

Robust Estimation and Adaptive Controller Tuning for Variance Minimization in Servo Systems*

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Abstract

For most servo systems, servo performance is quantified by the variance of a system dependent or user defined Servo Performance Signal (SPS). The smaller the variance of the SPS, the better is the performance of the servo control system. However, the variance of the SPS may be determined by both period and non-repeatable disturbances for which the characteristics are often not known a-priori during the servo algorithm design. Moreover, the servo control algorithm is often limited to a standard Proportional, Integral and Derivative (PID) controller as more complicated algorithms are viewed to be less robust. In this paper it is shown how a standard (PID) servo control algorithm can be augmented with an additional feedback loop that can be tuned automatically by estimating the actual disturbance spectra seen in a system dependent or user defined SPS. Adaptation to the disturbance spectra is done in lieu of possible model uncertainties in the servo actuator, guaranteeing stability robustness. As such, the control algorithm provides a Robust Estimation and Adaptive Controller Tuning (REACT) to disturbance spectra to maximize servo performance by minimizing SPS variance in high performance servo systems.

Key words: Servo Control, Adaptive Control, Data Storage Systems

1. Introduction

Optimizing a servo controller for a high performance servo system often requires an intricate tuning of the servo controller to minimize the variance a system dependent or user defined Servo Performance Signal (SPS), while maintaining robustness. As an example, the Position Error Signal (PES) in a Hard Disk Drive (HDD) or a Linear Tape Open (LTO) drive acts as a SPS that can be used to quantify the quality of the servo system during track following. The smaller the variance of the PES, the better is the performance of the servo control system. On the other hand, the servo controller should perform robustly in lieu of product variations or dynamic uncertainty of the servo actuator.

In data storage applications SPS variance minimization is required to improve storage density, while performance robustness is the mathematical concept of guaranteeing that a single and well designed algorithm is guaranteed to yield the same performance on a large class of servo actuators with similar dynamical properties. Obviously, performance and

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robustness are often conflicting requirements [1], unleashing the need to compromise performance to ensure, at least, stability robustness. In addition, for optimal tuning of the servo algorithm in terms of SPS variance minimization, a detailed model is needed of all possible disturbances that will be present in the servo loop and contribute to the variance of the SPS during servo operation. Such a model can be formulated reasonably well for repeatable disturbances, but the knowledge on random or non-repeatable disturbances is often lacking and only becomes available after the servo algorithm has been implemented.

To facilitate automatic tuning of the servo control algorithm to the disturbances seen in the servo loop this paper proposes a methodology for Robust Estimation and Adaptive Controller Tuning (REACT) [2] to minimize the variance of a servo performance signal (SPS) in a high performance servo system. Such an algorithm can readily be applied to data storage application such as a HDD or LTO drive in which the SPS can be measured via the PES and used for automatic tuning purposes. The algorithm formulates the tuning of the servo controller as a perturbation on a controller initially present in the servo system and formulates conditions for stability robustness by considering a special Youla [3] and dual-Youla [4] parametrization for possible uncertainty and product variations in the actuator dynamics. The initial controller may well be a standard Proportional, Integral and Derivative (PID) controller and provides a unique way of adjusting the controller to accommodate and adapt to undesirable disturbances.

This paper gives an overview of the main idea behind the REACT algorithm to tune servo controllers for data storage systems. Applications of REACT in the field of Active Noise Control can be found in [5], while an application to a Linear Tape Open (LTO) drive system is presented at the MIPE'09 conference [6] and the main ideas behind the adaptive method will be summarized in this paper. The outline of this paper is as follows. Section 2 first gives an overview of the Youla parametrization used to formulate the perturbation of the servo controller. This is followed in Section 3 by presenting the results on how model uncertainty in actuator dynamics can be incorporated to ensure stability robustness during servo controller tuning. Section 4 and 5 show how the parametrization can be used to tune the servo control algorithm directly on the basis of data measured directly in an operational servo loop and an example is given in Section 6.

2. Controller Parametrization

A well-known result in controller design and optimization is the Youla parametrization [3] that allows the parametrization of the class of all stabilizing feedback controllers C for a given dynamical system G by a single stable dynamical system Q . The parametrization is given in terms of coprime factorization of the dynamical system G and the feedback controller C that are defined as follows for Single Input, Single Output systems.

Definition 1 (right coprime factors): Consider the pair of stable transfer functions (N, D) . The pair (N, D) is a right coprime factorization (*rcf*) of G if the following three items hold:

- there exists a pair of stable transfer function (X, Y) such that $XN + YD = 1$,
- $\det\{D\} \neq 0$ and
- $G = ND^{-1}$

◇

According to Definition 1, coprime factors (N, D) are always stable transfer functions and item a. guarantees they do not have common unstable zeros, whereas item b. and c. guarantee that G can be written as the product of N and the inverse of D . A similar definition can also be given for the coprime factors (N_c, D_c) of the feedback controller $C = N_c D_c^{-1}$. With the definition of coprime factors, the Youla parametrization [3] applied to a servo system reads as follows.

