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ACTIVE NOISE CONTROL OF A COOLING FAN IN A SHORT DUCT

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ABSTRACT

In this paper we develop a data based model, design a high performance robust controller, and apply the controller in real-time to reduce narrowband acoustic noise from a cooling fan. A custom, portable enclosure houses the cooling fan. One end of the enclosure is fitted with four speakers and four microphones, inside of a short duct, connected to a data acquisition system and a personal computer. Passive materials mounted at the other end of the enclosure reduce backside noise. The frequency of the narrrowband noise is assumed unknown and therefore a control design that can be updated in realtime is needed. The control design that is presented uses a nominal model and a nominal controller. The nominal controller is enhanced by using closed loop signals and taking into account the modeling error. The end result is a data-based method for updating a nominal controller to improve performance.

INTRODUCTION

Many design methods have been developed to create feedback controllers that can remove periodic disturbances. In the 1960's and 70's much of the focus was on servocompensators [1–4]. In this work, exact knowledge of the disturbance frequency is required for cancellation to occur. The resulting compensator satisfies the internal model principle [2].

Today, the same principles are studied in discrete time repetitive and learning control literature [5]. The same constraints upon the system are needed as well as knowledge of the disturbance model. In practice, it is very difficult to precisely model the disturbance frequency and therefore many methods were developed to design controllers that were robust against this uncertainty in the disturbance model or to design controllers to adapt to the disturbance. In [6] adaptive repetitive control is used to suppress vibrations. In [7] an equivalence between time-varying internal models and adaptive feedforward control is shown. In [8] the internal model is updated to cancel an disturbance with an unknown frequency. In [9] the adaptive internal model principle is discussed.

Landau et al. [10] used the youla parameterization of all stabilizing controllers for a SISO system to update the controller online to reject the disturbance when the disturbance model was not completely known. It was assumed that the plant model was exact and the disturbances had poles on the unit circle. This method has several benifits, one of which is a linear least squares optimization that can be used to enhance the current controller for improved performance. The drawback was the lack of robustness that makes implementation difficult.

In this paper, we present an extention Landau's work by considering systems with uncertainty. The end result is a constrained least squares optimization to enhance the nominal controller. The constraint is used to gaurentee robustness and is based upon the

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small gain theorem [11]. Obviously, the goals of robustness and performance are conflicting and therefore there are many situations where complete cancellation in the presence of uncertainty is not possible. To deal with this problem, we present a design methodology to pick a model and nominal controller that can gaurentee regulation over a frequency range.

The control design is applied to cancel acoustic noise emitting from a cooling fan. Input/ouput data is used to fit a nominal model and design a nominal controller. Robustness and regulation analysis is used to verify the accuracy of the nominal system and a simplified algorithm is used to update the controller in realtime from closed loop signals. In the end, the cancellation of narrowband disturbances is accomplished in a system with uncertainty and without knowledge of the disturbance frequency.

PROBLEM FORMULATION

General Problem

The general problem we are considering is shown in Fig. 1. In this figure, the plant G_o is subjected to narrowband disturbances d with an unknown frequency, magnitude, and phase. The goal of the control design is to find a controller C_Δ that will stabilize the feedback system and rejects the narrowband disturbances.

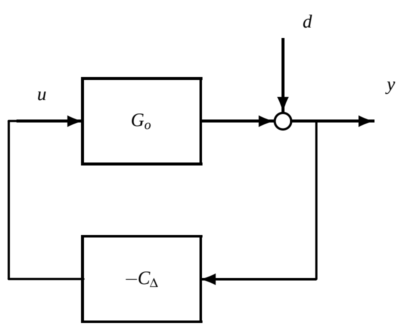

Figure 1. General control problem under consideration.

A model of the system G_x is obtained from data and a nominal controller C is designed that stabilizes the nominal feedback system. In the case of stable G_x, the controller can be $C = 0$. The idea is to perturb C to obtain the desired goals. To accomplish this, the youla and dual-youla parameterizations will be used simultaneously. Before we proceed with the control design we will first describe the acoustic system in more detail.

Acoustic System

Fig. 3 shows the layout of the acoustic system that we are considering in this paper. The fan is used to cool the enclosure, like a server or PC. However, due to the high speed of the fan, acoustic noise is created. To combat the acoustic noise, speakers are mounted near the fan and feedback microphones are placed near the speakers and downstream of the acoustic noise. To reduce vibrations and turbulent noise the microphones are mounted in acoustical foam. Additionally, for simplicity, the microphone signals are summed together and used as a single signal for feedback. Similarly, the same signal is sent to the speakers so that we are dealing with a single-input-single-output system.

The fan creates two types of noise: broadband and narrowband noise. The narrowband noise is due to the blade pass frequency (BPF) of the fan and is comprised of a fundamental frequency and several harmonics. The broadband noise is due to turbulence. Both types of noise are dependent upon the speed of the fan. When the RPM of the cooling fan is increased the BPF increases causing the fundamental frequency to increase. Likewise, when the RPM increases, the turbulence increases and therefore the broadband noise level will increase.

The fan noise is shown in Fig. 2. In this figure, it should be clear that the fundamental frequency is approximately 860 Hz and the broadband noise decreases as frequency increases. The goal of the active noise control system is to reduce the narrowband acoustic noise. The control design that will be used is model-based and therefore before we start a good model of the acoustic system is needed. The method used to obtain a model and to approximate the model uncertainty will be described in the following sections.

