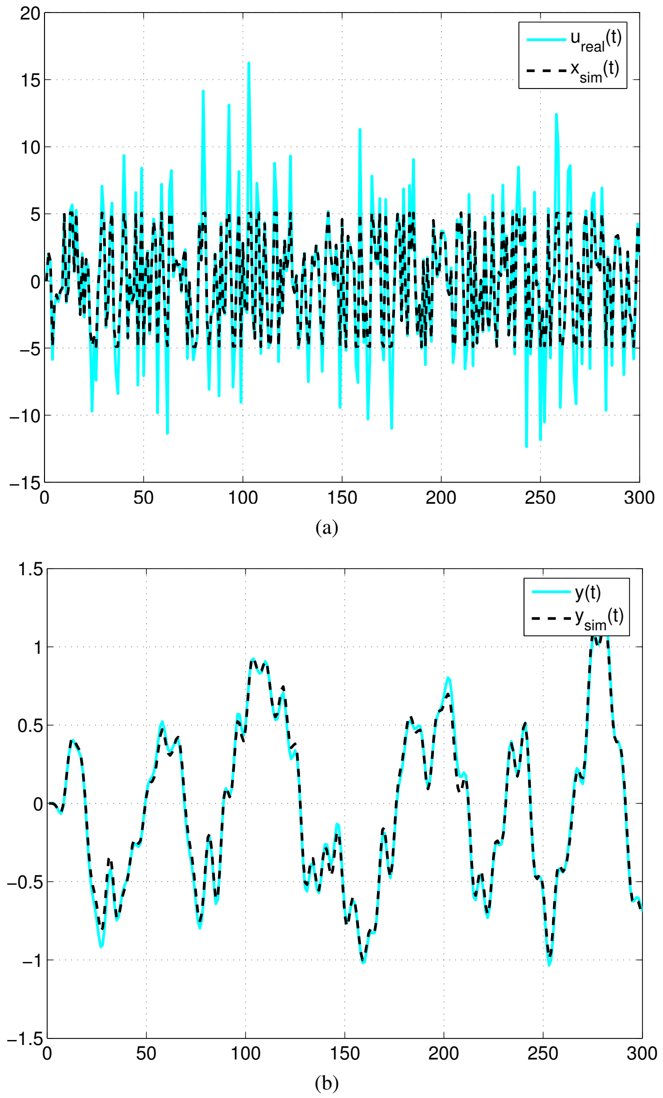


Fig. 6. (a) The plot of the measured input signal $u(t)$ and the simulated intermediate signal $x_{sim}(t)$. The $\pm 5V$ input saturation is very nicely estimated. (b) The plot of the measured output signal $y(t)$ and the simulated output signal $y_{sim}(t)$.

closed-loop signals and the filter, an iterative procedure that updates the knowledge of the static nonlinearity and the linear dynamic system is used. The effectiveness of the proposed algorithm is shown by an experimental study of the servo system in a Quantum LTO-3 tape drive in this paper.

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