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Robust Estimation for Automatic Controller Tuning with Application to Active Noise Control

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Abstract In this work, we show how the double-Youla parameterization can be used to recast the robust tuning of a feedback controller as a robust estimation problem. The formulation as an estimation problem allows tuning of the controller in real-time on the basis of closed loop data. Furthermore, robust estimation is obtained by constraining the parameter estimates so that feedback stability will be maintained during controller tuning in the presence of plant uncertainty. The combination of real-time tuning and guaranteed stability robustness opens the possibility to perform Robust Estimation for Automatic Controller Tuning (REACT) to slowly varying disturbance spectra. The procedure is illustrated via the application of narrow-band disturbance rejection in the active noise control of cooling fans.

1 Introduction

In the 60's and 70's there was a significant amount of work [8, 12, 14, 15] towards rejecting disturbances that satisfy differential equations. An interesting instance of this theory is when the differential equations have constant coefficients and poles on the imaginary axis. In this context, these results (known as the internal model principle) dictate what is required of the system so that a controller exists to reject disturbances that satisfy a known differential equation.

Today, the same principles are studied in discrete time repetitive and learning control literature [9]. The same constraints upon the system are needed as well as knowledge of the disturbance model. In practice, it is very difficult to precisely model the disturbance frequency and therefore many methods were developed to design controllers that were robust against this uncertainty in the disturbance model or to design controllers to adapt to the disturbance. Hillerström [13] used adaptive

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repetitive control to suppress vibrations. Bodson [3] showed an equivalence between time-varying internal models and adaptive feedforward control. Brown and Zhang [5] update the internal model to cancel a disturbance with an unknown frequency. In [4] the adaptive internal model principle is discussed. In [18], we show how the extended Kalman filter can be used to update parameters in a controller that satisfies the internal model principle. In [17], we find a family of controllers off-line that satisfy the internal model principle and a frequency estimator is used to switch between controllers online.

Landau et al. [20] used the Youla parameterization of all stabilizing controllers for a SISO system to update the controller online to reject the disturbance when the disturbance model was not completely known. It was assumed that the plant model was known exactly and the disturbances had poles on the unit circle. In [19], we added to this result by showing how to consider uncertainty to develop robust algorithms for updating the controller of SISO stable systems. The convergence of the algorithm was not analyzed.

In this work, we add to the work of [20] and [19] by considering a more general setting and proving convergence of the tuning algorithm. In this work, the plant is a MIMO system with uncertainty. The natural representation to express the uncertainty turns out to be the double-Youla parameterization [11]. This parameterization was studied by Schrama [24] in the context of cautious controller enhancement, where a controller perturbation is found off-line to improve nominal and robust performances. This parameterization is beneficial because it provides a clear strategy to maintain stability and gives access to many closed loop signals that aid in the control design. Our goal is to tune the controller in realtime to reduce the effect of the disturbances on the output and, if possible, achieve complete regulation. Attenuation of the disturbances will be accomplished in the presence of uncertainty in the plant and with very limited knowledge of the disturbance class. The only knowledge of the disturbance that will be used is the order of the generating system. The convergence of the realtime control algorithm will be analyzed be using Lyapunov theory and concepts from slowly-varying system [16]. The stability of the closed loop system will be analyzed via the Small Gain Theorem [26]. To demonstrate the effectiveness of the proposed Robust Estimation for Automatic Controller Tuning (REACT) algorithm, an experimental study based upon active noise control has been included.

2 Approach to Automatic Controller Tuning

2.1 Simultaneous Perturbation of Plant and Controller

We are considering the problem shown in Fig. 1. The output of the plant $y \in \mathbb{R}^{n_y}$, the reference signal $r \in \mathbb{R}^{n_r}$, the output of the controller $y_c \in \mathbb{R}^{n_u}$, the input disturbance $d_i \in \mathbb{R}^{n_u}$, and the output disturbance $d_o \in \mathbb{R}^{n_y}$.

The nominal plant model $G_x(s) \in \mathscr{R}_p(s)$ is a $n_y \times n_u$ transfer function matrix, where $\mathscr{R}_p(s)$ denotes the set of rational proper transfer function matrices [26], that admits a bicoprime factorization [26] given by $G_x = N_x D_x^{-1} = \tilde{D}_x^{-1}\tilde{N}_x$. The nominal controller $C(s) \in \mathscr{R}_p(s)$ is a $n_u \times n_y$ transfer function matrix that the bicoprime factorization $C = N_C D_C^{-1} = \tilde{D}_C^{-1}\tilde{N}_C$.

The uncertain system $G_\Delta \in \mathscr{R}_p(s)$ is a $n_y \times n_u$ transfer function matrix that has a dual-Youla parameterization given by

$$\begin{aligned} G_\Delta &= N_{G_\Delta} D_{G_\Delta}^{-1} = (N_x + D_c\Delta_G)(D_x - N_c\Delta_G)^{-1} \\ &= \tilde{D}_{G_\Delta}^{-1}\tilde{N}_{G_\Delta} = (\tilde{D}_x - \tilde{\Delta}_G\tilde{N}_c)^{-1}(\tilde{N}_x + \tilde{\Delta}_G\tilde{D}_c) , \end{aligned} \tag{1}$$

where N_{G_Δ} and D_{G_Δ} are right coprime factors $\tilde{N}_{G_\Delta}$ and $\tilde{D}_{G_\Delta}$ are the left coprime factors of the uncertain system.

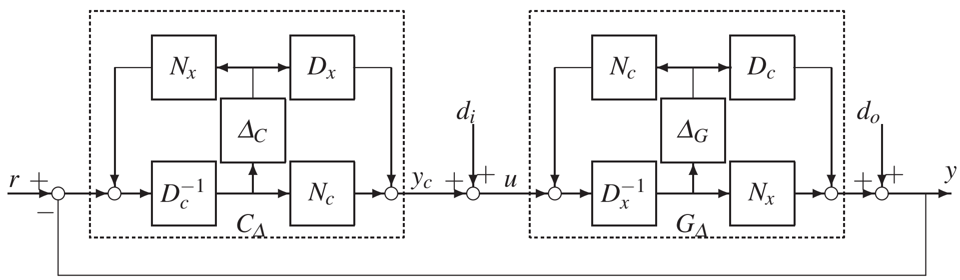

Fig. 1 Double-Youla parameterization of a feedback system with a MIMO controller C_Δ and a MIMO uncertain plant G_Δ. The controller C_Δ is expressed with the Youla parameterization and the plant G_Δ is expressed with the dual-Youla parameterization.

The uncertainty for the plant is considered to belong to the following set

$$\Xi = \{\Delta : \|\Delta\|_\infty < 1/\gamma,\ \ \Delta \in \mathscr{RH}_\infty\} , \tag{2}$$

for a given $\infty > \gamma > 0$. This class of uncertainty creates a set of plants Π, given by

$$\Pi = \left\{P : P = (N_x + D_c\Delta_G)(D_x - N_c\Delta_G)^{-1}, \Delta_G \in \Xi\right\} \tag{3}$$

for which a robustly stabilizing controller is sought.