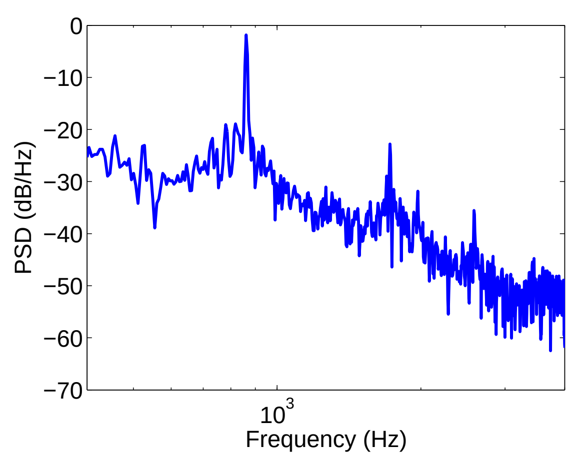

Figure 2. Power spectral density of the acoustic noise generated by the cooling fan.

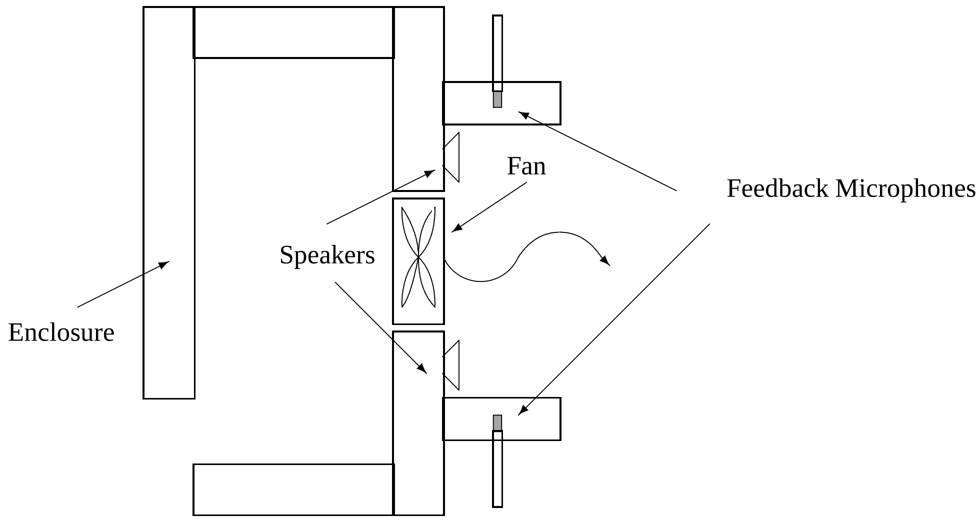

Figure 3. Active noise canceling system used to eliminate the unwanted noise from a cooling fan.

DUAL-YOULA PARAMETERIZATION

Consider representing a plant G_o as a perturbation from a nominal plant G_x with the coprime factors of the nominal controller $C = N_c D_c^{-1} = \tilde{D}_c^{-1}\tilde{N}_c$ that internally stabilizes the feedback system of the nominal controller and nominal plant. This gives the paramterization of all plants that are stabilized by a given controller C. The perturbed plant is given by

$$G_o = N_{G_o} D_{G_o}^{-1} \tag{1}$$
$$= \tilde{D}_{G_o}^{-1}\tilde{N}_{G_o} \tag{2}$$
$$\tag{3}$$

where N_{G_o} and D_{G_o} a right coprime factors (rcfs) of the uncertain plant and $\tilde{N}_{G_o}$ and $\tilde{D}_{G_o}$ are the left coprime factors.

The set of all plants stabilized by a given controller is given below. From [12] and others

Theorem 1 (Dual-Youla Parameterization). *Let G_x with rcf(N_x, D_x) be an auxiliary model that is stabilized by the controller C with rcf(N_c, D_c). Then a plant G_Δ is stabilized by C if and only if there exists an $R \in \mathcal{RH}_\infty$ such that*

$$G_\Delta = (N_x + D_c R)(D_x - N_c R)^{-1}.$$

For a specific plant G_o the dual-Youla parameter is given by $R_o = D_c^{-1}(I + G_o C)^{-1}(G_o - G_x)D_x$.

In this paper, we are dealing with a SISo open loop stable system and therefore we may choose $D_x = 1$ and $N_x = G_x$. This implies that

$$G_o = \frac{G_x + D_c R_o}{1 - N_c R_o} \tag{4}$$

This result will be combined with the parameterization of all stabilizing controllers, called the youla parameterization, to create a framework in which we may adjust the controller in the presence of uncertainty.

YOULA PARAMETERIZATION

Next, consider reversing the roles of plant and controller. This means perturbing a controller about the nominal factors of the plant. The perturbed or enhanced controller is given by

$$C_\Delta = N_{C_\Delta} D_{C_\Delta}^{-1} \tag{5}$$
$$= \tilde{D}_{C_\Delta}^{-1}\tilde{N}_{C_\Delta} \tag{6}$$
$$\tag{7}$$

where N_{C_Δ} and D_{C_Δ} a right coprime factors of the enhanced controller and $\tilde{N}_{C_\Delta}$ and $\tilde{D}_{C_\Delta}$ are the left coprime factors.

For a given plant, the set of all stabilizing controllers is below.

Theorem 2 (Youla Parameterization). *Let $C = N_c D_c^{-1}$ be an internally stabilizing controller for the plant $G_x = N_x D_x^{-1}$, where (N_c, D_c) and (N_x, D_x) are both rcfs. Then $C_\Delta = N_{C_\Delta} D_{C_\Delta}^{-1}$*

internally stabilizes the plant iff $\exists\, Q \in \mathcal{RH}_\infty$ *s.t.*

$$N_{C_\Delta} = N_c + D_x Q$$
$$D_{C_\Delta} = D_c - N_x Q.$$

For a stable SISO plant this parameterization can be specialized by choosing the nominal controller $N_c = 0$, $D_c = 1$ (which is stabilizing) and choosing the coprime factors of the plant as $N_x = G_x$ and $D_x = 1$. This gives the following parameterization of all stabilizing controllers

$$C_\Delta = \frac{Q}{1 - G_x Q} \tag{8}$$

In the next section the youla and dual-youla parameterizations will be used simultaneously. This is called the double-youla parmaeterization [13, 14].

