Desperately Seeking Sensor

In this issue of *IEEE Control Systems Magazine*, Raymond de Callafon and Gabe Graham respond to a query on using sensors for a robotics application.

Q. In a project for our controls course, my team needs to measure the angle of a robotic leg relative to the floor when the robot is walking in a straight line. We thought that all we had to do was attach an accelerometer and use it as a tilt sensor. But this didn't work since the leg moves horizontally and vertically and also rotates. So now we think we might need a gyro and maybe other sensors. We really need some advice.

Raymond and Gabe: We are happy to try to help since we encountered a similar problem in our System Identification and Control Laboratory (SICL) at the University of California, San Diego (UCSD) when working on the stabilization of the one- and two-dimensional moment-exchange inverted pendulum [1], [2] as well as a more recent design of the moment-exchange unicycle robot [3] depicted in Figure 1. In these applications a reliable tilt measurement was required for stabilization and control of the mechanical system. Solutions based on direct angle measurement were not possible due to the lack of a suitable mechanical rotation point on which an optical encoder or potentiometer could be mounted.

Relying on an accelerometer as a tilt sensor is based on the idea that the gravitational acceleration $g \approx 9.81\ \mathrm{m/s^2}$ under the tilt angle θ can be decomposed in the radial and tangential directions. Radial a_r^g and tangential a_t^g acceleration can be measured by commercially available two-axis accelerometers. Using trigonometry we see that

$$a_r^g = g\cos\theta,\quad a_t^g = g\sin\theta$$

and under a small-angle approximation we have $\sin\theta \approx \theta$. Based on this derivation, we conclude that a_t^g/g is a good angle measurement for small tilt angles θ. Having access to both the tangential and radial acceleration measurements allows θ to be computed by means of

$$\theta = \tan^{-1}\frac{a_t^g}{a_r^g},$$

avoiding the small-angle approximation.

Unfortunately, this derivation holds only for static measurements in which $\theta(t)$ does not change as a function of time t. Moreover, if the tilted object is also moving in the horizontal and vertical directions, additional time-dependent acceleration components occur. For the derivation of the additional acceleration components, we can rely on the superposition of the effects of time-dependent rotation and translation.

For the time-dependent rotation $\theta(t)$ we refer to the inverted pendulum in Figure 2, in which we ignore the effect of gravity g and linear acceleration $a_x(t)$ and $a_y(t)$ for now. Assuming a *fixed* distance L from the two-axis accelerometer to the rotation point p, the radial a_r^r and tangential a_t^r accelerations due to *rotation* are described by planar nonuniform circular motion [4] as given by

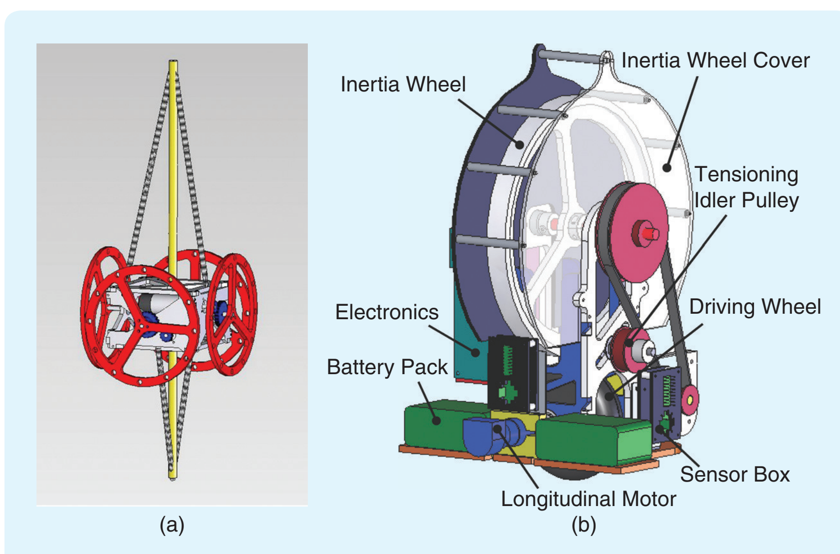

FIGURE 1 Two-dimensional inverted pendulum with (a) moment-exchange wheels and (b) moment-exchange unicycle robot.

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$$a_r^r(t) = L\omega^2(t), \qquad a_t^r(t) = L\alpha(t),$$

where

$$\omega(t) = \frac{d}{dt}\theta(t),$$
$$\alpha(t) = \frac{d}{dt}\omega(t) = \frac{d^2}{dt^2}\theta(t)$$

denote, respectively, the radial speed $\omega(t)$ and the radial acceleration $\alpha(t)$ of the rotational displacement $\theta(t)$.