Definition 2 (Youla parametrization): Let (N, D) be a *rcf* of an actuator transfer function G

and let (N_c, D_c) be a *rcf* of an *initial* feedback controller C that stabilizes G . Then all controllers C_Q of the form

$$\begin{aligned} C_Q &= N_Q D_Q^{-1} \text{ with} \\ N_Q &= N_c + DQ \text{ and} \\ D_Q &= D_c - NQ \end{aligned} \quad (1)$$

where Q is *any* stable transfer function, stabilize the same actuator transfer function G . \diamond

The Youla parametrization is a powerful result, as it allows us to parametrize all stable controllers for a specific servo actuator dynamics G via (right) coprime factors and a stable perturbation Q . Hence, as long as we keep the transfer function Q stable, the newly tuned or perturbed controller $C_Q = N_Q D_Q^{-1}$ in Definition 2 will still stabilize the actuator dynamics G .

The use of coprime factors is required for unstable actuator or controller dynamics to avoid the cancellation of unstable poles and zeros in our feedback loop during controller perturbations. However, the Youla parametrization can be simplified in case both the actuator dynamics G and the initial feedback controller C are known to be stable. With a stable actuator G , we obtain the trivial choice $(N, D) = (G, 1)$ for the *rcf* of G , while for a stable initial controller C we may choose $(N_c, D_c) = (C, 1)$ for the *rcf* of C . In that case, the newly tuned or perturbed controller $C_Q = N_Q D_Q^{-1}$ in Definition 2 will simplify to

$$C_Q = (C + Q)/(1 - GQ) \quad (2)$$

and writing the newly tuned or perturbed controller C_Q as a feedback connection of the stable transfer function Q and the actuator model G . Again, as long as Q is stable the controller C_Q stabilizes G and the feedback connection in (2) ensures this property.

As a final note, the assumption of a stable actuator G also allows the initial controller C to be chosen as $C = 0$ as a stable actuator G does not require a controller for stabilization. In that case C_Q can be simplified further to

$$C_Q = Q/(1 - GQ) \quad (3)$$

However, any information on an initially designed controller C used in the stabilization or control of the actuator dynamics G can be used in the parametrization of the newly tuned or perturbed controller C_Q . Moreover, the trivial choice $(N, D) = (G, 1)$ can be replaced by any right coprime factorization such as a *normalized* coprime factorization to ensure the uniqueness of the right coprime pair (N, D) .

3. Actuator Uncertainty

The Youla parametrization as given in Definition 2 assumes no modeling error or uncertainty on the (stable) actuator dynamics G to ensure stability of the newly tuned or perturbed controller C_Q in (1) or (2). As a result, the only requirement for stability robustness is the stability of the perturbation Q in (1). In case of actuator uncertainty, an additional constraint on the actual “size” of Q will have to be imposed to guarantee stability robustness.

The bound on the size of Q for stability robustness depends on how the uncertainty on the actuator dynamics G is described, e.g. additive or multiplicative uncertainty. However, describing the uncertainty on the actuator dynamics G also in a coprime framework allows a clear computation of the upper bound on the size of the perturbation Q . Following the ideas of a double-Youla parametrization [7], the following main result is given here.

Corollary 1 (stability robustness for dual-Youla): Let (N, D) be a *rcf* of an *nominal* actuator transfer function G and let (N_c, D_c) be a *rcf* of an *initial* feedback controller C that

stabilizes G . Now consider a stable uncertainty Δ perturbing $G = ND^{-1}$ to G_Δ given by

$$\begin{aligned} G_\Delta &= N_\Delta D_\Delta^{-1} \text{ with} \\ N_\Delta &= N + D_c \Delta \text{ and} \\ D_\Delta &= D - N_c \Delta \end{aligned} \tag{4}$$

where Δ is unknown but bounded by an H_∞ -norm $\|\Delta\|_\infty < \gamma^{-1}$. Then the uncertain actuator dynamics G_Δ with the controller perturbation C_Q given in (1) yields a stable feedback system for all $\|\Delta\|_\infty < \gamma^{-1}$ if and only if Q is stable and $\|Q\|_\infty \leq \gamma$. \diamond

The result in Corollary 1 indicates that next to the stability requirement of Q mentioned in Definition 2 we now also have a size constraint on Q measured by an H_∞ -norm. If we have no uncertainty, $\gamma = \infty$ and only the requirement on the stability of Q remains. The uncertainty Δ on the nominal actuator dynamics G is structured according to a dual-Youla parametrization, as indicated in Figure 1.