DOUBLE-YOULA PARAMETERIZATION

Putting both the youla and dual-youla parameterizations together gives the double youla paramterization. Suppose that there exists a nominal, internally stable, pair $G_x = N_x D_x^{-1}$ and $C = N_c D_c^{-1}$ then the parameterization of perturbed plants and controllers is given by

$$C_\Delta = N_{C_\Delta} D_{C_\Delta}^{-1} = (N_c + D_x Q)(D_c - N_x Q)^{-1}$$
$$G_\Delta = N_{G_\Delta} D_{G_\Delta}^{-1} = (N_x + D_c R)(D_x - N_c R)^{-1}$$

This parameterization is shown in Fig. 4. Here the controller is perturbed by the rcfs of the nominal plant and the plant is perturbed by the rcfs of the nominal controller.

From [13, 14]

Theorem 3. *Let G_x with rcf(N_x, D_x) be an auxiliary model that is stabilized by the controller C with rcf(N_c, D_c). Then a plant $G_o = (N_x + D_c R_o)(D_x - N_c R_o)^{-1}$ with $R_o \in RH_\infty$ is stabilized by $C_\Delta = (N_c + D_x Q)(D_c - N_x Q)^{-1}$ with $Q \in RH_\infty$ if*

$$\|QR\|_\infty < 1.$$

For stable SISO systems, we can choose $N_c = 0$, $D_c = 1$, $N_x = G_x$, and $D_x = 1$. From Eq. (4) this implies that

$$R_o = G_o - G_x \tag{9}$$

which is an additive uncertainty. This can be a restrictive representation and by choosing a different nominal controller one will arrive at a different expression for the uncertainty.

From Eq. (8) the controller parameterization is given by

$$C_\Delta = \frac{Q}{1 - G_x Q}.$$

Applying this controller to the true system gives the following sensitivity function

$$S_o(q) = \frac{1}{1 + G_o(q) C_\Delta(q)} \tag{10}$$
$$= \frac{1}{1 + G_o(q)\frac{Q(q)}{1 - G_x(q)Q(q)}} \tag{11}$$
$$= \frac{1 - G_x(q)Q(q)}{1 - G_x(q)Q(q) + G_o(q)Q(q)} \tag{12}$$
$$= \frac{1 - G_x(q)Q(q)}{1 + R_o(q)Q(q)} \tag{13}$$
$$= \frac{\frac{1}{1 + G_x(q)C_\Delta(q)}}{1 + R_o(q)Q(q)} \tag{14}$$
$$= \frac{S_x(q)}{1 + R_o(q)Q(q)} \tag{15}$$

where $S_x = \frac{1}{1 + G_x(q)C_\Delta(q)}$ is the sensitivity function for the nominal plant.

From Eq. (15), it can be seen that nominal and robust performance are coupled. For periodic disturbance, this is actually an instance of the internal mode principle [2] since $|S_x(e^{j\omega_o})| = 0$ implies $S_o(e^{j\omega_o}) = 0$ if $\frac{1}{1 + R_o(q)Q(q)}$ is stable. This says that the internal model principle is robust against any non-destabilizing perturbations from the nominal system. This parameterization of $S_o(q)$ and $R_o(q)$ will be exploited to reduce disturbances while maintaining stability.

CONTROLLER DESIGN

Controller Enhancement

The goal of the control system is to reduce the effect that the disturbance has upon the output of the plant. From Eq. (15), the output is given by

$$y(t) = \frac{1 - G_x(q)Q(q)}{1 + R_o(q)Q(q)} d(t) \tag{16}$$

where $d(t)$ is the acoustic noise from the cooling fan. For our design, we will consider $d(t)$ to be periodic. Even though the true system is not known, we can estimate $d(t)$.

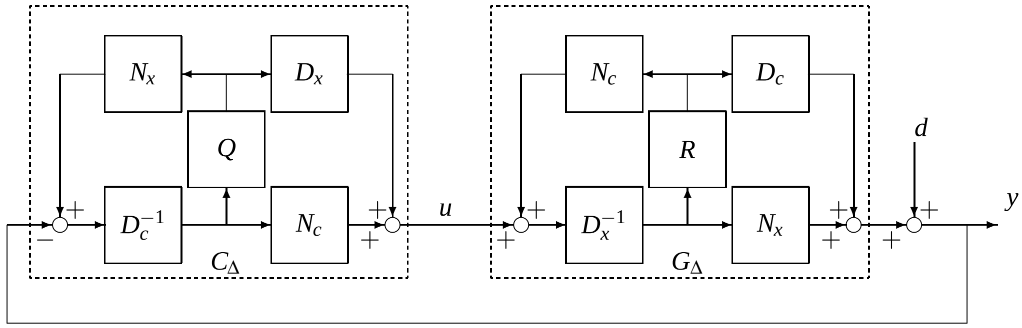

Figure 4. Double-youla parameterization of C_Δ and G_Δ.

