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CLOSED-LOOP INPUT SHAPING IN DISCRETE-TIME LTI SYSTEMS

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Introduction

For linear time-invariant (LTI) setpoint control systems, input shaping is a powerful technique to reduce residual vibrations in those systems as shown in [1]. The targeting trajectory can be optimized (e.g. minimize targeting time or energy consumption) through convex optimization techniques. A broad overview of real-time or nearly real-time applications has been given in [2]. Input shaping is usually formulated as an open-loop problem where linear constraints on input and output signals are imposed to formulate a convex optimization problem to find optimal and possible minimal time input profiles. Commonly finite impulse response (FIR) filters are used to pre-filter input signals as e.g. shown in [3] or [4]. Some closed-loop approaches are given in [5] where input shaping based on FIR filters is also applied to closed-loop systems. Another approach to closed-loop input shaping that is often applied to the seeking process in a Hard Disk Drive (HDD) is the shaped time-optimal servomechanism (STOS) that has been developed in [6]. Here, mode switching control turns off the feedback during the targeting stage. In [7], the reference signal generation for constrained closed-loop systems based on piecewise affine functions of state and reference vector is shown.

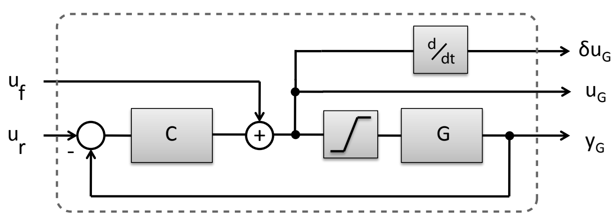

Figure 1. Closed loop LTI system

In [8] and [9], the reference signal generation is shown for a closed-loop system although time-minimal control is not addressed. Limited results are available on performing input shaping on closed-loop systems where reference and feedforward signals are computed in the presence of constraints on control and output signals. The computation of optimal reference profiles in closed-loop systems has direct application in high performance servo systems such as HDDs where short-time tracking of set-point values is required. In this study, we show an input shaping technique for closed-loop multi-input multi-output (MIMO) LTI systems that use full degree-of-freedom control such as the one shown in Fig. 1. The developed algorithm computes the optimal reference signals u_r and u_f given linear constraints on the output signal y_G, the plant control signal u_G and the reference signals u_r and u_f.

System definition

We consider an LTI model of the plant G in Fig. 1 with p inputs and m outputs of order n_G and an LTI model of the controller C with p outputs and m inputs of order n_C. The closed-loop system which is indicated by the dashed box in Fig. 1 is formulated in state space form. The input vector $u(k) = \begin{bmatrix} u_r & u_f \end{bmatrix}^T$ incorporates the computed reference signals. The output vector $y(k) = \begin{bmatrix} y_G & u_G & \delta u_G \end{bmatrix}^T$ combines the plant output y_G, the plant input u_G and its rate of change δu_G on which constraints will be imposed. For writing the linear constraints we follow [10] to compute the output assuming a fixed and pre-specified control horizon M and optimization horizon $N \geq M$.

$$\mathbf{y} = \Psi \mathbf{u} + \underbrace{\Omega x(0) + \Delta}_{q} \tag{1}$$

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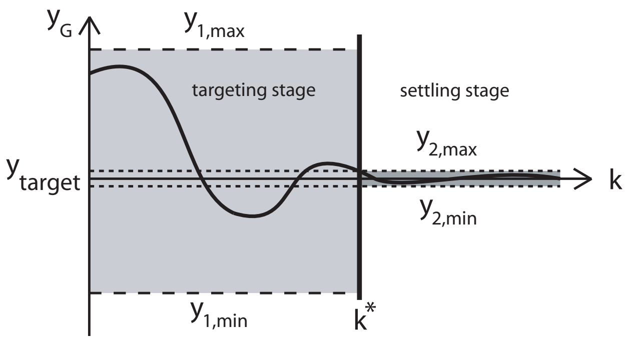

Figure 2. Definition of the output constraints

where Ψ is a matrix that contains the input/output relationship for all samples $k < M$ and Δ contains the contribution of a residual reference signal which is set to a constant value u_s for $M \leq k \leq N$. The term $\Omega x(0)$ incorporates the initial conditions which will be set to zero in this study.

Convex Optimization

The plant output y_G is subject to two different amplitude constraints as indicated in Fig. 2. One constraint is a large amplitude constraint during the targeting stage. Once the target is reached, a tolerance ε of the output from the desired target is specified creating a tight amplitude constraint during the settling stage. In Fig. 2, k^* denotes the number of samples to reach the target. Furthermore, we specify amplitude constraints on the plant input u_G, the maximum rate of change of the input signal δu_G and on the reference signals u_r and u_f. All constraints are combined in one single linear matrix inequality (LMI):

$$\mathbf{Lu} \leq \mathbf{W}(\mathbf{k}^*) - \mathbf{Q} \qquad (2)$$

We can check whether or not the constraints are feasible for a given k^* by solving the following linear program (LP) [12, 13]:

$$\begin{aligned} \min_{u,z} \;& \mathbf{1}^T\mathbf{z} \\ \text{subject to} \quad & \mathbf{Lu} - \mathbf{z} \leq \mathbf{W}(\mathbf{k}^*) - \mathbf{Q} \\ & \mathbf{z} \geq 0 \end{aligned} \qquad (3)$$

If $\mathbf{z} = 0$ is the optimal solution then the inequality (2) is feasible, otherwise infeasible. We use a bisection method [11] to find the minimum sample number k^*_{min} ($1 \leq k^*_{min} \leq M$) for a feasible set of constraints. The computed reference signal $\mathbf{u}$ based on k^*_{min} represents a time-optimal solution