The perturbed or tuned controller is described with the Youla parameterization given by

$$\begin{aligned} C_\Delta &= N_{C_\Delta} D_{C_\Delta}^{-1} = (N_C + D_x\Delta_C)(D_C - N_x\Delta_C)^{-1} \\ &= \tilde{D}_{C_\Delta}^{-1}\tilde{N}_{C_\Delta} = (\tilde{D}_C - \tilde{\Delta}_C\tilde{N}_x)^{-1}(\tilde{N}_C + \tilde{\Delta}_C\tilde{D}_x) , \end{aligned} \tag{4}$$

where N_{C_Δ} and D_{C_Δ} a right coprime factors of the tuned controller and $\tilde{N}_{C_\Delta}$ and $\tilde{D}_{C_\Delta}$ are the left coprime factors. The controller perturbation Δ_C is used to improve performance while maintaining robust stability in the presence of Δ_G. The perturbation

$\Delta_C \in \mathscr{RH}_\infty$ is a stable system, such that $(I - D_C^{-1} N_x \Delta_C)(\infty)$ and $(I - \tilde{\Delta}_C \tilde{N}_x \tilde{D}_C^{-1})(\infty)$ are invertible.

2.2 Disturbance Model

Each element of the input disturbance d_i and the output disturbance d_o is assumed to satisfy

$$\dot{x} = \text{diag}\left(\begin{bmatrix} 0 & \omega_1 \\ -\omega_1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & \omega_2 \\ -\omega_2 & 0 \end{bmatrix}, \dots, \begin{bmatrix} 0 & \omega_{n_d} \\ -\omega_{n_d} & 0 \end{bmatrix}\right) x = A_d x, \qquad x(0) = x_o$$
$$y_d = C_d x \ ,$$

such that the system is observable. This implies that every input and every output channel of the plant is subjected to a sum of periodic disturbance. However, the magnitude and phase are not known since we assume that the initial conditions are unknown.

For notational simplicity, we will assume that d_i and d_o are produced by

$$\begin{aligned} \dot{x}_i &= A_{H_i} x_i & \dot{x}_o &= A_{H_o} x_o \\ d_i &= C_{H_i} x_i & d_o &= C_{H_o} x_o \ , \end{aligned}$$

where

$$\begin{aligned} A_{H_i} &= \text{diag}(\underbrace{A_d, A_d, \dots, A_d}_{n_u \text{ times}}) & A_{H_o} &= \text{diag}(\underbrace{A_d, A_d, \dots, A_d}_{n_y \text{ times}}) \\ C_{H_i} &= \text{diag}(\underbrace{C_d, C_d, \dots, C_d}_{n_u \text{ times}}) & C_{H_o} &= \text{diag}(\underbrace{C_d, C_d, \dots, C_d}_{n_y \text{ times}}) \ . \end{aligned}$$

When $\omega_i, i = 1, \dots, n_d$ are known constants, this problem can be solved with the servocompensator theory developed by Davison [7, 8], Johnson [15], Francis and Wonham [12], and more recently de Roover et al. [9]. However, the precise knowledge of the frequencies is sometimes not feasible. Therefore, in this work we will create a realtime control algorithm to reject the disturbances without knowledge of the frequencies $\omega_i, i = 1, \dots, n_d$ and maintain robust stability.

2.3 Overview of REACT

The aim of the REACT algorithm is to update the controller without violating the robustness constraint imposed by the plant uncertainty Δ_G and without complete knowledge of the disturbance model. To facilitate the automatic or realtime controller tuning, the double-Youla parameterization is used to create a robust estimation problem to find the controller perturbation Δ_C.

The REACT algorithm inputs signals from the closed loop system and outputs the controller perturbation Δ_C by using information regarding the nominal plant model and nominal controller to formulate an estimation problem. In the end, REACT provides a method for updating Δ_C to improve performance and maintain robust stability. The derivation of REACT is presented in Sec. 3, where we define and minimize a cost function. The stability and convergence of the algorithm are considered in Sec. 4. Finally, in Sec. 5 the REACT algorithm is applied to reduce the periodic noise emitted by a pair of cooling fans.

3 REACT Algorithm

3.1 Defining an Error Function

In this section, we will define an error function based upon the how the disturbances d_o and d_i effect the output y. The equation that relates the disturbances and output is given by

$$\begin{aligned} y &= (I+G_\Delta C_\Delta)^{-1}d_o + G_\Delta(I+C_\Delta G_\Delta)^{-1}d_i \\ &= (I+G_\Delta C_\Delta)^{-1}d_o + (I+G_\Delta C_\Delta)^{-1}G_\Delta d_i \\ &= D_{C\Delta}\tilde{\Lambda}^{-1}\tilde{D}_{G_\Delta}(d_o + G_\Delta d_i)\ , \end{aligned}$$

where $\Lambda := \tilde{D}_{C_\Delta}D_{G_\Delta} + \tilde{N}_{C_\Delta}N_{G_\Delta}$ and $\tilde{\Lambda} := \tilde{D}_{G_\Delta}D_{C_\Delta} + \tilde{N}_{G_\Delta}N_{C_\Delta}$. After some tedious algebra we can write Λ and $\tilde{\Lambda}$ as $\Lambda = \Lambda_0 + \tilde{\Delta}_C\tilde{\Lambda}_0\Delta_G$ and $\tilde{\Lambda} = \tilde{\Lambda}_0 + \tilde{\Delta}_G\Lambda_0\Delta_C$, where $\Lambda_0 = \tilde{D}_C D_x + \tilde{N}_C N_x$ and $\tilde{\Lambda}_0 = \tilde{D}_x D_C + \tilde{N}_x N_C$.

From [23, Lemma 3.2], we get $\tilde{\Lambda} = \tilde{\Lambda}_0 + \tilde{\Lambda}_0\Delta_G\Delta_C = \tilde{\Lambda}_0(I+\Delta_G\Delta_C) = (I+\tilde{\Delta}_G\tilde{\Delta}_C)\tilde{\Lambda}_0$ therefore it is easy to see that we require the feedback connection of Δ_C and Δ_G to be stable if it is desired that the feedback system of G_Δ and C_Δ is stable.

Next, notice that

$$\begin{aligned} \tilde{\Lambda}_o^{-1}(\tilde{D}_x y - \tilde{N}_x y_c) &= \tilde{\Lambda}_o^{-1}(\tilde{D}_x D_{C\Delta} + \tilde{N}_x N_{C\Delta})D_{C\Delta}^{-1} y = D_{C\Delta}^{-1} y \\ &= \tilde{\Lambda}^{-1}\tilde{D}_{G_\Delta}(d_o + G_\Delta d_i)\ , \end{aligned}$$

which is obtained by substituting y and rearranging. Thus the output satisfies $y = D_{C\Delta}\tilde{\Lambda}_o^{-1}(\tilde{D}_x y - \tilde{N}_x y_c)$, where $\tilde{\Lambda}_o^{-1}$ is stable since the nominal feedback system of C and G is stable, D_x and N_x are stable, and the signals y and y_c are known.

