

# Multichannel Adaptive Stochastic Filtering for Active Noise Control in Personal Computers

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**Abstract:** In this paper, the feasibility of using active noise control inside personal computers is demonstrated by applying an adaptive filter to Hewlett-Packard's Blackbird 002 Gaming PC. Conditions relating the control signals and the sound heard by an external listener are presented as a means of evaluating the hardware design before applying the adaptive filter. A multichannel stochastic adaptive filter is derived by vectorizing the equations and applying gradient descent and Newton's method. For implementation purposes, data-based approximations to the gradient and hessian are provided. It is shown that a 6dB reduction in sound pressure level is obtainable by adding reference microphones, error microphones, a speaker, and some acoustical foam into the system even though the length of the noise cancellation system is small, approx. 6", compared to the fundamental wavelength of the noise, approx 23".

Keywords: Noise Control; Feedforward Control; Multichannel; Adaptive Filters; Stochastic Approximation.

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## 1. INTRODUCTION

Active noise control is concerned with reducing annoying or harmful noise to a safe and comfortable level. In many of these applications, adaptive filtering (Haykin [2002], Kay [1993]) is used to update filter coefficients that use a measurement of the noise to produce anti-noise. Most of these applications make use of first derivative information and possibly second derivative information about the cost function. For example, the Fx-LMS filter uses the gradient and the RLS algorithm uses the hessian. In multichannel applications (Chen and Gibson [2001], Elliott et al. [1987], Esmailzadeh et al. [2004], Coradine et al. [1997], Bouchard and Quednau [2000], Djigan [2006]) it is difficult to derive the hessian because of notational reasons. Additionally, many of these applications consider long ducts (Esmailzadeh et al. [2004]) that are smooth and have relatively simple dynamics.

In addition to the lack of higher order information, most application assume that the error signal, the signal that measures the performance of the system, is located at the listener and that the anti-noise speaker is near the noise source. Locating the sensors and actuators in this manner imply that the performance of the system can be evaluated by monitoring the error signal. In some applications, such as personal computers, the error signal cannot be located in the same position as the listener. In this scenario, the

performance of the system is not always directly related to the error signal and the relationship between the reference, error, and listener signals should be studied for proper system design.

In this paper, the conditions for cancelation at a location other than the error location are studied and the results are interpreted as design constraints for the noise control system. After the system is properly designed, a suitable algorithm must be derived and for improved performance it is desirable to use information about the second derivative of the cost function. To accomplish this task, we use a technique called *vectorization* and derive the first and second derivatives of the cost function(s). We also consider a wide class of cost functions so that the algorithm is as general as possible. Finally, the algorithm is applied to HP's Blackbird 002 Gaming System to demonstrate the feasibility of active noise control in a personal computer; a system with small size, complicated geometry, and high airflow.

## 2. PRELIMINARIES

In this paper we are considering a multichannel adaptive algorithm and as a result it will be beneficial to use the Kronecker product for notational purposes. The Kronecker product is defined below.

**Definition** The Kronecker product (a special case of a tensor product) of two matrices  $A \in \mathbb{R}^{m \times n}$  and  $B \in \mathbb{R}^{p \times q}$  is given by

$$B^T \otimes A = \begin{bmatrix} b_{11}A & b_{21}A & \dots & \dots & b_{q1}A \\ b_{12}A & \ddots & \dots & & \vdots \\ \vdots & \ddots & \ddots & & \vdots \\ b_{1p}A & b_{2p}A & \dots & & b_{qp}A \end{bmatrix}$$

where  $b_{ij}$  is the  $(i^{th}, j^{th})$  element of  $B$ .

**Proposition 1.** Consider the matrices  $A, B, C$  and  $D$  with the appropriate dimensions and the scalar  $c$  then the following hold

- (1)  $(A \otimes B)^T = (A^T \otimes B^T)$
- (2)  $(A + B) \otimes C = (A \otimes C) + (B \otimes C)$
- (3)  $A \otimes (B + C) = A \otimes B + A \otimes C$
- (4)  $(cA) \otimes B = c(A \otimes B) = A \otimes (cB)$
- (5)  $(A \otimes B) \otimes C = A \otimes (B \otimes C)$
- (6)  $(A \otimes B)(C \otimes D) = AC \otimes BD$  and if in addition  $A$  and  $B$  are invertible then
- (7)  $(A \otimes B)^{-1} = A^{-1} \otimes B^{-1}$

Matrices can be vectorized, stacked column-wise into a large vector, by using the  $\text{vec}(\cdot)$  operator. This process is very useful since the multichannel problem requires optimization over a set of matrices. If one desires to calculate the hessian to improve performance over standard gradient methods then this operator is necessary.