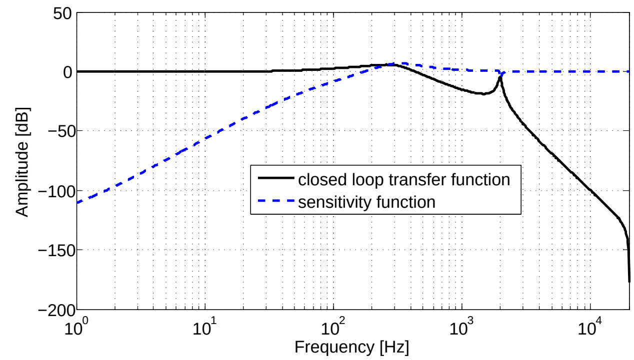

Figure 3. Amplitude plot of closed-loop transfer function and sensitivity function for both controllers

to the problem which is not unique. Further optimization e.g. in the form of quadratic programming (QP) is needed to obtain a unique solution or e.g. to further improve the energy properties of the signals. Hence, one can pose a quadratic criterion involving both u_G, u_r and u_f.

$$\begin{aligned} \min_{u, u_G} \;& \mathbf{u_G}^T\mathbf{P_1}\mathbf{u_G} + \mathbf{u}^T\mathbf{P_2}\mathbf{u} \\ \text{subject to} \;\; & \mathbf{Lu} \leq \mathbf{W}(\mathbf{k}^*) - \mathbf{Q} \\ & \mathbf{u_G} = \mathbf{S}\left(\Psi\mathbf{u} + \mathbf{q}\right) \end{aligned} \qquad (4)$$

where $\mathbf{P_1}$ and $\mathbf{P_2}$ are semi-positive definite matrices with dimensions of $\mathbf{u_G}$ and $\mathbf{u}$, respectively. With $\mathbf{P_1} \geq 0$ and $\mathbf{P_2} \geq 0$, the QP problem is convex. The QP in (4) consists of a quadratic cost function, an inequality constraint linear in $\mathbf{u}$ and and equality constraint linear in $\mathbf{u}$ and $\mathbf{u_G}$. In (4), $\mathbf{S}$ is a selection matrix. The QP in (4) represents only one possible optimization objective, although a very relevant one but there are many other possible objectives.

Simulation Example

We consider the seeking process in a Hard Disk Drive (HDD) as an example. The servo servo actuator in a HDD is a voice coil motor (VCM) that incorporates a double integrator behavior with a low frequency spring and a set of high frequency resonance modes. For simplification we only assume one main resonance mode at 2 kHz and a well damped low frequency resonance mode at 1 Hz. The system is converted to discrete time using zero order hold (ZOH) with a sampling frequency of 40 kHz. A simple PID controller with high frequency roll-off was designed in discrete-time. The amplitude plots of closed-loop transfer function from reference input to output and the sensitivity function (error rejection function) are shown in Fig. 3. In

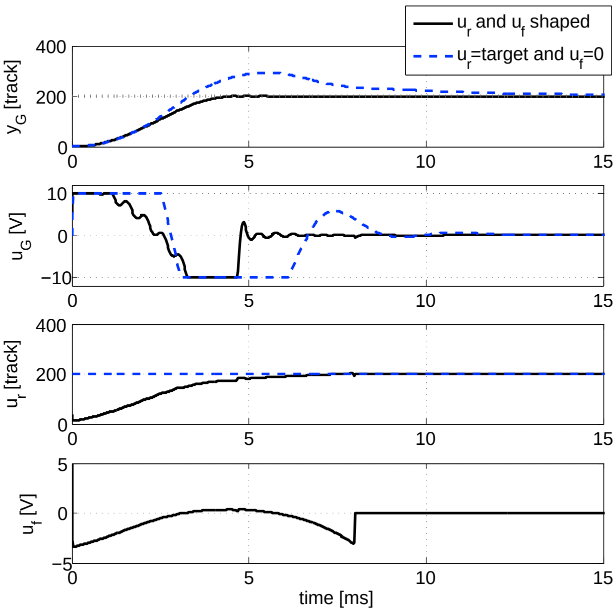

Figure 4. Simulated seek of 200 tracks

this simulation example the feed through-terms of both the controller and the plant are zero. We only assume constraints on u_G and δu_G in addition to the constraints on y_G. We solve the QP according to (4) with $\mathbf{P_1}$ and $\mathbf{P_2}$ being the identity matrix, respectively. When only using feedback and a step-wise change of u_r will saturate the actuator input and yield a slow response. Input shaping can alleviate this problem. The values of the constraints are: $u_{G,max} = 10\,\text{V}$, $\delta u_{G,max} = 5\,\text{V}$, $y_{1,max} = 2y_{\text{target}}$ and $u_{G,min} = -u_{G,max}$, $\delta u_{G,min} = -\delta u_{G,max}$, $y_{1,min} = -y_{1,max}$. A simulated seek of 200 tracks is shown in Fig. 4. It can be observed that a step-wise change of u_r (blue dashed line) will yield actuator saturation and a much larger targeting time compared to the shaped reference signal that uses one extra degree of freedom.

Conclusions

An reference signal shaping algorithm for closed-loop discrete-time LTI systems has been described in this study. It was shown that reference signal shaping significantly reduces targeting time and residual vibrations compared to output responses obtained by standard reference signals such as steps. Reference signal shaping may improve the response of systems whether or not plant saturation is present. To draw more detailed conclusions about a practical implementation, further theoretical and experimental studies are necessary. The system was simulated without considering noise. It is anticipated that by assuming a sufficiently high SNR, the noise could be included by simply adjusting (loosening) the constraints as the feedback controller will remove the steady-state error in most cases.

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