$$d(t) = y(t) - G_o u(t) \tag{17}$$
$$= y(t) - (G_x + R_o)u(t) \tag{18}$$
$$= y(t) - G_x u(t) - R_o u(t) \tag{19}$$

and since the nominal controller $C = N_c D_c^{-1} = 0$ we get that $d(t)$ is trivially measurable with the current controller. Thus the current controller, the nominal model, and the closed loop signals can be used to measure $d(t)$. After the update has occurred $d(t)$ will be biased. This bias term δd is given below

$$\delta d(t) = -\frac{R_o Q}{1 - QG_x} y(t)$$

which gives

$$d(t) = y + \frac{GxQ}{1 - QG_x} y - \delta d$$

We define an estimate of $d(t)$ by $\hat{d}(t)$ given by

$$\hat{d}(t) = y(t) - G_x u(t) \tag{20}$$
$$= R_o u(t) + d(t) \tag{21}$$

Let $C_{\Delta_k} := \frac{Q_k}{1 - G_x Q_k}$ be the controller that is implemented, then using the sensitivity function, $\hat{d}$ can be written as

$$\hat{d}(t) = y(t) - G_x u(t) \tag{22}$$
$$= (1 + G_x C_{\Delta_k}) y(t) \tag{23}$$
$$= (1 + G_x C_{\Delta_k}) \frac{1 - G_x Q_k}{1 + R_o Q_k} d(t) \tag{24}$$
$$= \frac{1}{1 + R_o Q_k} d(t) \tag{25}$$

From theorem 3, Q should be constraint such that $|Q_k(e^{j\omega})| < 1/|R_o(e^{j\omega})|$ for all ω. Since $R_o, Q_k \in RH_\infty$ and if we impose that $\|R_o Q_k\|_\infty < 1$ then $\hat{d}(t)$ will be nonzero and asymptotically the same frequency as $d(t)$. Hence $\hat{d}(t)$ is a good estimator of $d(t)$, in that $\hat{d}(t)$ contains the same frequencies.

We want to minimize the variance of $y(t)$ that is given by

$$y(t) = \frac{1 - G_x(q)Q(q)}{1 + R_o(q)Q(q)} d(t)$$

However, $Q(q)$ appears nonlinearly in the output equation and $d(t)$ is not known. This means that we cannot pose an optimization in this form. To deal with this let us denote $Q_k(q)$ by the current filter that is implemented, then

$$y(t) = \frac{1 - G_x(q)Q_k(q)}{1 + R_o(q)Q_k(q)} d(t) \tag{26}$$
$$= (1 - G_x(q)Q_k(q))\hat{d}(t) \tag{27}$$

Now we can parameterize Q such that $Q_k(q) = Q(\theta_k, q)$. In this form we can search for a ϕ such that the variance of

$$\varepsilon(t, \phi) = (1 - G_x(q)Q(\phi, q))\hat{d}(t) \tag{28}$$
$$\hat{d}(t) = \frac{1}{1 + R_o(q)Q(\theta_k, q)} d(t) \tag{29}$$
$$= y(t) - G_x u(t) \tag{30}$$

is minimized. Note that θ_k is fixed and we are searching over ϕ. This is not equivalent to minimizing $y(t)$, but is strikingly close to a bootstrapping approach. In fact, $\varepsilon(t, \phi)$ can be rewritten as

$$\varepsilon(t, \phi) = \frac{1 - G_x(q)Q(\phi, q)}{1 - G_x(q)Q(\theta_k, q)} y(t) \tag{31}$$

where it can easily be seen that if $\phi = \theta_k$ then $\varepsilon(t,\phi) = y(t)$.

We now have two options:

1) Gather a batch of data and update the controller once each batch.
2) Update the controller every time step.

For the first option, define the following cost function

$$V(N,\theta) := \frac{1}{2N}\sum_{t=1}^{N}[(1-G_x(q)Q(\theta,q))\hat{d}(t)]^2$$

θ_{k+1} is found by the following minimization

$$\theta_{k+1} = \underset{|Q(\theta,e^{j\omega})|<1/|R_o(e^{j\omega})|}{\operatorname{argmin}} V(N,\theta) \tag{32}$$

This θ_{k+1} minimizes the output variance of the nominal system when disturbed by $\hat{d}(t)$. Before we proceed, we need to pick a suitable parameterization for Q.

Proposition 1. *Suppose that $Q(\theta,q)$ is a such that $Q(\theta,q) = \theta_0 + \theta_1 z^{-1} + \ldots + \theta_{2N_d} z^{-2N_d+1}$ then given a set of distinct frequencies $0 < \omega_i < \pi$, $i = 1,2,\ldots,N_d$ and a set of complex numbers $Q_i \in \mathbb{C}$ there exists θ_i, $i = 1,2,\ldots,2N_d$ such that*

$$Q(\theta,e^{j\omega_i}) = Q_i, \quad i = 1,2,\ldots,N_d$$

Proof. Define

$$X := \begin{bmatrix} 1 & e^{-j\omega_1} & \ldots & e^{-(2N_d-1)j\omega_1} \\ 1 & e^{-j\omega_2} & \ldots & e^{-(2N_d-1)j\omega_2} \\ \vdots & & & \\ 1 & e^{-j\omega_{N_d}} & \ldots & e^{-(2N_d-1)j\omega_{N_d}} \end{bmatrix}$$

$$Y := \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_{N_d} \end{bmatrix} \quad \theta := \begin{bmatrix} \theta_0 \\ \vdots \\ \theta_{2N_d-1} \end{bmatrix}$$

then form

$$\begin{bmatrix} \mathrm{Re}(X) \\ \mathrm{Im}(X) \end{bmatrix} \theta = \begin{bmatrix} \mathrm{Re}(Y) \\ \mathrm{Im}(Y) \end{bmatrix}$$

X is full rank and invertible and theta is given by $\theta = X^{-1}Y$.