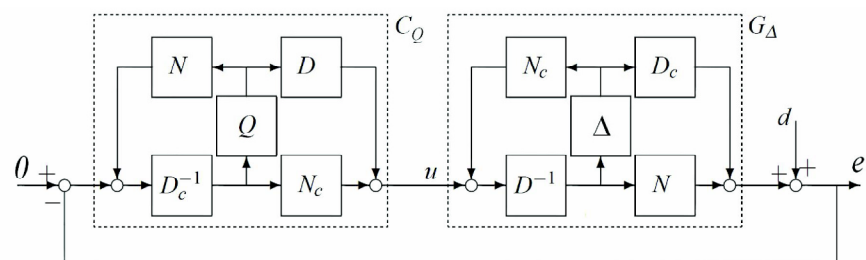


Fig. 1: Block diagram of combined controller perturbation according to (1) and actuator uncertainty given in (4). The signal e indicates the Servo Performance Signal (SPS).

4. Affine Optimization

Next to providing a parametrization of all stabilizing controllers, the Youla parametrization in (1), and its dual form for uncertainty representation, provides an another advantage for controller adaptation: all closed-loop transfer function are linear in the controller perturbation Q of (1). This can be seen as follows.

Consider the output sensitivity function or transfer function S from the input disturbance d to the output signal or PES e in Figure 1 given by

$$S = 1/(1 + G_\Delta C) \tag{5}$$

Assuming the nominal actuator model G is stable, allows substitution of $C = C_Q$ given in (2) to modify the sensitivity function S in (5) to S_Q

$$S_Q = (1 - G_\Delta Q)/(1 + G_\Delta C) = S(1 - G_\Delta Q) \tag{6}$$

and indicating that the closed-loop transfer function that models the disturbance rejection (sensitivity function) is *linear* in the controller perturbation parameter Q . A similar result can also be obtained when using the full freedom in right coprime factorization, allowing the nominal actuator model also to be unstable.

The Youla parametrization allows affine optimization techniques to, for example, minimize the 2-norm or ∞ -norm of the (weighted) sensitivity function S as a function of Q . Moreover, direct minimization of the variance of the SPS e in Figure 1 can be used for a 2-norm specification, allowing direct data-based closed-loop tuning of a minimum variance controller [8] computed via the controller perturbation parameter Q . For that purpose, the transfer function $Q(q)$ must also parametrized in a linear affine form and here $Q(q)$ is chosen to be parametrized via a n th tap discrete-time Finite Impulse Response (FIR) filter

$$Q(q, \theta) = \sum_{k=1}^n \theta_k q^{-k} \quad (7)$$

where q denotes the time shift operator, n is the order of the FIR filter and θ_k the parameters to be estimated in the FIR filter. The parametrization of the controller $C_Q(q)$ in (2) and $Q(q, \theta)$ in (7) provides a rich class of controllers that does not necessarily require a specific parametrization of noise models to tune the feedback controller [9].

We like to point out that the perturbation $Q(q, \theta)$ in (7) being added as a perturbation on the initial controller $C(q)$ in (2) is simply a tapped delay filter. FIR filtering can be implemented relatively easily in existing DSP technology for fast data processing. Obviously, the larger the order n of the FIR filter $Q(q, \theta)$ in (7), the better the resulting servo performance will be. This is due to the fact that more FIR filter parameters θ_k can be used to tune the controller $C_Q(q)$ and minimize the variance of the servo performance signal $e(t)$. We will illustrate this effect in the simulations study given in Section 7 where the order n is varied and the effect on the SPS $e(t)$ is documented.

5. Closed-Loop and Data-Based Tuning

Based on the favorable properties of having an affine parametrization of $Q(q, \theta)$ in any closed-loop transfer function, the proposed controller structure based on the Youla parametrization allows direct tuning of the Youla parameter Q on the basis of closed-loop data. As indicated before, for optimization purposes the variance of the Servo Performance Signal (SPS) e given in Figure 1 will be considered as an important indication of servo performance. The variance of the SPS e is given by

$$\text{var}\{e\} = \sum_{t=1}^N e^2(t) \quad (8)$$

and computed over a final time interval of N data points. To minimize the variance of the SPS e as depicted in Figure 1 and to use the affine parametrization result given in (6) and (7) we need access to specific closed-loop signals for direct data-based tuning of the controller perturbation parameter $Q(q, \theta)$. Assuming disturbances that influence the variance of the SPS e occur as an additive output disturbance d as indicated in Figure 1, we see that

$$e(t) = S_Q(q)d(t) \quad (9)$$

and substitution of S_Q in (9) yields

$$e(t, \theta) = S(q)d(t) - Q(q, \theta)G_\Delta(q)S(q)d(t) \quad (10)$$

so that minimization of the 2-norm of the time domain signal $e(t, \theta)$ using the affine parametrization in (7) requires access to the two closed-loop signals

$$\begin{aligned} y(t) &= S(q)d(t) \quad \text{and} \\ v(t) &= G_\Delta(q)S(q)d(t) = G_\Delta(q)y(t) \end{aligned} \quad (11)$$