Let η be defined as $\eta := \tilde{\Lambda}_o^{-1}(\tilde{D}_x y - \tilde{N}_x y_c)$ then the output is simply $y = (D_c - N_x\Delta_C)\eta$.

At this point, we will introduce two parameters: θ and ψ. This will facilitate in the analysis of the control algorithm by separating the optimization problem from the feedback system. Let $\Delta_C(\theta)$ indicate the parameter that will be used in the optimization problem and let $\Delta_C(\psi)$ indicate the controller perturbation that is implemented in the feedback system. In this case, we have the following error signal ε, defined as

$$\varepsilon(t,\theta,\psi) := (D_c(t) - N_x(t) * \Delta_C(t,\theta)) * \eta(t,\psi) \quad (5)$$

$$\eta(t,\psi) = \Lambda_o^{-1}(t) * (\tilde{D}_x(t) * y(t) - \tilde{N}_x(t) * y_c(t,\psi)) \,, \quad (6)$$

where $*$ is the convolution operator.

The separation between the optimization and the feedback system is needed for convergence of the adaptive algorithm. The separation will be used to create two different time-scales, one time-scale for the feedback system and one for the optimization algorithm. The separation of time-scales is a common theme in the robustness analysis of adaptive systems [2, 22] and the same idea will be used here to prove convergence in the presence of uncertainty.

Notice that $\varepsilon(t,\psi,\psi) = y(t,\psi)$ with $\Delta_C(\psi)$ implemented. Thus, this error signal has fixed points in common with $y(t)$ but is not always equal. Also, $\varepsilon(t,\psi,\psi)$ is a nonlinear function of $\Delta_C(\psi)$ and it depends upon the uncertainty Δ_G. For these reasons it is not possible to use $\varepsilon(t,\psi,\psi)$ as an error signal, but we can use $\varepsilon(t,\theta,\psi)$ and minimize over θ.

3.2 Derivation of Algorithm

Since Δ_C appears affinely in Eq. (5), if we pick an affine parameterization of $\Delta_C(\theta)$ then $\varepsilon(t,\theta,\psi)$ will be an affine function of θ. This will aid in the analysis of robust stability when the small gain theorem is applied.

The controller perturbation has a parameterization given by

$$\begin{aligned}\Delta_C[u(t)] &= \Theta(t)^T \int_0^t (C_D e^{A_D(t-\tau)} B_D + D_D) u(\tau) d\tau \\ &= \Theta(t)^T (\mathscr{D}(t) * u(t)) \,, \end{aligned} \quad (7)$$

where $\Theta \in \mathbb{R}^{n_y n_\theta \times n_u}$, $\mathscr{D}$ is the stable LTI system given by

$$\mathscr{D}^T = \begin{bmatrix} \mathtt{I}\left(\frac{s-p_0}{s+p_0}\right) & \mathtt{I}\left(\frac{s-p_0}{s+p_0}\right)^2 & \dots & \mathtt{I}\left(\frac{s-p_0}{s+p_0}\right)^{n_\theta} \end{bmatrix} ,$$

and $p_0 > 0$. The state space realizations for $\mathscr{D}$ and Δ_C are given by

$$\mathscr{D} : \left[\begin{array}{c|c} A_D & B_D \\ \hline C_D & 0 \end{array}\right] \qquad \Delta_C : \left[\begin{array}{c|c} A_D & B_D \\ \hline \Theta^T(t) C_D & 0 \end{array}\right]$$

with zero initial conditions. Notice that since Δ_C is strictly proper, this parameterization guarantees that $(I - D_C^{-1} N_x \Delta_C)(\infty)$ is invertible.

If the Θ is stacked into a vector instead of a matrix then define θ as $\theta^T := \mathrm{vec}(\Theta)^T = \begin{bmatrix}\theta_1^T & \dots & \theta_{n_\theta}^T\end{bmatrix}$, where $\mathrm{vec}(\cdot)$ is the vectorization operator and θ_i is the i^{th} column of Θ. With this notation we get

$$\varepsilon(t,\Theta,\psi) = \left(D_c(t) - N_x(t) * \left(\Theta^T(t)\mathscr{D}(t)\right)\right) * \eta(t,\psi) \tag{8}$$

$$= v(t,\psi) - N_x(t) * \left(\Theta^T(t)\eta_D(t,\psi)\right) \ , \tag{9}$$

where $v(t,\psi) := D_c(t) * \eta(t,\psi)$ and $\eta_D(t,\psi) := \mathscr{D}(t) * \eta(t,\psi)$.

Suppose that we want to minimize $V(t,\theta,\psi) = \frac{1}{2}\|\varepsilon(t,\theta,\psi)\|_2^2$ then ∇V is given by

$$\frac{\partial V(t,\theta,\psi)}{\partial \theta} = -\begin{bmatrix} [N_x]_{11} * \eta_D & \dots & [N_x]_{n_y 1} * \eta_D \\ \vdots & \ddots & \vdots \\ [N_x]_{1 n_u} * \eta_D & \dots & [N_x]_{n_y n_u} * \eta_D \end{bmatrix} \varepsilon(t,\theta,\psi) \tag{10}$$

$$= -\Phi(t,\psi)\varepsilon(t,\theta,\psi) \ . \tag{11}$$

Thus, update equation is given by

$$\begin{aligned}\frac{d\theta}{dt} &= \mu\Phi(t,\psi)\varepsilon(t,\theta,\psi) \\ &= \mu r(t,\psi) - \mu\Phi(t,\psi)\Phi(t,\psi)^T\theta \ ,\end{aligned}$$

where $r(t,\psi) := \Phi(t,\psi)v(t,\psi)$. This is the LMS algorithm for updating θ when ψ is constant. Thus, if ψ is constrained to vary slowly then we expect similar properties to the standard LMS algorithm [2, 22].

4 Stability and Convergence of the Tuning Algorithm

In this section the stability and convergence of the realtime tuning algorithm is considered. The stability of the feedback system during tuning will be analyzed via the Small Gain Theorem [25, 26]. The convergence of the algorithm will be analyzed with Lyapunov theory [16].

4.1 Stability of the Feedback System

In this section, we investigate two different scenarios. The first is where the controller is updated very quickly. In this case, a Small Gain Theorem for time-varying systems can be used. In the second scenario, we will constrain ψ to vary slowly. In this case, we will be able to impose an LTI Small Gain Theorem for stability.

Suppose that $\Psi(t)$ is a time-varying function. If the magnitude of $\Psi(t)$ is constrained to be small enough then stability of the feedback system is assured. The following theorem clarifies this point.