The fact that X is full rank comes from

$$\begin{bmatrix} \mathrm{Re}(X) \\ \mathrm{Im}(X) \end{bmatrix} = \begin{bmatrix} \frac{1}{2}I & \frac{1}{2}I \\ \frac{1}{2j}I & -\frac{1}{2j}I \end{bmatrix} \begin{bmatrix} X \\ X^* \end{bmatrix}$$

$\begin{bmatrix} X \\ X^* \end{bmatrix}$ is a vandermonde matrix and since $0 < \omega_i < \pi$ are distinct it is full rank. $\begin{bmatrix} \frac{1}{2}I & \frac{1}{2}I \\ \frac{1}{2j}I & -\frac{1}{2j}I \end{bmatrix}$ is full rank, therefore $\begin{bmatrix} \mathrm{Re}(X) \\ \mathrm{Im}(X) \end{bmatrix}$ is full rank by sylvesters inequality.

□

This proposition implies that an fir parameterization for Q with $N_\theta \geq 2$ is sufficient for regulation of a sinusoid.

Using the batch method of updating the controller, we arrive at the following theorem.

Theorem 4 (Batch Update). *Suppose that $\bar{\theta}$ is fixed with*

$$C(q) = \frac{Q(\bar{\theta},q)}{1-G_x(q)Q(\theta,q)}$$

$Q(\theta,q) = \sum_{i=1}^{N_\theta}\theta_i q^{-i}$ with $N_\theta \geq 2$, $d(t) = \sin(\omega_o t)$, $y(t) = G_o(q)u(t) + d(t)$, $u(t) = -C(q)y(t)$, and the feedback system is stable.

Choose θ^* by

$$\theta^* = \underset{|Q(\theta,e^{j\omega})|<1/|R_o(e^{j\omega})|}{\operatorname{argmin}} \lim_{N\to\infty}\frac{1}{2N}\sum_{t=1}^{N}\varepsilon(\theta,t)^2 \tag{33}$$

with $\varepsilon(\theta,t) = (1-G_x(q)Q(\theta,q))\hat{d}(t)$ and $\hat{d}(t) = y(t) - G_x u(t)$.

Then $C_\Delta(q) = \frac{Q(\theta^*,q)}{1-G_x(q)Q(\theta^*,q)}$ stabilizes $G_o(q)$ and if

$$\theta^* = \operatorname{argmin} \lim_{N\to\infty}\frac{1}{2N}\sum_{t=1}^{N}\varepsilon(\theta,t)^2$$

then

$$\lim_{t\to\infty}\left|\frac{1-G_x(q)Q(\theta^*,q)}{1+R_o(q)Q(\theta^*,q)}d(t)\right| = 0$$

Proof. Suppose that $d(t) = \sin(\omega_o t)$ and that

$$\theta^* = \operatorname{argmin} \lim_{N\to\infty}\frac{1}{2N}\sum_{t=1}^{N}\varepsilon(\theta,t)^2$$

happens to stabilize the system. Then since $N_\theta \geq 2$ and $\hat{d}(t)$ is a sine wave, $(1 - G_x(q)Q(\theta,q))\hat{d}(t) \to 0$ as $N \to \infty$. This implies that $1 - G(e^{j\omega_o})Q(e^{j\omega_o}) = 0$ which means that $C_\Delta = Q(\theta^*,q)(1 - G_xQ(\theta^*,q))^{-1}$ satisfies the internal model principle. This implies that C_Δ has poles on the unit circle located at ω_o or equivalently $1 - GxQ(\theta^*,q)$ has zeros on the unit circle at the same frequency.

When we apply this controller to the true system $y(t) \to 0$ since the internal model principle is robust against all non-destabilizing uncertainty. To see this fact recall from Eq. (15) that

$$y(t) = \frac{1 - GxQ(\theta^*,q)}{1 + R_oQ(\theta^*,q)} d(t)$$

which implies that the sensitivity function of the true system has zeros at the frequency of the disturbance and hence blocks the disturbance. This proves the second assertion.

The first assertion is proved by applying the small gain principle to the output equation. The sufficient condition for stability is $\|R_oQ(\theta^*,q)\|_\infty < 1$, which is the condition used in the theorem. Therefore regulation of the nominal system is the same as regulation for the true system when robustness is satisfied.

□

The batch update theorem says that regulation is guaranteed if the batch size is large enough and the uncertainty is small enough. When there is no uncertainty, a large batch size is still needed. Therefore, batch updates are similar with or without uncertainty. The case where the controller is updated every time step is strikingly different and will be considered next.

Consider the situation where a new controller is updated every time step. In this case, $C_{\Delta_t} = \frac{Q(\theta_t,q)}{1-G_x(q)Q(\theta_t,q)}$ and $Q(\theta_{t+1},q)$ is found by minimizing the variance of

$$\varepsilon(t,\phi) = (1 - G_x(q)Q(\phi,q))\hat{d}(t) \tag{34}$$

$$\hat{d}(t) = \frac{1}{1 + R_o(q)Q(\theta_t,q)} d(t) \tag{35}$$

$$= y(t) - G_x u(t) \tag{36}$$

In this case, the calculation of Q and implementation of the new controller every time step results in a bootstrap algorithm. Suppose, for a moment, that $R_o(q) = 0$ then the resulting algorithm is a RLS algorithm and convergence is guaranteed [10]. The case that we are considering $R_o(q) \neq 0$ results in a algorithm that is very similar to a recursive pseudo linear regression (RPLR) [15].