The signal $y(t)$ given in (11) is readily available, as this is the output signal $e(t)$ when the initial controller $C(q)$ is implemented in the feedback loop for $Q(q, \theta) = 0$. For the computation of the signal $v(t)$ we need the (perturbed) actuator model $G_\Delta(q)$ that can be approximated by the nominal model $G(q)$ that is used also for the computing the perturbation of the controller C to C_Q given in (2). The availability of the signals $y(t)$ and $v(t)$ defined in (11) now allows the minimization of the variance of the SPS as function

of the parameter θ of the FIR filter $Q(q, \theta)$ via

$$\hat{\theta} = \arg \min_{\theta} \sum_{t=1}^N [y(t) - Q(q, \theta)v(t)]^2 \quad (12)$$

and becomes a standard Least Squares (LS) optimization for the affine parametrization of $Q(q, \theta)$ given in (7). The LS optimization can be implemented recursively [8] to facilitate adaptive tuning of the controller, but special conditions on the rate of change in the parameters θ has to be imposed to guarantee stability of the resulting time varying system, as indicated in [2].

6. Robustness and Control Signals

Tuning of the controller $C_Q(q)$ in (1) or (2) can be done *off-line* based on measurements of the closed-loop signal present in the servo loop or *in-situ* using recursive Least Squares minimization. Although the Least Squares minimization in (12) based on the closed-loop signals $y(t)$ and $w(t)$ given in (11) allows for a direct minimization of the variance of the SPS $e(t)$, any model uncertainties restrict the freedom in updating the parameters θ_k in the FIR filter $Q(q, \theta)$ given in (7) to guarantee stability robustness. As summarized in Corollary 1, any unknown but H_∞ -norm bounded uncertainty Δ modeled as a dual-Youla perturbation Δ on the plant dynamics G requires

$$\|Q(q, \theta)\Delta\|_\infty \leq 1 \quad (13)$$

to guarantee stability robustness while minimizing the variance of the SPS $e(t)$. For scalar systems, the optimization of $Q(q, \theta)$ under the constraint (13) can be approximated over a user-specified (dense) frequency grid

$$\Omega = \{\omega \mid \omega = \omega_k, k = 1, 2, \dots, N\}$$

if a frequency dependent upper bound $W(\omega_k)$ with

$$|W(\omega_k)\Delta(\omega_k)| \leq 1 \quad \forall \omega_k \in \Omega \quad (14)$$

is available to capture the frequency dependency of the unknown but H_∞ -norm bounded uncertainty dual-Youla Δ . With the frequency dependent upper bound $W(\omega)$ in (14) we obtain

$$|Q(e^{j\omega_k}, \theta)\Delta(\omega_k)| \leq |Q(e^{j\omega_k}, \theta)W^{-1}(\omega_k)| \leq 1 \quad \forall \omega_k \in \Omega$$

allowing us to approximate the constraint in (13) with

$$|Q(e^{j\omega_k}, \theta)W^{-1}(\omega_k)| \leq 1 \quad \forall \omega_k \in \Omega \quad (15)$$

over a sufficiently dense frequency grid Ω . Due the linear parametrization of $Q(q, \theta)$ given in (7), the constrained minimization that combines (12) and (15) via

$$\hat{\theta} = \arg \min_{\theta} \sum_{t=1}^N [y(t) - Q(q, \theta)v(t)]^2 \quad \text{s.t.} \quad |Q(e^{j\omega_k}, \theta)W^{-1}(\omega_k)| \leq 1 \quad \forall \omega_k \in \Omega \quad (16)$$

can easily be rewritten into a quadratically constrained quadratic programming (QCQP) problem

$$\hat{\theta} = \arg \min_{\theta} \frac{1}{2} \bar{\theta}^T P_0 \bar{\theta} + q_0 \bar{\theta} \quad \text{s.t.} \quad \frac{1}{2} \bar{\theta} P_k \bar{\theta} + q_k^T \bar{\theta} + r_k \leq 0 \quad \forall k \quad \text{and} \quad A \bar{\theta} = b \quad (17)$$

where the parameter $\bar{\theta}^T = [\theta^T \quad 0]$ and the constraint $A \bar{\theta} = b$ with

$$A = \begin{bmatrix} 0_{n \times n} & 0_{n \times 1} \\ 0_{1 \times n} & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0_{m \times 1} \\ 1 \end{bmatrix}$$

ensures that indeed $\bar{\theta}^T = [\theta^T \quad 0]$. For the quadratic minimization and the amplitude constraint in (16), it is easy to verify that the matrices P_0 and P_k for all values k of the frequency $\omega_k \in \Omega$ are positive definite. This makes the QCQP problem in (16) and (17) convex, allowing the computation of a unique minimum using interior point methods, similar as in semidefinite programming [10].