Theorem 1. *Consider the feedback system depicted in Fig. 1 where the uncertain plant $G_\Delta \in \Pi$ has a representation given by Eq. (1) and the controller has a Youla-parameterization given by Eq. (4) with Δ_C given by Eq. (7) with Θ replaced by Ψ. If the time-varying matrix $\Psi(t)$ satisfies*

$$\|\Psi(t)\|_2 \leq \frac{\gamma}{\|\mathscr{D}\|_\infty} \quad \forall t$$

then the feedback system of G_Δ and C_Δ is L_2-stable for all $G_\Delta \in \Pi$.

Proof. The feedback system of G_Δ and C_Δ is stable iff the feedback system of Δ_G and Δ_c is stable since $\tilde{\Lambda}^{-1} = (I+\Delta_G\Delta_C)^{-1}\tilde{\Lambda}_0^{-1}$ and $\tilde{\Lambda}_0^{-1}$ is stable since the feedback system of G_x and C is stable. Let $u_D(t) := \mathscr{D}(t) * u(t)$ then

$$\begin{aligned}\|\Delta_C[u]\|_{L2}^2 &= \int_0^\infty \|\Delta_C[u]\|_2^2 dt = \int_0^\infty \|\Psi(t)^T u_D(t)\|_2^2 dt \\ &\leq \int_0^\infty \|\Psi(t)^T\|_2^2 \|u_D(t)\|_2^2 dt\ .\end{aligned}$$

Since $\|u_D\|_{L2} \leq \|\mathscr{D}\|_\infty \|u\|_{L2}$ holds, if $\|\Psi\|_2 \leq \frac{\gamma}{\|\mathscr{D}\|_\infty}$ then

$$\begin{aligned}\|\Delta_C[u]\|_{L2}^2 &\leq \frac{\gamma^2}{\|\mathscr{D}\|_\infty^2} \int_0^\infty \|u_D(t)\|_2^2 dt = \frac{\gamma^2}{\|\mathscr{D}\|_\infty^2} \|u_D(t)\|_{L2}^2 \\ &\leq \frac{\gamma^2}{\|\mathscr{D}\|_\infty^2} \|\mathscr{D}\|_\infty^2 \|u\|_{L2}^2 = \gamma^2 \|u\|_{L2}^2\end{aligned}$$

implies that the $L2/L2$ gain of Δ_C is not greater than γ. Since $\|\Delta_G\|_\infty < 1/\gamma$ the closed loop system of Δ_G and Δ_C is L_2-stable [25]. □

The result in Theorem 1 is similar to [11, Proposition 1], except here we are considering the case where the controller perturbation is a time-varying operator.

The preceding work constrains the magnitude of $\Psi(t)$ only. If a less restrictive bound is sought then additionally constraining $\dot{\Psi}(t)$ is an option. Under the right conditions, if $\dot{\Psi}(t)$ is small enough then the stability constraint is equivalent to the LTI case. Before stating the result, a lemma from obtained from [10] will be presented.

Lemma 1. *Consider $\dot{x} = A(t)x$ where $A(t)$ is a piecewise continuous function on $\mathbb{R}^+$. Suppose that $\sup_{t\geq 0}\|A(t)\| < \infty$ and $\sup_{t\geq 0}\mathscr{R}\mathrm{e}(\mathrm{eig}(A(t))) < 0$ then there exists an $\varepsilon > 0$ such that $\dot{x} = A(t)x$ is stable if $\sup_{t\geq 0}\|\dot{A}(t)\| \leq \varepsilon$.*

Theorem 2. *Suppose that $\|\Delta_G\|_\infty < 1/\gamma$, the frozen time system Δ_C satisfies $\|\Delta_C(\Psi)\|_\infty \leq \gamma$ for each $t \geq 0$, and that $\sup_{t\geq 0}\|\Psi(t)\| < \infty$. Then there exits an $\varepsilon > 0$ such that the feedback system of G_Δ and C_Δ is stable if $\sup_{t\geq 0}\|\dot{\Psi}(t)\| \leq \varepsilon$.*

Proof (Outline) The proof shows that the conditions for Lemma 1 are upheld. $\sup_{t\geq 0}\|\Psi(t)\| < \infty$ implies that $\sup_{t\geq 0}\|A_{cl}(t)\| < \infty$ is true, where A_{cl} is the "A" matrix of the closed loop system. $\sup_{t\geq 0}\|\Delta_G\|_\infty\|\Delta_C(\Psi)\|_\infty \leq \|\Delta_G\|_\infty\gamma < 1$ implies that $\sup_{t\geq 0}\mathscr{R}\mathrm{e}(\mathrm{eig}(A(t))) < 0$.

By Lemma 1 there exists an ε^* such that $\sup_{t\geq 0}\|\dot{A}_{cl}(t)\| \leq \varepsilon^*$ implies that the closed loop is stable. Due to the affine structure of the controller parameterization, for each ε^* there exits an $\varepsilon > 0$ such that $\sup_{t\geq 0}\|\dot{\Psi}(t)\| \leq \varepsilon$ implies $\sup_{t\geq 0}\|\dot{A}_{cl}(t)\| \leq \varepsilon^*$. □

4.2 Convergence of the Tuning Algorithm

In this section, we analyze the convergence of the tuning algorithm by separating the time-scale of the feedback system and the update algorithm. So far, we have defined how the parameters in the optimization are updated but have not defined how the controller is updated. First it is necessary to constrain the controller perturbation parameters to be in the set of parameters that stabilize the feedback system. This can be done via the following algorithm.

$$\begin{aligned}\dot{\theta} &= \mu r(t,\psi) - \mu\Phi(t,\psi)\Phi(t,\psi)^T\theta \\ \dot{\psi} &= \operatorname*{Proj}_{\psi\in\mathscr{S}}\left(-\lambda\frac{\psi-\theta}{1+\|\psi-\theta\|}\right),\end{aligned}$$

where $\mathrm{Proj}_{\psi\in\mathscr{S}}(\cdot)$ is the projection operator [2, 22] that guarantees that ψ never leaves the set of stabilizing parameters $\mathscr{S}$. The set $\mathscr{S}$ can either be

$$\mathscr{S} = \{\psi : \|\Psi\|_2 \leq \gamma/\|D\|_\infty\} \tag{12}$$

in agreement with Theorem 1 or

$$\mathscr{S} = \{\psi : \|\Delta_C(\Psi)\|_\infty \leq \gamma\}\bigcap\{\psi^T\psi \leq N_\psi < \infty\} \tag{13}$$

in agreement with Theorem 2, where N_ψ is a large number that guarantees that $\sup_{t\geq 0}\|\Psi(t)\| < \infty$. In the latter case, λ must be chosen small enough to satisfy Theorem 2. In either case, the set $\mathscr{S}$ is a compact set. Also, note that the approximation $\|\Psi\|_2 \leq \|\Psi\|_F = \|\psi\|_2$ can be used for computational speed.