As indicated in Figure 2, in the case of time-dependent translation $x(t)$ and $y(t)$ of the inverted pendulum, additional radial a_r^t and tangential a_t^t acceleration components need to be included. We thus find

$$a_r^t(t) = a_x(t)\sin\theta(t) - a_y(t)\cos\theta(t),$$
$$a_t^t(t) = -a_x(t)\cos\theta(t) - a_y(t)\sin\theta(t),$$

where

$$a_x(t) = \frac{d^2}{dt^2}x(t), \quad a_y(t) = \frac{d^2}{dt^2}y(t)$$

are the linear accelerations, respectively, in the x- and y-directions. Combining the rotational accelerations, $a_r^r(t), a_t^r(t)$, translation accelerations $a_r^t(t), a_t^t(t)$, and the time-dependent gravitational acceleration components $a_r^g(t), a_t^g(t)$ now leads to

$$a_r(t) = L\omega(t)^2 + a_x(t)\sin\theta(t) + (g - a_y(t))\cos\theta(t),$$
$$a_t(t) = L\alpha(t) - a_x(t)\cos\theta(t) + (g - a_y(t))\sin\theta(t),$$

The small-angle approximations $\sin\theta(t) \approx \theta(t)$, $\cos\theta(t) \approx 1$ allow an estimate of the tilt angle $\theta(t)$ based on the tangential acceleration measurement $a_t(t)$ only by means of

$$\theta(t) = \frac{a_t(t) - L\alpha(t) + a_x(t)}{g - a_y(t)}. \qquad (1)$$

However, having access to both the tangential $a_t(t)$ and radial $a_r(t)$ acceleration measurements allows $\theta(t)$ again to be computed without a small-angle approximation. To see this, we first define

$$\sin\psi(t) = -a_x(t),$$
$$\cos\psi(t) = g - a_y(t),$$

allowing us to rewrite

$$\begin{aligned} a_r(t) - L\omega(t)^2 &= -\sin\psi(t)\sin\theta(t) + \cos\psi(t)\cos\theta(t) \\ &= \cos(\psi(t) + \theta(t)), \\ a_t(t) - L\alpha(t) &= \sin\psi(t)\cos\theta(t) + \cos\psi(t)\sin\theta(t) \\ &= \sin(\psi(t) + \theta(t)), \end{aligned}$$

using the trigonometric angle sum and difference identities. From this trigonometric identity we see

$$\theta(t) = \tan^{-1}\frac{a_t(t) - L\alpha(t)}{a_r(t) - L\omega(t)} + \tan^{-1}\frac{a_x(t)}{g - a_y(t)},$$

which shows that independent measurements of linear accelerations $a_x(t)$, $a_y(t)$, the rotational speed $\omega(t)$, and rotational acceleration $\alpha(t)$ are required to obtain a tilt-angle measurement.

Independent measurements of rotational speed $\omega(t)$ can be obtained by adding an angular gyroscope, whereas rotational acceleration $\alpha(t)$ can be obtained by computing an approximate derivative of $\omega(t)$ by using Kalman filtering [5]. It should be noted that $\alpha(t)$ can also be estimated directly from a torque applied to the inverted pendulum depicted in Figure 2. Assuming a rigid body with a *known* rotational inertia I_p around the point p allows $\alpha(t)$ to be computed from an applied torque $T(t)$ by means of $\alpha(t) = I_p^{-1}T(t)$.

In particular situations, the use of additional sensors for measuring linear accelerations $a_x(t)$ and $a_y(t)$ can be avoided. In the absence of linear acceleration in the x-direction, that is, $a_x(t) = 0$, the tilt angle $\theta(t)$ can be computed as

$$\theta(t) = \tan^{-1}\frac{a_t(t) - L\alpha(t)}{a_r(t) - L\omega(t)^2},$$

which is independent of the linear acceleration $a_y(t)$ in y-direction, as long as $a_y(t)$ is not equal to g, that is, free fall. This result is not found when using the small-angle approximation in (1). The situation of zero linear acceleration in the x-direction is applicable in the stabilization of a one- and two-dimensional moment-exchange inverted pendulum [1], [2] as done in our laboratory at UCSD.