**Definition** For a matrix  $A \in \mathbb{R}^{m \times n}$ , given by

$$A = [a_1 \ a_2 \ \dots \ a_n],$$

where  $a_i \in \mathbb{R}^m$  is a column vector, the  $\text{vec}(\cdot)$  is given by

$$\text{vec}(A) = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}.$$

**Proposition 2.** Consider the matrices  $A, B, C$  with appropriate sizes then the following identities hold

- (1)  $\text{vec}(ABC) = (C^T \otimes A)\text{vec}(B)$
- (2)  $\text{trace}(A^T A) = \text{vec}(A)^T \text{vec}(A)$

In this paper we will consider the optimization of a cost function that consists of a mixture of p-norms. The p-norm of a vector is defined below.

**Definition** Consider the vector  $x \in \mathbb{R}^n$ , the p-norm is given by

$$\|x\|_p^p = \sum_{i=1}^n |x_i|^p,$$

for all  $p \in [1, \infty)$ .

**Lemma 3.** For any p-norm, with  $p < \infty$ , the following properties holds

$$\frac{d\|x\|_p^p}{dx} = p\tilde{x}$$

$$\frac{d\|x\|_p}{dx} = \|x\|_p^{1-p}\tilde{x}$$

where  $\tilde{x}$  is defined element wise by  $\tilde{x}_i = |x_i|^{p-2}x_i$ .

It is also beneficial to introduce derivative operators for notational reason. The following operators will be used throughout the paper.

**Definition** The differential operator  $\frac{\partial}{\partial x}$ , where  $x$  is a column vector of size  $n$ , is a column operator. That is

$$\frac{\partial}{\partial x} = \begin{bmatrix} \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_1} \\ \vdots \\ \frac{\partial}{\partial x_n} \end{bmatrix}$$

**Definition** The differential operator  $\frac{\partial}{\partial x^T}$ , where  $x$  is a column vector of size  $n$ , is a row operator. That is

$$\frac{\partial}{\partial x^T} = \left[ \frac{\partial}{\partial x_1} \ \frac{\partial}{\partial x_1} \ \dots \ \frac{\partial}{\partial x_n} \right]$$

### 3. ACTIVE NOISE CONTROL SYSTEM

#### 3.1 Basic System Requirements

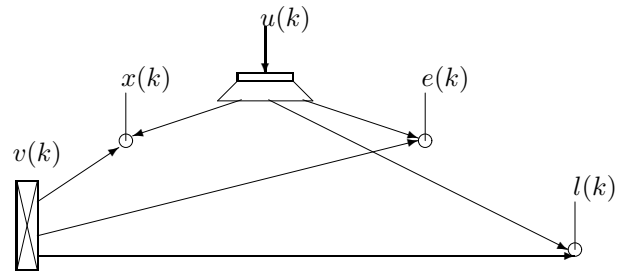


Fig. 1. Depiction of the relationship between signals in the noise cancellation system.

The problem that is presented in this paper is summarized in Fig. 1. It is desired that by using the reference signal  $x(t)$  and an error signal  $e(t)$ , we can generate anti-noise  $u(t)$  to cancel the noise  $v(k)$  at the listener  $l(k)$ . In order to accomplish this task, the reference signal  $x(k)$  must be correlated with  $v(k)$  and the error signal  $e(k)$  needs to reflect the sound measured by the listener  $l(k)$ . In most active noise cancellation systems the error signal is driven to zero. However, this may not be the best choice if the system is not one-dimensional and the noise  $v(k)$  and control source  $u(k)$  are not point sources. The remainder of this section explores this situation.