To analyze this system consider for the moment ψ as being a fixed parameter and recall the error signal

$$\begin{aligned}\varepsilon(s,\theta,\psi) &= (D_c - N_x\Delta_C(\theta))\eta(s,\psi) \\ \eta(s,\psi) &= \Lambda_o^{-1}(\tilde{D}_x y(s) - \tilde{N}_x y_c(s,\psi)) \\ &= (D_{C\Delta}(\psi) - G_\Delta N_{C\Delta}(\psi))^{-1}(d_o + G_\Delta d_i)\ .\end{aligned}$$

The purpose the the tuning algorithm is to place blocking zeros [26] in the transfer function from the disturbances to the output. Thus, it is required that there exists a θ^* such that

$$D_C(j\omega_i) - N_x(j\omega_i)\Delta_C(j\omega_i,\theta^*) = \bar{D}_C(j\omega_i) = 0 \tag{14}$$

for some $\bar{D}_C$ that internally stabilizes the system and has blocking zeros at the given ω_i. From this equation it can be seen that it is required that $N_x(j\omega_i)$ have full row rank. This agrees with general servocompensator theory [15, 8, 12, 6, 9] since $N_x(j\omega_i)$ having full row rank is the same as G_x having no zeros on the $j\omega$-axis located at ω_i and at least as many inputs as outputs.

In this case, one such controller perturbation is given by

$$\Delta_C(j\omega_i,\theta^*) = N_x(j\omega_i)^T(N_x(j\omega_i)N_x(j\omega_i)^T)^{-1}D_C(j\omega_i,\theta^*)\ .$$

When $\Delta_C = \Theta^T\mathscr{D}$ then Θ given by

$$\Theta^T = N_x(j\omega_i)^+ D_C(j\omega_i,\theta^*)\mathscr{D}(j\omega_i)^+$$

will satisfy Eq. (14), where it is required that $\mathscr{D}(j\omega_i) \in \mathbb{C}^{n_\theta n_y \times n_y}$ has full column rank, $N_x(j\omega_i)^+ = N_x(j\omega_i)^T(N_x(j\omega_i)N_x(j\omega_i)^T)^{-1}$, and
$\mathscr{D}(j\omega_i)^+ = (\mathscr{D}(j\omega_i)^T\mathscr{D}(j\omega_i))^{-1}\mathscr{D}(j\omega_i)^T$.

Theorem 3. *Consider the controller perturbation Δ_C given by Eq. (7). Suppose that $N_x(j\omega_i)$ has full row rank for all $0 \le i \le n_d$ and that $n_\theta \ge 2n_d$.*

Define the following matrices: $D := \begin{bmatrix}\mathscr{D}(j\omega_1) & \mathscr{D}(j\omega_2) & \dots & \mathscr{D}(j\omega_{n_d})\end{bmatrix}$,

$$b^T := \begin{bmatrix} D_C(j\omega_1)^T(N_x(j\omega_1)^+)^T \\ D_C(j\omega_2)^T(N_x(j\omega_2)^+)^T \\ \vdots \\ D_C(j\omega_{n_d})^T(N_x(j\omega_{n_d})^+)^T \end{bmatrix},\ A := \begin{bmatrix}\mathscr{R}\mathrm{e}\,(D^T) \\ \mathrm{im}\,(D^T)\end{bmatrix},\ \textit{and}\ B := \begin{bmatrix}\mathscr{R}\mathrm{e}\,(b^T) \\ \mathrm{im}\,(b^T)\end{bmatrix}\ .$$

Then A is full rank if $\omega_i \ne \omega_j$ for all $i \ne j$ and in this case the optimal parameter given by $\Theta^* = A^+B$ satisfies $D_C(j\omega_i) - N_x(j\omega_i)(\Theta^*)^T\mathscr{D}(j\omega_i) = 0$, for all $0 \le i \le n_d$.

Proof. For the controller perturbation given in Eq. (7) D can be written as

$$D = I_{n_y\times n_y}\otimes\begin{bmatrix} \frac{j\omega_1-p_o}{j\omega_1+p_o} & \frac{j\omega_2-p_o}{j\omega_2+p_o} & \cdots & \frac{j\omega_{n_d}-p_o}{j\omega_{n_d}+p_o} \\ \left(\frac{j\omega_1-p_o}{j\omega_1+p_o}\right)^2 & \left(\frac{j\omega_2-p_o}{j\omega_2+p_o}\right)^2 & \ddots & \left(\frac{j\omega_{n_d}-p_o}{j\omega_{n_d}+p_o}\right)^2 \\ \vdots & \vdots & \ddots & \vdots \\ \left(\frac{j\omega_1-p_o}{j\omega_1+p_o}\right)^{n_\theta} & \left(\frac{j\omega_2-p_o}{j\omega_2+p_o}\right)^{n_\theta} & \cdots & \left(\frac{j\omega_{n_d}-p_o}{j\omega_{n_d}+p_o}\right)^{n_\theta} \end{bmatrix},$$

where $\otimes$ is the Kronecker product.

Thus, $[D\ \bar{D}]$ is the first $n_d n_y$ columns of a block Vandermonde matrix which can be written as $[D\ \bar{D}] = I\otimes d$ where d is the first n_d columns of a Vandermonde matrix and $\bar{(\cdot)}$ is the complex conjugate. The Vandermonde matrix is invertible if $\omega_i \ne \omega_j$ for all $i \ne j$. Thus $[D\ \bar{D}]$ is full rank. And since

$$[D\ \bar{D}]\begin{bmatrix} \mathtt{I}\frac{1}{2} & \mathtt{I}\frac{1}{2j} \\ -\mathtt{I}\frac{1}{2} & -\mathtt{I}\frac{1}{2j} \end{bmatrix} = [\mathscr{R}\mathrm{e}(D)\ \ \mathrm{im}(D)]$$

then $[\mathscr{R}\mathrm{e}(D)\ \ \mathrm{im}(D)]$ is full rank.

The equation $(\Theta^*)^T[\mathscr{R}\mathrm{e}(D)\ \ \mathrm{im}(D)] - [\mathscr{R}\mathrm{e}(b)\ \ \mathrm{im}(b)] = 0$ can be written as

$$(\Theta^*)^T[D\ \bar{D}]\begin{bmatrix} \mathtt{I}\frac{1}{2} & \mathtt{I}\frac{1}{2j} \\ -\mathtt{I}\frac{1}{2} & -\mathtt{I}\frac{1}{2j} \end{bmatrix} = [b\ \bar{b}]\begin{bmatrix} \mathtt{I}\frac{1}{2} & \mathtt{I}\frac{1}{2j} \\ -\mathtt{I}\frac{1}{2} & -\mathtt{I}\frac{1}{2j} \end{bmatrix} ,$$

and since $\begin{bmatrix} \mathtt{I}\frac{1}{2} & \mathtt{I}\frac{1}{2j} \\ -\mathtt{I}\frac{1}{2} & -\mathtt{I}\frac{1}{2j} \end{bmatrix}$ is invertible then we get $(\Theta^*)^T[D\ \bar{D}] = [b\ \bar{b}]$. From the first part of this matrix, Θ^* satisfies $(\Theta^*)^T D = b$ which is the same as