Recursive solutions for the *in-situ* computation of a solution to the QCQP problem would be challenging to implement in case of limited computational resources in the servo system. For on-line tuning in the presence of model uncertainty and one can use a projection method [2] to enforce the constraints in (17). Otherwise the QCQP problem formulation should be used to compute an optimal controller perturbation $Q(q, \theta)$ using off-line measurements of the closed-loop signals $y(t)$ and $v(t)$ in (11).

Finally, it should be noted that the LS optimization aims at minimizing the variance of the SPS, possibly resulting in large control signals for the minimum variance controller [8]. To avoid large control signals, an additional penalty on the control signal

$$u(t) = C_o(q)S(q)d(t) \quad (18)$$

can be imposed by including a (filtered version) of $u(t)$ in (18) in the minimization of (11). Alternatively, control signals can be limited by adding an additional fixed filter stable and stably invertible $F(q)$ into the controller perturbation

$$Q_f(q, \theta) = Q(q, \theta)F(q) \quad (19)$$

and possibly including the inverse of $F(q)$ in the filtering of $y(t)$ as used in the LS optimization. Penalizing the control signal avoids the solution of the LS minimization that computes the controller perturbation $Q(q, \theta)$ to converge to the solution of a minimum variance controller, with possible large control signals.

7. Illustration of REACT on Example System

Practical implementations of REACT have been reported in [5] for active noise control and at the MIPE'09 conference for a Linear Tape Open (LTO) servo system [6]. To illustrate the main concepts in this paper, a simulation example is used to illustrate the power of the REACT algorithm, as it allows direct tuning of the feedback controller with respect to disturbances.

With the simulation study in this paper we illustrate how the design of a simple initial controller used to stabilize a poorly damped servo system can be augmented with an adaptation strategy based on the REACT algorithm. The adaptation strategy perturbs the initial controller to a more advanced servo controller that has tuned itself optimally towards the disturbances present on the servo system. As such knowledge on the disturbances present on the servo system is not known beforehand while designing the initial stabilizing controller, the proposed REACT algorithm in this paper allows *in-situ* tuning of the servo controller while the servo system is in operation. As will be illustrated by the example, the resulting servo performance will be greatly improved due to additional tuning of the initial controller with the REACT algorithm.

For the example in this paper we consider the Zero Order Hold (ZOH) equivalent of a continuous-time fourth order servo actuator model sampled at 10 kHz that models a low frequency friction or flex cable mode and a high frequent poorly damped resonance mode. A continuous-time fourth order model with two (lightly damped) resonance modes is given by the transfer function model

$$G(s) = \frac{K\omega_1^2\omega_2^2}{(s^2 + 2\beta_1\omega_1s + \omega_1^2)(s^2 + 2\beta_2\omega_2s + \omega_2^2)}$$

with

$$K = 100, \quad \omega_1 = 10 \text{ rad/s}, \quad \omega_2 = 10^3 \text{ rad/s},$$

$$\beta_1 = \sqrt{1/2} \quad \text{and} \quad \beta_2 = 0.1$$

where the numerical values for the undamped resonance frequencies ω_1 , ω_2 the damping ratios β_1 , β_2 and the DC-gain K are just chosen for illustration purposes. The example in this section can easily be reproduced for different (more detailed) actuator models. A Bode plot of the discrete-time actuator model used in this example is given in Figure 2.

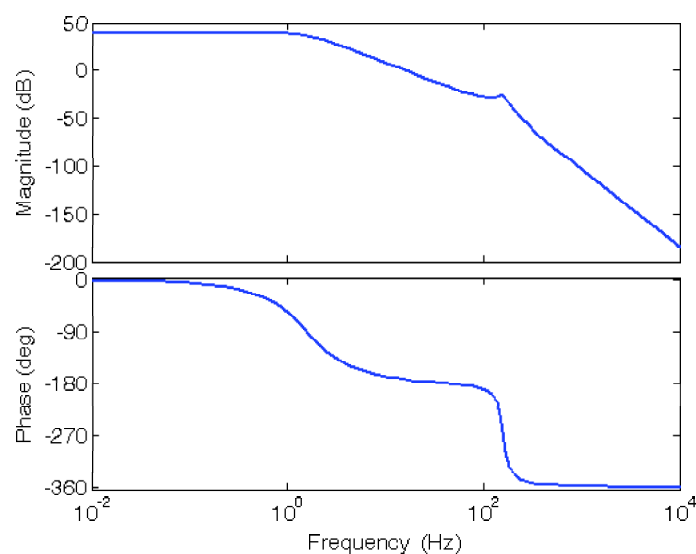


Fig. 2: Bode plot of discrete-time Zero Order Hold equivalent of continuous-time 4th order servo actuator model given in (20). Top figure: Amplitude Bode plot. Bottom figure: Phase Bode plot.