First, rewrite ε in a familiar prediction-error form as

$$\varepsilon(t,\phi) = \tilde{y}(t) - Q(\phi,q)\tilde{u}(t) \tag{37}$$

with $\tilde{y} = \hat{d}$ and $\tilde{u} = G_x\hat{d}$. Since $\tilde{y}$ and $\tilde{u}$ are measurable signals (after filtering with known filters) we can pose a standard RLS algorithm on $\varepsilon(t,\phi)$. However, this algorithm does not take into account the robustness requirement $\|QR\|_\infty < 1$ and therefore a modification is needed. The modification that will be used is presented in the **Online Enhancement Algorithm** section. Before we proceed with the algorithm a nominal model is needed.

System Identification

Let $G_o(q)$, a linear discrete-time transfer function, denote the true model for regulation. The model of the system is found wia standard system identification techniques [15]. To generate data that can be used for the identification, a white noise signal was sent into the speakers and the resulting signal was recorded with the feedback microphones. The input and output data is shown in Fig. 5 (recall that the microphone signals are averaged and the speakers are sent the same signal so that a SISO system results). The microphone signal has some periodic components indicating that the acoustic system has some resonance modes. Note that, the acoustic system is composed of the speaker amplifier, the speakers, that acoustic between the speaker and microphone, the microphone, and the microphone filter.

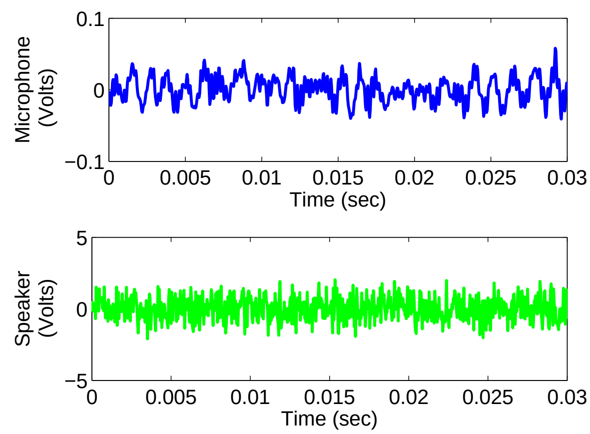

Figure 5. Input and output data from the acoustic system.

The frequency response $G_o(e^{j\omega_k})$, $k = 1,2,...,N$, between the feedback microphones and the speakers is shown in Fig. 6. In this figure it can be seen that the acoustic system has many resonance modes and even some non-minimum phase zeros that will limit performance. This frequency response function, obtained with standard non-parametric methods, will be considered to be the frequency response of the true system.

From the analysis of the Q filter given in the previous sections, it can be seen that we desire Q to satisfy two conditions

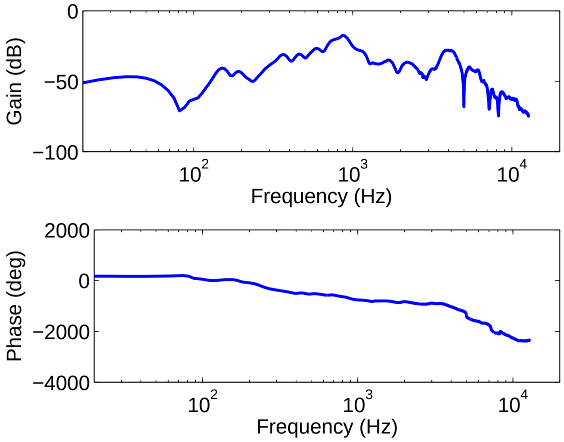

Figure 6. Frequency response between the feedback microphone and speaker.

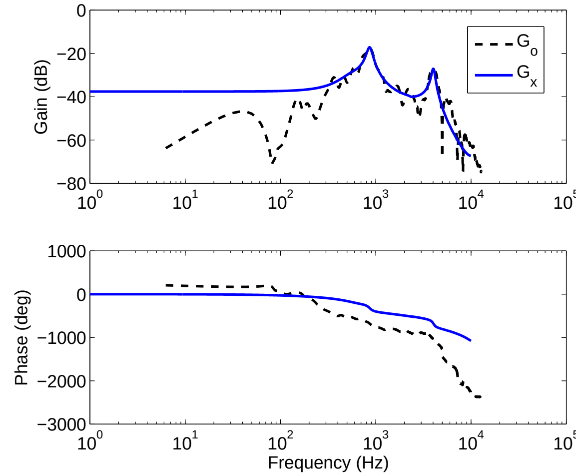

Figure 7. Frequency response of model G_x and the true system G_o.

1. $|Q(e^{j\omega})| < 1/|R_o(e^{j\omega})|$ for all ω.
2. $Q(e^{j\omega_o}) = 1/G_x(e^{j\omega_o})$ where ω_o is the frequency of the disturbance.

If these two conditions are satisfied then the feedback system is stable in the presence of the modeling error and regulation is achieved.

This criteria also provides a method for determining which nominal controller and system should be used if regulation is wanted. The design methodology is outlined below:

1. Fit a model upon the input/output data.
2. Design a nominal controller.
3. If $1/G_x(e^{j\omega}) < 1/R_o(e^{j\omega})$ for the frequency range of interest then quit. If not then goto 1.

After several iterations a suitable model was chosen by the above design method. Using an ARX model structure [15] and a Steiglitz-Mcbride iteration a 10^{th} order model of the acoustic system was found. The frequency response of the model G_x and the true system G_o is shown in Fig. 7. It can be seen that the model captures the general trend but does not capture every single feature of the true system, this would take a very high order system.

Step 3) of this design method is shown in Fig. 8. In this figure, it can be seen that regulation is possible from 400 to 4000 Hz. Notice that with this model regulation at high frequencies is not possible.

To validate the model a chirp signal was sent into the speaker. The simulation error is shown in Fig. 9. Again, here it can be seen that the model captures the general trend of the data but does not capture every single detail.