$$(\Theta^*)^T = N_x(j\omega_i)^+ D_C(j\omega_i, \theta^*)\mathscr{D}(j\omega_i)^+ ,$$

for all $0 \le i \le n_d$ since $N_x(j\omega_i)$ has full row rank and $\mathscr{D}(j\omega_i)$ has full column rank when $\omega_i \neq \omega_j$ for all $i \neq j$. This implies that

$$D_C(j\omega_i) - N_x(j\omega_i)(\Theta^*)^T \mathscr{D}(j\omega_i) = 0 ,$$

for all $0 \le i \le n_d$. □

In the case that the ω_i's are known then this theorem provides an equation for θ^* that can be implemented. In this work, we are considering the ω_i's as unknown and are using the realtime tuning algorithm to converge to this point θ^*. Using this optimal point θ^*, the error signal is written as

$$\begin{aligned} \varepsilon(s,\theta,\psi) &= (D_c - N_x\Delta_C(\theta))\eta(s,\psi) \\ &= (\bar{D}_C(s) + N_x(\Delta_C(\theta^*) - \Delta_C(\theta)))\eta(s,\psi) , \end{aligned}$$

where $\bar{D}_C$ is given in Eq. (14). In the time domain, this can be written as

$$\varepsilon(t,\theta,\psi) = \bar{D}_C(t) * \eta(t,\psi) + N_x(t) * \left((\Theta^* - \Theta(t))^T \eta_D(t,\psi)\right) ,$$

where $\bar{D}_C(t) * \eta(t,\psi) \to 0$ since $\bar{D}_C(s) \in \mathscr{RH}_\infty$ and has blocking zeros at ω_i.

Hence, the point θ^* such that $D_c(j\omega_i) - N_x(j\omega_i)\Delta_C(j\omega_i,\theta^*) = 0$ is an equilibrium point for all fixed $\psi \in \Gamma$. Next, change variables $x = \theta - \theta^*$ to get

$$\begin{aligned} \dot{x} &= \mu r(t,\psi) - \mu\Phi(t,\psi)\Phi(t,\psi)^T(x+\theta^*) \\ &= -\mu\Phi(t,\psi)\Phi(t,\psi)^T x + \nu(t,\psi) , \end{aligned}$$

where $\nu(t,\psi)$ is an exponentially decreasing signal due to initial conditions. The solution $x \to 0$ if $\nu(t,\psi) \to 0$ exp. fast and $\dot{x} = -\mu\Phi(t,\psi)\Phi(t,\psi)^T x$ is exp. stable. So, by studying the stability of

$$\dot{z} = -\mu \Phi(t,\psi)\Phi(t,\psi)^T z$$

we can determine when the optimization will converge to the desired solution.

To analyze the convergence of the algorithm, we will find a Lyapunov function. To this end we first start by examining when a suitable Lyapunov function exists for a given system.

Lemma 2. *Let $x = 0$ be an equilibrium for*

$$\dot{x} = f(t,x,\alpha) \; ,$$

where $f : [0,\infty) \times D \times \gamma \to \mathbb{R}^n$ is continuously differentiable, $D = \{x \in \mathbb{R}^n : \|x\| < r\}$, and

$$\left\| \frac{\partial f}{\partial x} \right\| \leq L_1 \qquad \left\| \frac{\partial f}{\partial \alpha} \right\| \leq L_2 \|x\|$$

on D, uniformly in t and α. Let k, λ, and r_o be positive constants with $r_o < r/k$. Let $D_o = \{x \in \mathbb{R}^n : \|x\| < r_o\}$. Assume that the trajectories of the system satisfy

$$\|x(t)\| \leq k\|x(t_0)\| e^{-\lambda(t-t_0)} \;\; \forall x \in D, t \geq t_0 \geq 0, \alpha \in \Gamma$$

then there exists a function $V : [0,\infty) \times D_o \times \gamma \to \mathbb{R}$ such that the following hold

$$c_1\|x\|^2 \leq V(t,x,\alpha) \leq c_2\|x\|^2, \; \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t,x,\alpha) \leq -c_3\|x\|^2, \; \left\| \frac{\partial V}{\partial \alpha} \right\| \leq c_4\|x\|^2 \; .$$

Proof (Outline) Let $\phi(\tau,t,x,\alpha)$ indicate the solution to $\dot{x}(t) = f(t,x(t),\alpha)$ starting at (t,x) for a given $\alpha \in \Gamma$. Thus $\phi(t,t,x,\alpha) = x$. Choose

$$V(t,x,\alpha) = \int_t^{t+\delta} \phi(\tau,t,x,\alpha)^T \phi(\tau,t,x,\alpha) d\tau$$

as the Lyapunov function. It can be shown that this Lyapunov function satisfies the inequalities, and the proof is similar to proofs of Theorem 4.14 and Lemma 9.18 in [16]. □

Now, we can state the convergence result.

Theorem 4. *Suppose that $\Phi(t,\psi)$ is uniformly P.E., i.e. there exists constants β, δ, and T such that*

$$\infty > \beta I \geq \int_{t_o}^{t_o+T} \Phi(t,\psi)\Phi(t,\psi)^T dt \geq \delta I > 0$$

holds for all $t_o \geq 0$, all fixed $\psi \in \mathscr{S}$, and where β and δ are independent of ψ. Then there exists an $\varepsilon^* > 0$ such that

$$\dot{z} = -\mu \Phi(t,\psi(t))\Phi(t,\psi(t))^T z, \qquad \psi : \mathbb{R}^+ \to \mathscr{S}$$

converges to the origin exponentially fast if $\|\dot{\psi}\| \leq \varepsilon^*$.

Proof (Outline) Since the system is continuously differentiable and driven by a bounded periodic signal it can be shown that the assumptions of Lemma 2 are upheld. Therefore, there exists a Lyapunov function s.t.

$$\begin{aligned}
&c_1\|z\|^2 \leq V(z,\psi,t) \leq c_2\|z\|^2 \\
&\frac{\partial V}{\partial t} + \frac{\partial V}{\partial z}\dot{z} \leq -c_3\|z\|^2 \\
&\left\|\frac{\partial V}{\partial \psi}\right\| \leq c_4\|z\|^2
\end{aligned}$$

hold. Using this function $V(z,\psi(t))$ as the Lyapunov function for

$$\dot{z} = -\mu\Phi(t,\psi(t))\Phi(t,\psi(t))^T z\ ,$$

where now $\psi(t)$ is a function of t instead of a fixed parameter, yields

$$\begin{aligned}
\dot{V}(z,\psi,t) &= \left\|\frac{\partial V}{\partial t} + \frac{\partial V}{\partial z}\dot{z} + \frac{\partial V}{\partial \psi}\dot{\psi}\right\| \leq -c_3\|z\|^2 + \left\|\frac{\partial V}{\partial \psi}\dot{\psi}\right\| \\
&\leq -c_3\|z\|^2 + \left\|\frac{\partial V}{\partial \psi}\right\| \|\dot{\psi}\| \leq -c_3\|z\|^2 + c_4\|z\|^2\|\dot{\psi}\| \\
&= (c_4\|\dot{\psi}\| - c_3)\|z\|^2\ .
\end{aligned}$$

Therefore if $\|\dot{\psi}\| < c_3/c_4 := \varepsilon^*$ then $z = 0$ is exponentially stable. □

This theorem states when θ converges to the correct point θ^*. To conclude when regulation will occur the set $\mathscr{S}$ must contain θ^* and the point θ^* must exist in the first place. Thus we have the following.