We will assume throughout the paper that the system depicted in Fig. 1 has a linear discrete-time finite-dimensional representation. In other words the finite-dimensional model of the system is exact even though the true system is infinite dimensional. In practice this is a good assumption and most of the time the system is approximated even further with a FIR model. With this assumption, we can describe the relationship between the signals of the system.

$$\begin{bmatrix} x(k) \\ e(k) \\ l(k) \end{bmatrix} = \begin{bmatrix} T_{xv}(q) & T_{xu}(q) \\ T_{ev}(q) & T_{eu}(q) \\ T_{lv}(q) & T_{lu}(q) \end{bmatrix} \begin{bmatrix} v(k) \\ u(k) \end{bmatrix} \quad (1)$$

The signal  $x(k)$  can be written as

$$x(k) = T_{xv}(q)v(k) + T_{xu}(q)u(k) \quad (2)$$

and  $u(k)$  is given by

$$u(k) = F(q)x(k) \quad (3)$$

$$= F(q)T_{xv}(q)v(k) + F(q)T_{xu}(q)u(k) \quad (4)$$

The block diagram of the system is shown in Fig. 2. Here, it should be clear that knowledge about  $x(k)$  and  $u(k)$  can replace  $v(k)$ .

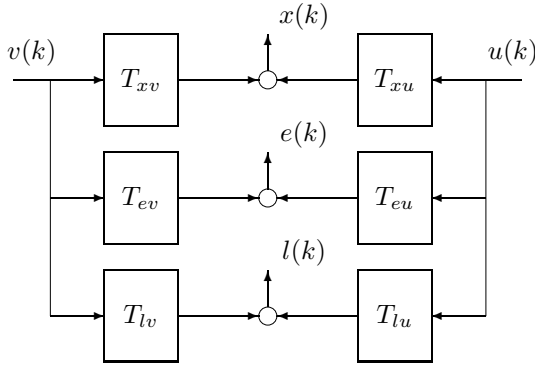


Fig. 2. Block diagram of noise cancellation system.

If the feedback term  $F(q)T_{xu}(q)u(k)$  is small enough then we can neglect this term for the feedforward design. If not, then a good model for  $T_{xu}(q)$  is needed and design techniques, such as (Zeng and de Callafon [2006]), are available to deal with this situation. In either case the resulting design can be accomplished with an adaptive filter and therefore we will not consider this term in the current work. For the remainder of the paper we will assume for simplicity that  $T_{xu}(q) = 0$ .

With this assumption in hand and enforcing a mapping between  $x(k)$  and  $u(k)$  the system that we will consider is given by

$$\begin{bmatrix} e(k) \\ l(k) \end{bmatrix} = \begin{bmatrix} T_{ev}(q) & T_{eu}(q) \\ T_{lv}(q) & T_{lu}(q) \end{bmatrix} \begin{bmatrix} v(k) \\ F(q)x(k) \end{bmatrix} \quad (5)$$

*Theorem 4.* Suppose that  $T_{ev}(q)$  is stably invertible then  $e(k) = 0$  implies  $l(k) = 0$  iff

$$(T_{lu}(q) - T_{lv}(q)T_{ev}(q)^{-1}T_{eu}(q))u(k) = 0 \quad (6)$$

**Proof**

$$\begin{bmatrix} T_{ev}(q)^{-1} & 0 \\ -T_{lv}(q)T_{ev}(q)^{-1} & I \end{bmatrix} \begin{bmatrix} e(k) \\ l(k) \end{bmatrix} \quad (7)$$

$$= \begin{bmatrix} T_{ev}(q)^{-1} & 0 \\ -T_{lv}(q)T_{ev}(q)^{-1} & I \end{bmatrix} \begin{bmatrix} T_{ev}(q) & T_{eu}(q) \\ T_{lv}(q) & T_{lu}(q) \end{bmatrix} \begin{bmatrix} v(k) \\ F(q)x(k) \end{bmatrix} \quad (8)$$

$$= \begin{bmatrix} I & T_{ev}(q)^{-1}T_{eu}(q) \\ 0 & T_{lu}(q) - T_{lv}(q)T_{ev}(q)^{-1}T_{eu}(q) \end{bmatrix} \begin{bmatrix} v(k) \\ u(k) \end{bmatrix} \quad (9)$$

This implies that  $l(k) - T_{lv}(q)T_{ev}(q)^{-1}e(k) = (T_{lu}(q) - T_{lv}(q)T_{ev}(q)^{-1}T_{eu}(q))u(k)$ .