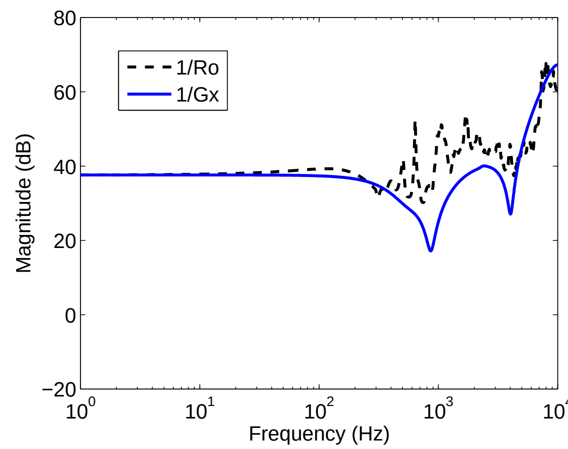

Figure 8. Criteria used to choose nominal model and controller.

Online Enhancement Algorithm

In this section, a simplified algorithm is developed to reduce the online computational burden. The optimization that we desire to solve is

$$\theta^* = \underset{|Q(\theta,e^{j\omega})|<1/|R_o(e^{j\omega})|}{\text{argmin}} \frac{1}{2N}\sum_{k=1}^{N}[(1-G_x(q)Q(\theta,q))\hat{d}(t)]^2$$

however the constraint $|Q(\theta,e^{j\omega})| < 1/|R_o(e^{j\omega})|$ increases the computational complexity greatly. Instead of solving this problem, we will approximate this optimization by applying the constraint on N_f different grid points in the frequency domain and by augmenting the cost function. Finally, the gridded constraint will be checked before updating the filter.

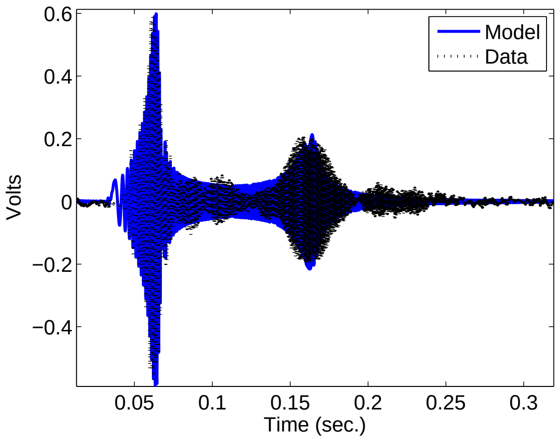

Figure 9. Simulation of model and ouput of true system.

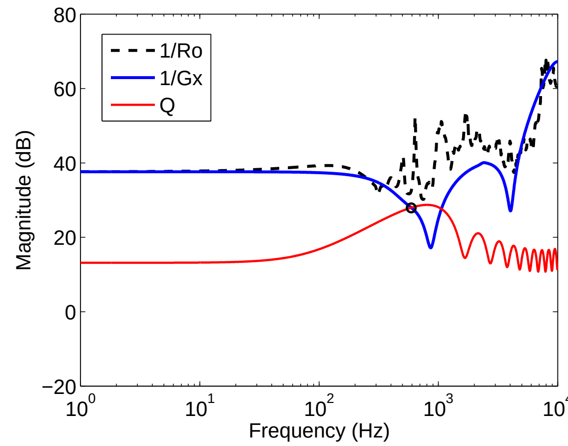

Figure 10. Controller enhancement filter Q for a frequency of 592 Hz.

The modified optimization problem is given by

$$\theta^* = \underset{Q(\theta,q)\in RH\infty}{\text{argmin}} \frac{1}{2N}\sum_{k=1}^{N}[(1-G_x(q)Q(q))\hat{d}(t)]^2 + \frac{\lambda}{2N_f}\sum_{i=1}^{N_f}|R_o(e^{j\omega_i})Q(e^{j\omega_i})|^2$$

and if we choose Q to be an FIR filter as given below

$$Q = \sum_{i=1}^{N_\theta}\theta_i q^{-i}$$

then the optimization becomes

$$\theta^* = \text{argmin}\frac{1}{2N}\sum_{k=1}^{N}[(1-G_x(q)Q(q))\hat{d}(t)]^2 + \frac{\lambda}{2N_f}\sum_{i=1}^{N_f}|R_o(e^{j\omega_i})|^2\theta^T\left[\text{real}(f_i)\ \text{imag}(f_i)\right]\begin{bmatrix}\text{real}(f_i)\\ \text{imag}(f_i)\end{bmatrix}\theta$$

where $\theta = [\theta_1\ \ \theta_2\ ,...,\ \theta_{N_\theta}]^T$ and $f_i = \left[e^{j\omega_i}\ e^{2j\omega_i}\ \ldots\ e^{N_\theta j\omega_i}\right]^T$.

If $d(t) = \sin(\omega_o t)$, where $\omega = 592$ Hz then the resulting $Q(\theta^*,q)$ for $N_\theta = 20$ is shown in Fig. 10. It is easily verified that $Q(\theta^*,q)$ satisfies all of the conditions for stability and regulation. The fact that $|Q(\theta^*,q)(e^{j\omega})| < 1/|R_o(e^{j\omega})|$ implies stability and the fact that $Q(\theta^*,q)(e^{j\omega_0}) = 1/G_x(e^{j\omega_o})$ implies regulation. It is also important to point out that $N_\theta = 2$ will achieve regulation will not achieve stability. Therefore, for robustness, an increase order for Q is needed.