Theorem 5. *Consider the feedback system depicted in Fig. 1 where the uncertain plant* $G_\Delta \in \Pi$ *has a representation given by Eq. (1) and the controller has a Youla-parameterization given by Eq. (4) with* Θ *replaced by* Ψ*. Assume that the controller perturbation parameters* ψ *are updated with the following tuning algorithm*

$$\begin{aligned}
\dot{\theta} &= \mu r(t,\psi) - \mu\Phi(t,\psi)\Phi(t,\psi)^T\theta \\
\dot{\psi} &= \operatorname*{Proj}_{\psi\in\mathscr{S}}\left(-\varepsilon\frac{\psi-\theta}{1+\|\psi-\theta\|}\right)\ ,
\end{aligned}$$

where $\mathscr{S}$ *is defined in either Eq. (12) or Eq. (13). If* $N_x(j\omega_i)$ *has full row rank for all* $0 \leq i \leq n_d$, $n_\theta \geq 2n_d$, *and* $\Phi(t,\psi)$ *is uniformly P.E. then there exists an* $\varepsilon^* > 0$ *such that* $\lim_{t\to\infty}\|\theta(t)-\theta^*\| = 0$ *whenever* $\varepsilon \leq \varepsilon^*$*, where* θ^* *is given in Theorem 3, and if* $\theta^* \in \mathscr{S}$ *then* $\lim_{t\to\infty}\|\psi(t)-\theta^*\| = 0$ *which implies* $\lim_{t\to\infty}\|y(t)\| = 0$.

Proof. Straightforward combination of Theorem 3 and Theorem 4, where either Theorem 1 or Theorem 2 is used to show that the set $\mathscr{S}$ is a set of robustly stabilizing parameters. □

5 Application to ANC

5.1 Description of System

Fig. 2 shows the layout of the acoustic system that we are considering. Two variable speed cooling fans connected in series are used to cool the enclosure, similar to a server or PC. However, due to the high speeds of the fans, acoustic noise is created. To combat the acoustic noise, 4 speakers are mounted near the fan and 4 feedback microphones are placed near the speakers and downstream of the acoustic noise. To reduce vibrations and turbulent noise, the microphones are mounted in acoustical foam. Additionally, for simplicity, the microphone signals are summed together and used as a single signal for feedback. Similarly, the same signal is sent to the speakers so that we are dealing with a single-input-single-output (SISO) system.

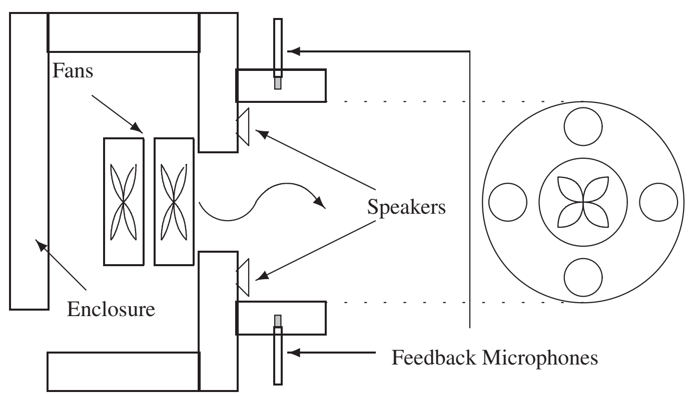

Fig. 2 Custom fan housing with active noise canceling speakers and feedback microphones.

The fans create two types of noise: broadband and narrowband noise. The narrowband noise is due to the blade pass frequency (BPF) of each fan and is comprised of a fundamental frequency and several harmonics. The broadband noise is due to turbulence. Both types of noise are dependent upon the speed of the fans. When the RPM of a cooling fan is increased the BPF increases causing the fundamental frequency to increase. Likewise, when the RPM increases, the turbulence increases and therefore the broadband noise level will increase.

The goal of the active noise control system is to reduce the narrowband acoustic noise without a priori knowledge of the fan speeds and with the presence of modeling errors. The REACT algorithm described in Sec. 2.3 is used to this end. The identification of a nominal model for the REACT algorithm is presented in Sec. 5.2 and the experimental results of applying REACT to the ANC system are presented in Sec. 5.3.

5.2 Identification of Plant Model

The nominal plant model G_x is found via standard system identification techniques [21]. To generate data that can be used for the identification, a white noise signal was sent into the speakers and the resulting signal was recorded with the feedback microphones. Recall that the microphone signals are averaged and the speakers are sent the same signal so that a SISO system results, and note that the acoustic system is comprised of the speaker amplifier, the speakers, that acoustic between the speaker and microphone, the microphone, and the microphone filter.

At steady state, if $\|\Delta_C \Delta_G\|_\infty < 1$ then the closed loop system is stable by the small gain theorem [26]. In addition, the internal model principle requires that $D_C(j\omega_k) - N_x(j\omega_k)\Delta_C(j\omega_k) = 0$, $k = 1,2,...,n_d$ for complete regulation, where ω_k is the frequency of the disturbances. Since we are dealing with a SISO, open-loop stable system we can choose $N_x = G_x$, $D_x = 1$, $N_C = 0$, and $D_C = 1$. Let G_o denote the "true" system then the requirements for stability and performance are given by the following:

1. $|G_o(e^{j\omega}) - G_x(e^{j\omega})| = |\Delta_{G_o}(e^{j\omega})| < 1/|\Delta_C(e^{j\omega})|$ for all ω.
2. $G_x(e^{j\omega_o}) = 1/\Delta_C(e^{j\omega_k})$ where ω_o is the unknown frequency of the disturbance.

Hence, these requirements can be satisfied if $|\Delta_{G_o}(e^{j\omega})| < |G_x(e^{j\omega})|$ holds over the frequency range that the disturbance is expected. A nominal model that satisfies this bound is deemed acceptable.

After several iterations a suitable model was chosen by the above design method. Using an ARX model structure [21] and a Steiglitz-Mcbride iteration a 25^{th} order model of the acoustic system was found. The frequency response of the model G_x and the true system G_o is shown in Fig. 3. It can be seen that the model neglects some low and high frequency dynamics and is a close approximation of the frequency response measurements in the middle frequencies. The nominal controller is chosen as $C = 0$ since the model is open-loop stable.

The magnitude of the uncertainty $|\Delta_{G_o}|$ is also shown in Fig. 3. In this figure, it can be seen that regulation is possible from 300 to 5000 Hz. However, the Δ_C that is found must satisfy $|\Delta_{G_o}(e^{j\omega})| < 1/|\Delta_C(e^{j\omega})|$ for all ω and one must be careful not to violate this bound while tuning the controller.