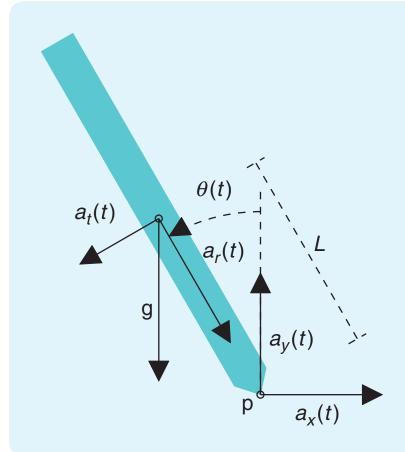

FIGURE 2 Tangential and radial gravity components.

In the case of a Segway [6] or the moment-exchange unicycle robot [3], [7] driving on a flat surface, we have $a_y(t) = 0$. Instead of measuring the linear acceleration $a_x(t)$ independently, this variable can be computed as the product of the radius and the rotational acceleration of the driving wheels. An encoder or tachometer on the driving wheels can be used to estimate the rotational acceleration of the driving wheels, eliminating the need for an additional accelerometer.

AUTHOR INFORMATION

Raymond A. de Callafon received the M.Sc. and Ph.D. in mechanical engineering from Delft University of Technology, The Netherlands, in 1992 and 1998, respectively. From 1997 to 1998, he was a research assistant with the Structural Systems and Control Laboratory in the Mechanical and Aerospace Engineering Department, University of California at San Diego, La Jolla. Since 1998, he has been a faculty member with the Dynamic Systems and Control Group, University of California at San Diego. His research interests include system identification, structural damage detection, feedback control design, model reduction, high-precision data storage systems, and active noise and vibration control.

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the depth of their torpedo because, given a high auxiliary shaft angular speed and the spatial confinement, this type of governor is more stable than its better-known cousin. The tangential governor is also a neater solution than the multiple-component diaphragm-and-pendulum method, assuming the availability of an auxiliary shaft whose angular speed changes with depth.

One possible objection to the idea of a flyball governor solution to the torpedo depth-control problem, or rather to our analysis of it, is that we have not considered the equations of motion for the pitch of the torpedo. That is, we have assumed that the auxiliary shaft of Figure 3 is vertical. In fact, from the data we have about torpedo "porpoising," it is not difficult to show that the maximum deviation of this axle from the vertical direction when placed inside a porpoising torpedo, is about 3º, and this angle changes slowly compared with the timescale of the feedback mechanism. Thus we can expect that the influence of torpedo pitch angle and its rate of change of this angle is small.

Other possible flyball governor designs might be considered for regulating torpedo depth. For example, what about a hybrid governor with a ball-and-socket joint connecting horizontal and swinging arms, that is, at locations D and E of Figure 2? Such a free-swinging arrangement might be expected to display the properties of both the centrifugal governor and the tangential governor, but in fact it does not. For the hybrid governor the centrifugal force dominates, and the angular-speed-dependent tangential equilibrium positions of Figure 4 disappear. The only tangential equilibrium angle is at $\theta = 0^{\circ}$. Perhaps this dominance of centrifugal force points to a difficulty of implementation for a tangential governor. The design shown in Figure 3 is subject to stress, due to centrifugal forces, at the joints where the horizontal and swinging arms are connected; the flyballs want to swing outward but are not able to do so. Perhaps careful design of these joints was needed to ensure that, for example, the friction coefficient is not sensitive to flyball speed. Whether this is the case or not, it is clear that a tangential governor requires better engineering than does a centrifugal governor, and this fact alone explains why we hear little of the tangential governor. Once gyros took over from centrifugal governors, there was a transition period of a decade or so during which engineers searched for more capable regulators than currently existed, to satisfy the more demanding requirements of new technology.

AUTHOR INFORMATION

Mark Denny earned a Ph.D. in theoretical physics from Edinburgh University, Scotland, and then pursued research at Oxford University from 1981 to 1984. He was subsequently employed by industry, where he worked as a radar systems engineer. He has written 50 papers on radar signal processing and physics, plus five popular science books. He is semiretired and lives on Vancouver Island.

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ASK THE EXPERTS *(continued from page 17)*

Gabe Graham received the M.S. in mechanical engineering from the University of California in San Diego in 2007. His thesis project was the design, fabrication, and control of a one-legged balancing robot. He was subsequently employed by Rasirc, a startup company developing steam-purification systems for the semiconductor industry. He is currently employed by General Atomics Aeronautical, developing laser and camera systems for airborne applications.

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