If we further approximate the solution by using the instantaneous gradient, like the normalized LMS filter [16], then we get the following update algorithm.

$$\theta_{k+1} = G\theta_k - \mu e(t)\frac{X(t)}{1+X(t)^T X(t)} \tag{38}$$

where $e(t) = \hat{d}(t) + \theta_k^T X(t)$, $X(t) = G_x(q)[d(t-1)\ \ d(t-2)\ ,...,\ d(t-N_\theta)]^T$,

$$G = I - \frac{\mu\lambda}{N_f}\sum_{i=1}^{N_f}|R_o(e^{j\omega_i})|^2\left[\text{real}(f_i)\ \text{imag}(f_i)\right]\begin{bmatrix}\text{real}(f_i)\\ \text{imag}(f_i)\end{bmatrix}$$

and μ is the step size. This creates an algorithm with low computational complexity that aims to remove narrowband disturbances in the presence of uncertainty. To gaurentee stability a check can be added. If $|Q(\theta_{k+1},e^{j\omega_k})| < 1/|R_o(e^{j\omega_k})|$, $k = 1,2,...,N_f$, then implement the new Q_{k+1} otherwise not. This gives the following algorithm for periodic noise reduction.

Algorithm 1 (Robust Periodic Noise Cancellation).
Define

$$f_i = \left[e^{j\omega_i}\ e^{2j\omega_i}\ \ldots\ e^{N_\theta j\omega_i}\right]^T$$
$$G = I - \frac{\mu\lambda}{N_f}\sum_{i=1}^{N_f}|R_o(e^{j\omega_i})|^2\left[real(f_i)\ imag(f_i)\right]\begin{bmatrix}real(f_i)\\ imag(f_i)\end{bmatrix}$$

then for t=0,1,2,...

$$\hat{d}(t) = y(t) - G_x(q)u(t)$$
$$X(t)^T = G_x(q)[\hat{d}(t-1),\ \hat{d}(t-2), \ldots, \hat{d}(t-N_\theta)]$$
$$\theta'_{t+1} = G\theta_k - \mu e(t)\frac{X(t)}{1+X(t)^T X(t)}$$
$$\theta_{t+1} = \begin{cases} \theta'_{t+1} & \text{if } |Q(\theta'_{k+1}, e^{j\omega_i})R(e^{j\omega_i})| < 1\ \forall i \\ \theta_t & \text{if } |Q(\theta'_{k+1}, e^{j\omega_i})R(e^{j\omega_i})| \not< 1\ \forall i \end{cases}$$
$$C_{\Delta_{t+1}} = \frac{Q(\theta_{t+1}, q)}{1 - G_x(q)Q(\theta_{t+1}, q)}$$

where μ is the step size.

EXPERIMENTAL RESULTS

To analyze the control algorithm a separate, external microphone was used to measure the performance. The microphone was located approximatly 6” from the outlet and slightly to the side to reduce windage.

The algorithm described in the previous section was applied to the system acoustic system described in the beginning of the paper. The acoustic system is shown in Fig. 3. The fan produces broadband and narrowband disturbances. The goal of the feedback system is to maintain stability and reduce the narrowband disturbances in the presence of modeling error and without knowledge of the disturbance frequency.

The microphone signals are summed together and sampled with a MultiQ3 12 bit signed AD/DA card, a Pentium based P.C. is used to calculate the control signal, and the control signal is sent out of the AD/DA card to each of the speakers.

Since the speed of the fan in not known, the closed loop signals are used to update the nominal controller. Even in the presence of uncertainty, shown in Fig. 8, the controller is able to maintain stability and reduce the narrowband disturbances. The power spectral density (PSD) before and after control is applied is shown in Fig. 11. It can be seen that the controller reduces the fundamental frequency and third harmonic without excessive amplification of the broadband noise. Note that this result is obtained from an external microphone and therefore this performance is a good indicator of the audible difference that a listener would experience.

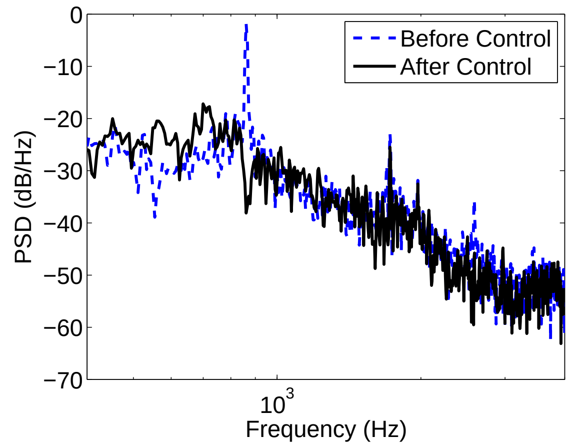

Figure 11. Power spectral density of the acoustic noise generated by the cooling fan before and after feedback control.

CONCLUSIONS

In this paper, we presented a method of reducing narrowband disturbance in the presence of plant and disturbance uncertainty. The plant uncertainty was in the form of modeling error and the disturbance uncertainty was the unknown frequency of the disturbance. A constrained optimization resulted to update a nominal controller based upon closed loop signals.

It was shown that the update (or enhancement) filter should be of high order to satisfy the robustness and performance constraints simultaneously. It was also shown how robustness and performance can be conflicting, and a design method for picking a nominal system and controller was given so that robustness and performance is possible over a frequency range.

The design was applied to cancel narrowband disturbances emitted from a cooling fan. Small microphones were embedded into acoustical foam to measure the error signal and miniature speakers were mounted near the cooling fan to produce anitnoise for cancellation. Experimental results showed the ability of the control algorithm to cancel noise in the presence of modeling errors and without knowledge of the disturbance frequency.

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