□

*Corollary 5.* Suppose that either  $(T_{lu}(q) = T_{lv}(q)$  and  $T_{eu}(q) = T_{ev}(q))$  or that  $(T_{ev}(q) = T_{lv}(q)$  and  $T_{eu}(q) = T_{lu}(q))$  then

$$(T_{lu}(q) - T_{lv}(q)T_{ev}(q)^{-1}T_{eu}(q)) = 0$$

**Proof** Straightforward.

□

This corollary implies that perfect cancellation will occur if the cancellation speaker is in exactly the same position as the noise source or the error microphone is where the listener is located. In reality, this will not occur. Therefore, if one seeks to optimize the speaker and microphone locations a good performance measure is given by  $J$

$$J = \|T_{lu}(q) - T_{lv}(q)T_{ev}(q)^{-1}T_{eu}(q)\| \quad (10)$$

where  $\|\cdot\|$  is a system norm like the  $H_2$  or  $H_\infty$  norm. If the frequency range of operation is known a priori then this

measure can be used to apply this criteria in the frequency domain.

After selecting the best location for speakers and microphones then the signal that should be minimized is  $l(k)$ , not  $e(k)$ . This gives the following error signal  $\varepsilon(k)$  for control design

$$\varepsilon(k) = l(k) = T_{lv}(q)T_{ev}(q)^{-1}e(k) + (T_{lu}(q) - T_{lv}(q)T_{ev}(q)^{-1}T_{eu}(q))u(k) \quad (11)$$

$$= T_{lv}(q)T_{xv}(q)^{-1}x(k) + T_{lu}(q)u(k) \quad (12)$$

$$= H(q)x(k) + S(q)F(q, \Theta)x(k) \quad (13)$$

*Corollary 6.* Suppose that  $J = 0$ , defined in Eq. 10, then  $\varepsilon(k) = 0$  whenever  $e(k) = 0$ .

**Proof** If  $J = 0$  then

$$\varepsilon(k) = T_{lv}(q)T_{ev}(q)^{-1}e(k)$$

□

This theorem shows that when  $J = 0$ , minimizing  $e(k)$  is the same as minimizing  $\varepsilon(k)$ . This indicates that the standard criteria is valid when  $J$  is small compared to  $\|T_{lv}(q)T_{ev}(q)^{-1}\|$ . For this comparison a better choice for the cost would be given by

$$J^*(\omega) = \frac{\bar{\sigma}(T_{lu}(e^{j\omega}) - T_{lv}(e^{j\omega})T_{ev}(e^{j\omega})^{-1}T_{eu}(e^{j\omega}))}{\underline{\sigma}(T_{lv}(e^{j\omega})T_{ev}(e^{j\omega})^{-1})}$$

where  $\bar{\sigma}(\cdot)$  is the maximal singular value and  $\underline{\sigma}(\cdot)$  is the minimal. In that case it is still important to weight the optimization by  $T_{lv}(q)T_{ev}(q)^{-1}$ .

Also, if  $J$  is large then a model of  $H(q)$  and  $S(q)$  is needed to generate  $\varepsilon(k)$ . In this case, the benefit of adaptation is removed and an off-line control problem can be solved in lieu of adaptation. Here it would be necessary for very accurate modeling for good performance. From this point on we will assume that the  $J$  is small, and without loss of generality, we can focus our attention on deriving algorithms for  $\varepsilon(k)$ . We note that it is very important that  $J$  be small for the adaptation to work correctly and be beneficial to the listener. This point demonstrates the importance of hardware design for control systems and provides a methodology of evaluating hardware before the control design.