Without *a-priori* information on disturbances or performance constraints, the actuator model is controlled by a simple first order discrete-time Proportional Derivative (PD) controller sampling at 10kHz and given by

$$C(q) = 10 \cdot \frac{q - 0.995}{q - 0.95} \quad (21)$$

creating a feedback loop with a gain margin of 9dB at approximately 823 rad/s and a phase margin of 55.8 deg at approximately 200 rad/s.

For simulating the controller tuning effects of REACT due to unknown and unanticipated external disturbances during the operation of the servo system, a low-pass filtered unit variance white noise $\varepsilon(t)$ together with a sinusoidal signal of 10Hz are used to create an additive disturbance

$$d(t) = L(q)\varepsilon(t) + 0.1\sin(2\pi \cdot 10t) \quad (22)$$

where the low-pass filter $L(q)$ is a fourth order discrete-time Butterworth filter with a cutoff frequency of 125 rad/s.

Simulating the performance of the initial controller $C(q)$ in (21) in feedback with the ZOH equivalent of $G(s)$ in (20) yields the disturbance $d(t)$ and SPS $e(t)$ as depicted in Figure 3 with an estimated variance of the SPS $e(t)$ of 0.14717. Although there is significant disturbance attenuation, the existing PD controller $C(q)$ in (21) obviously has not been optimized for the non-repeatable and low frequent periodic disturbances.

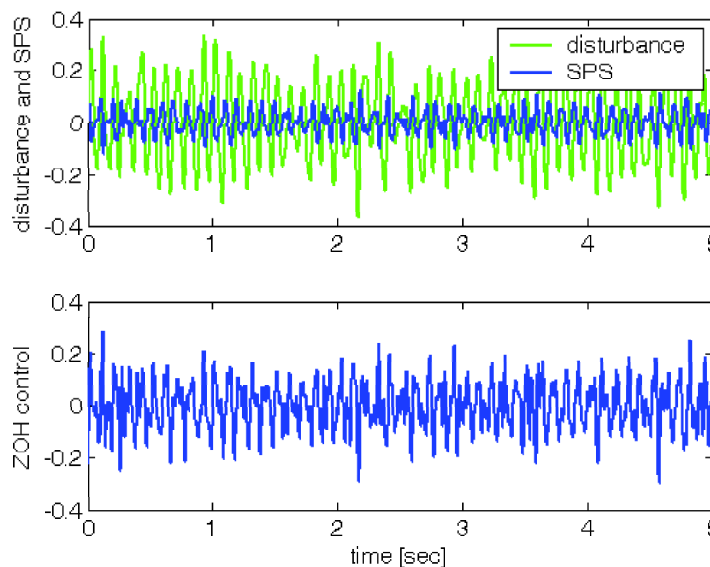


Fig. 3: Simulation results for rejection of the disturbance $d(t)$ given in (22) using the initial controller $C(q)$ given in (21). Top figure: actual disturbance $d(t)$ and PES or SPS $e(t)$. Bottom figure: control signal $u(t)$. Estimated variance of $e(t)$ is 0.14717.

To optimize the feedback controller using the REACT algorithm, the controller $C(q)$ in (21) is perturbed to $C_o(q)$ in (2) with a FIR filter $Q(q, \theta)$ in (7). To illustrate the effect of the order n of the FIR filter on the achievable servo performance, we choose different orders n for the FIR filter $Q(q, \theta)$ in (7) varying from $n=2$ tapped delays lines to $n=7$ tapped delay lines. For each of FIR filters the controller $C_o(q)$ is adjusted using the REACT algorithm. Furthermore, to limit the control signal $u(t)$ during the variance minimization of REACT, the filter $F(q)$ in (19) is chosen as a low-pass 4th order discrete-time Butterworth filter with a cutoff frequency of 1000π rad/s.

Running the closed-loop simulation and using the signals $y(t)$ and $v(t)$ as given in (11) now allows an direct tuning of the controller perturbation $Q(q, \theta)$ on the basis of the available closed-loop signals. The tuning aims at minimizing the variance of the Servo Performance Signal or PES $e(t)$ in the presence of the unknown disturbances $d(t)$ given in (22). The result will be a perturbed feedback controller $C_Q(q)$ that will be better tuned towards the disturbances experienced during the closed-loop operation of the servo system. Simulating the performance of the perturbed controller $C_Q(q)$ in (2) in feedback with the ZOH equivalent of $G(s)$ in (20) for different orders n of the FIR filter $Q(q, \theta)$ used in the REACT algorithm yields the variance improvement listed in Table 1.