5.3 Experimental Results

The REACT algorithm described in Sec. 2.3 is applied to the ANC system described in Sec. 5.1. The results of the controller tuning are shown in Fig. 4. In this figure, the A-weighted[1] output $y(t)$ and control $u(t)$ signals are shown. On the left, the signals are shown for a duration of 6.4 sec. The first 3.2 sec. are without controller tuning. The last 3.2 sec. are the signals after the convergence of REACT. On the right, the

[1] A-weighting is used to reflect the sensitivity of human hearing [1].

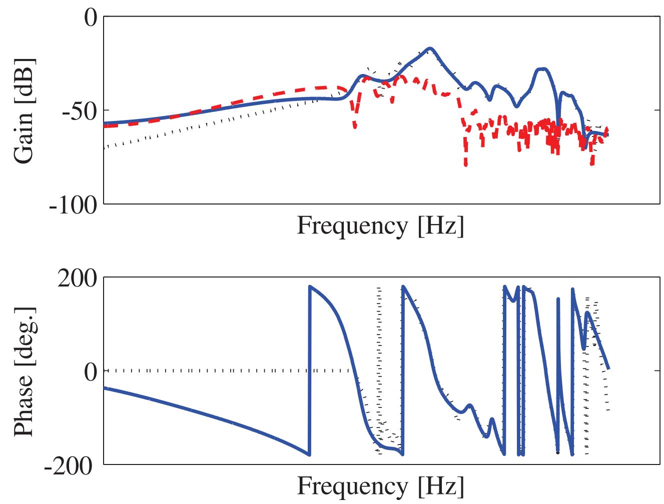

Fig. 3 Frequency response data of G_o (dotted), bode plot the nominal system G_x (solid) found with system identification techniques [21], and the magnitude of the additive uncertainty $|\Delta_{G_o}|$ (dashed). This figure is used to evaluate the quality of the nominal model G_x. For complete regulation in the presence of uncertainty, it is required that $|\Delta_{G_o}(e^{j\omega})| < |G_x(e^{j\omega})|$ for the frequency range of interest.

signals are shown for the first and last 0.0195 sec. In the plots on the left, it is clear that the level of the output signal is reduced when the controller tuning is switched on. The removal of the narrowband disturbances can be seen clearly in the plots on the right.

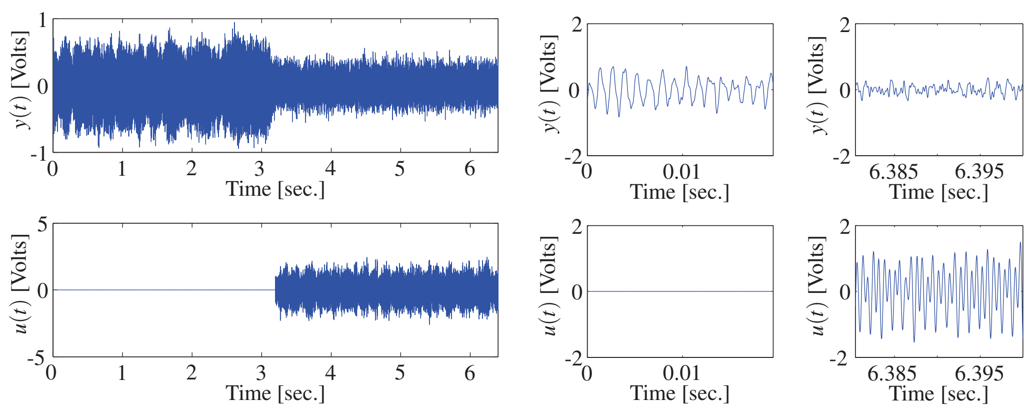

Fig. 4 The A-weighted output $y(t)$ and input $u(t)$ of the plant before and after tuning of the nominal controller. On the left is the entire 6.4 sec of data. On the right is the output and input for the first and last 0.0195 sec. The tuning algorithm reduces the output variance by 79%, or a 6.7 dB drop in SPL, by eliminating narrowband disturbances.

The A-weighted sample variance of the signal is reduced by 79%, which is a 6.7dB drop in the A-weighted sound pressure level (SPL)[2]. The A-weighted spectrum of the output $y(t)$ before and after tuning is shown in Fig. 5. In this figure, it can be seen that largest harmonics are reduced. The reduction of these harmonics

[2] SPL is defined as $SPL := 10\log_{10}(P^2/P_0^2)$ where P^2 is the sample variance of the microphone signal and P_0^2 is the reference level. If weighting is applied then the signal is filtered before the calculation of the sample variance.

can also be seen in Fig. 4. Note that both fans are running at similar speeds and therefore two harmonics from each fan, for a total of 4 harmonics, are reduced by tuning the controller.

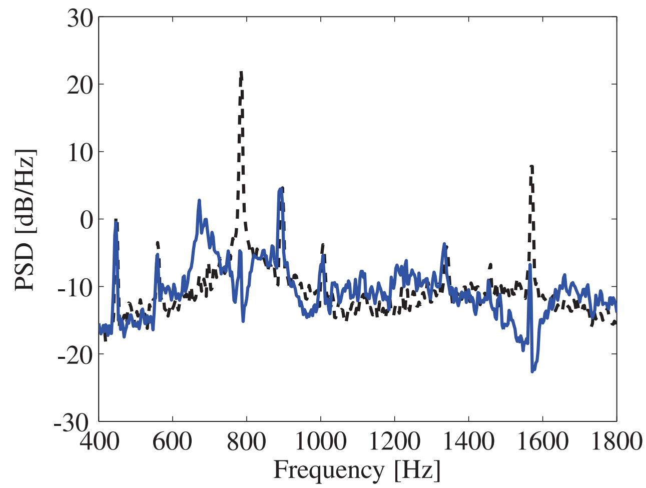

Fig. 5 A-weighted spectrum of the plant output $y(t)$ (the microphone signal) before (dashed) and after (solid) controller tuning. The tuning of the controller reduces the most prominent harmonics of both cooling fans. The result is a reduction in the A-weighted output variance by 79%, or a 6.7 dB drop in A-weighted SPL.

6 Conclusions

We have shown how to tune a given nominal controller in realtime to reject disturbances and to preserve robust stability. The uncertain system was assumed to belong to a set of systems described with the dual-Youla parameterization. To facilitate the tuning of the controller, the Youla parameterization was used along with a real-time optimization that utilizes signals from the closed loop system and information regarding the nominal system. The magnitude and velocity of the controller perturbation parameters are constrained to preserve robustness, and to guarantee that the optimization converges the controller in the feedback system is changed gradually. An application to active noise control was used to demonstrate the power of the proposed tuning algorithm. In the experimental results, it was shown how the harmonic noise of cooling fans can be reduced in the presence of modeling errors and measurement noise without a priori knowledge of the harmonic noise frequency.

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