Table 1: Simulation results for rejection of the disturbance $d(t)$ given in (16) using a tuned controller $C_Q(q)$ given in (2) based on REACT algorithm.

order	$Q(q, \theta)$	variance $e(t)$	Improvement [%]
1		0.067276	54.3
2		0.065114	56.3
3		0.013086	91.1
4		0.010706	92.7
5		0.010448	92.9
6		0.010405	92.9
7		0.010401	92.9
8		0.010401	92.9

From Table 1 it can be observed that even simple first order FIR filter with only a single parameter θ_1 already improves the variance of the PES with 54%. Adding more tapped delay lines to the FIR filter improves the variance minimization of $e(t)$ even further and levels off at only a sixth order FIR filter. The perturbation $Q(q, \theta)$ in (7) being added as a perturbation on the initial controller $C(q)$ in (2) is simply a tapped delay or FIR filter that can be implemented relatively easily in an embedded system for fast data processing. The results in Table 1 illustrate that indeed a better servo performance is obtained as the order n of the FIR filter $Q(q, \theta)$ in (7) is increased, as more FIR filter parameters θ_k can be used to tune the controller $C_Q(q)$ and minimize the variance of $e(t)$.

The simulation results for a seventh order FIR filter $Q(q, \theta)$ is depicted in Figure 4, where a significant improvement of the servo performance signal SPS can be observed.

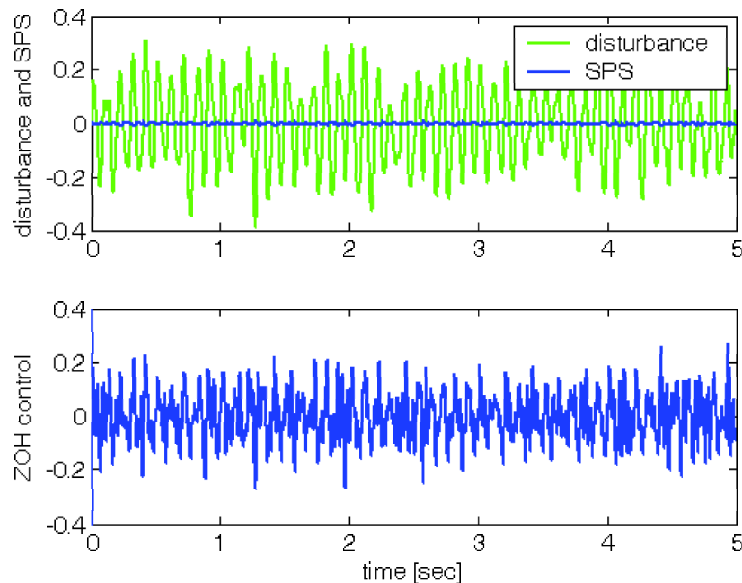


Fig. 4: Simulation results for rejection of disturbance $d(t)$ given in (22) for the optimized REACT controller $C_Q(q)$ using a $n=7$ order FIR filter $Q(q, \theta)$ in (7).

It can be seen from the simulation in Figure 4 that REACT improves the variance of the SPS $e(t)$, at only a small increase of the control signal $u(t)$. To illustrate the effect of tuning of the controller $C_Q(q)$ achieved by the REACT algorithm, a comparison of the Bode plots of the initial controller $C(q)$ in (21) and the tuned controller $C_Q(q)$ in (2) for different choices of the model order n of $Q(q, \theta)$ in (7) is given in Figure 5. Only the choice of order $n=2$ and the final order $n=7$ is plotted to keep the Bode plots readable.

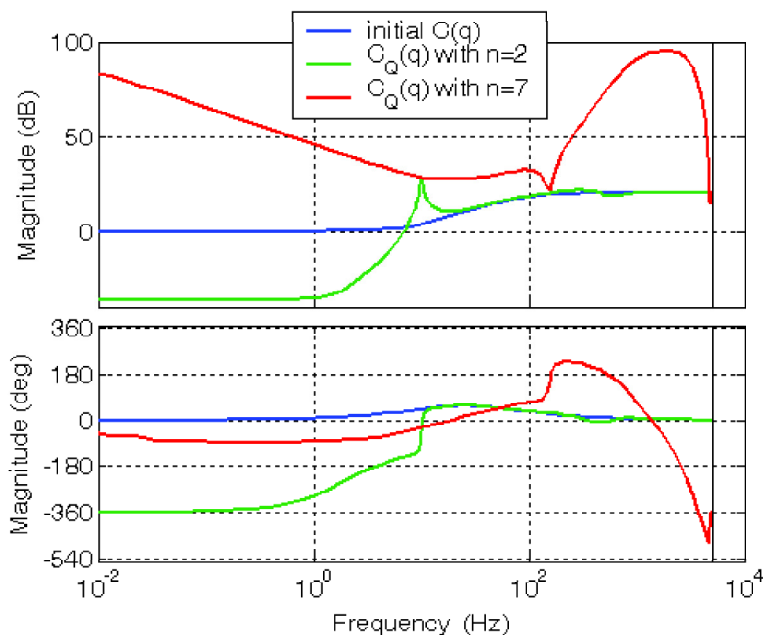


Fig. 5: Amplitude (top) and phase (bottom) Bode plot of the initial controller $C(q)$ in (21) and the optimized controller $C_Q(q)$ obtained via REACT for different orders n for the FIR filter $Q(q, \theta)$ in (7).

It can be observed from Figure 5 that a simple second order FIR filter $Q(q, \theta)$ allows the creation of an additional complex pole pair in the controller $C_Q(q)$. It can be seen that this additional complex pole pair is used to create a resonance in the controller at 10Hz with enough gain to reduce the periodic part of the disturbance $d(t)$ given in (22). Further increase of the allowable model order for the FIR filter to $n=7$ creates both an integrating action in the controller and an additional increase of the overall gain. This allows the same gain reduction of the periodic disturbance and additional reduction of the low frequency parts in the disturbance $d(t)$ given in (22).

When inspecting the shape of the frequency response of the controller $C_Q(q)$ as given in Figure 5, one could also design a good (low order) controller that achieves the same level of disturbances attenuation. However, it should be pointed out that for such a design, the disturbances $d(t)$ in (22) should be known *a priori*. The power of the REACT algorithm is that the initial controller $C(q)$ is able to tune itself to $C_Q(q)$ on the basis of the (unknown) spectral contents of the disturbance. Henceforth, REACT enables an *in situ* tuning of the controller *a posteriori*.

The improvement in disturbance rejection can also be seen in Figure 6 in which a comparison is made between the sensitivity functions $S(q)$ in (5) and $S_Q(q)$ in (6). Again, a comparison is made for the choice of order $n=2$ and the final order $n=7$ for the FIR filter $Q(q, \theta)$ in (7) to keep the Bode plot of the sensitivity function readable. The additional gain at low frequencies in $C_Q(q)$ emulates an integrator that was missing in the PD controller $C(q)$ and now creates a feedback loop with a gain margin of 8.25dB at approx.

3500 rad/s and a phase margin of 52.3 deg at approx. 865 rad/s. The advantage is that the resulting controller $C_Q(q)$ was found by automatic tuning based on data obtained from the closed-loop system $C(q)$. The tuning has caused optimal disturbance rejection in the servo performance signal (SPS) $e(t)$ by direct minimization of the variance of the SPS.

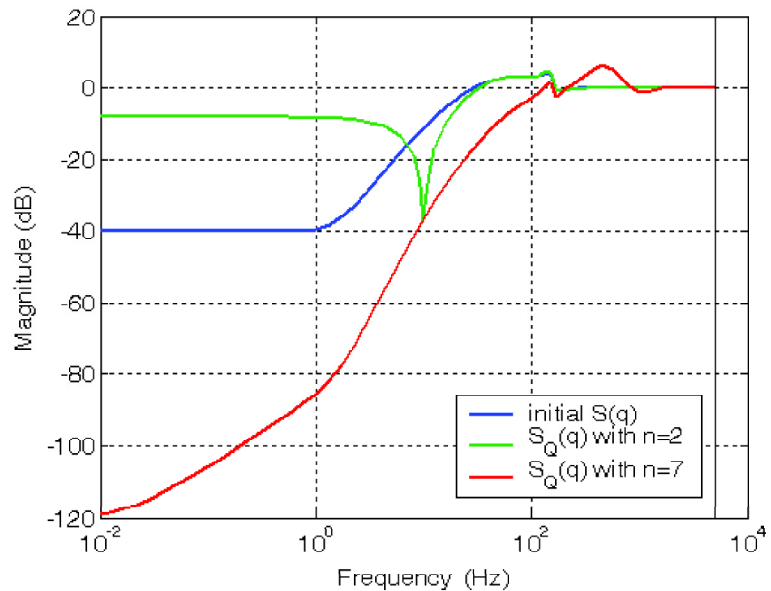


Fig. 6: Amplitude Bode plot of the initial sensitivity function $S(q)$ in (5) and the optimized sensitivity function $S_Q(q)$ in (6) found via REACT for different orders n for the FIR filter $Q(q, \theta)$ in (7).

8. Summary and Conclusions

An initial (PID) servo control algorithm can be augmented with a Youla parametrization based perturbation to formulate a self-tuning algorithm for a servo controller. For actuator dynamics that can be modeled by a stable transfer function, the parametrization is formulated as a feedback loop that uses the actuator model and a free, but stable, perturbation transfer function given by a Finite Impulse Response (FIR) filter.

It is shown in this paper that the parameters of the FIR filter can be found by an affine optimization based on closed-loop data to optimally tune the perturbed feedback controller. Uncertainty on the actuator model can be incorporated by bounding the allowable controller perturbation to provide a Robust Estimation and Adaptive Controller Tuning (REACT) to disturbance spectra to minimize SPS variance in high performance servo systems in data storage applications.

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