### 3.2 Stochastic Optimization Problem

The problem under consideration is depicted in Fig. 3. In

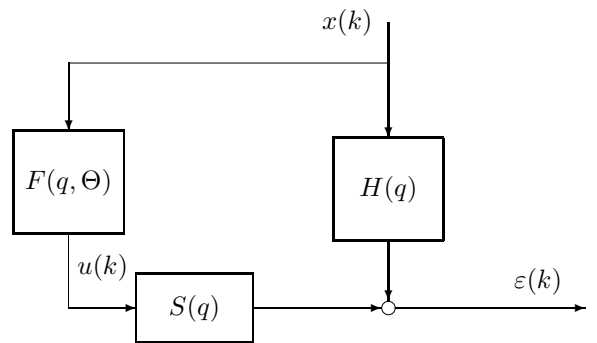


Fig. 3. Block diagram of the optimization problem used to design the feedforward controller for ANC.

this figure, the signal  $\varepsilon(k)$  is defined by  $\varepsilon(k) = H(q)x(k) +$

$S(q)F(q, \Theta)x(t)$ , where  $x(k) \in \mathbb{R}^{n_x}$  for each  $k$ . The multichannel FIR filter of interest  $F(q, \Theta)$  is given by

$$F(q, \Theta) = \theta_0 + q^{-1}\theta_1 + q^{-2}\theta_2 + \dots + q^{-L+1}\theta_{L-1} \quad (14)$$

$$= \Theta^T [I \quad Iq^{-1} \quad \dots \quad Iq^{-L+1}]^T \quad (15)$$

$$= \Theta^T \mathcal{D}, \quad (16)$$

where each  $\theta_i \in \mathbb{R}^{n_u \times n_x}$ . The discrete-time linear dynamic system  $S(q)$  is a  $(n_e \times n_u)$  transfer matrix in the delay operator  $q$ .  $H(q)$  is a  $(n_e \times n_x)$  transfer matrix.

Throughout the paper the following sizes will be used:

- $L$  = number of taps.
- $n_x$  = number of reference signals.
- $n_e$  = number of error sensors.
- $n_u$  = number of actuators.

We are interested in minimizing  $\varepsilon(k)$  and therefore will attempt to minimize  $\|\varepsilon(k)\|$ . For practical purposes and to ensure that the problem will be regular (Haykin [2002]), we will also try to simultaneously minimize  $\|\text{vec}(\Theta)\|$ . This gives the following two cost functions  $V_i(N, \Theta)$ ,  $i=1,2$ .

$$V_1(k, \Theta) = \mathbb{E}\{\|\varepsilon(k)\|_p + \alpha\|\text{vec}(\Theta)\|_q\}, \quad (17)$$

$$V_2(k, \Theta) = \mathbb{E}\{\|\varepsilon(k)\|_p^p + \alpha\|\text{vec}(\Theta)\|_q^q\}, \quad (18)$$

where  $p < \infty$  and  $q < \infty$ .

As we will see in the following section, the two cost functions  $V_1(k, \Theta)$  and  $V_2(k, \Theta)$  will result in similar expressions for the gradient and hessian. The difference will be in how the gradient and hessian are scaled. Just like the difference in  $V_1(k, \Theta)$  and  $V_2(k, \Theta)$  is seen for small and large signals. For large signals  $V_2(k, \Theta)$  will be greater than  $V_1(k, \Theta)$  and for small signals will be much smaller. This will be reflected in the gradient and hessian. We could also introduce some weighting for the norms we are considering, but will not consider this case for notational purposes.

#### 4. GRADIENT DESCENT AND NEWTON'S METHOD

In this section we seek to minimize the cost function(s)  $V_i(N, \Theta)$  by applying the following algorithm

$$\text{vec}(\Theta)(k+1) = \text{vec}(\Theta)(k) - \mu(k)\mathcal{H}_i(k)^{-1}\nabla V_i(k) \quad (19)$$

where

$$\nabla V_i(k) = \frac{\partial V_i(k, \Theta)}{\partial \text{vec}(\Theta)} \quad (20)$$

$$\mathcal{H}_i(k) = \frac{\partial}{\partial \text{vec}(\Theta)} \frac{\partial V_i(k, \Theta)}{\partial \text{vec}(\Theta)^T} \quad (21)$$

If we approximate  $\mathcal{H}$  with  $I$ , let  $\mu(k)$  be a small constant, set  $\alpha = 0$ , set  $p = 2$ , and consider  $V_2(k, \Theta)$  then we get a gradient descent method

$$\text{vec}(\Theta)(k+1) = \text{vec}(\Theta)(k) - \mu\nabla V_2(k) \quad (22)$$

commonly known as LMS or Fx-LMS depending upon the implementation. If, in addition, we set  $\alpha$  to equal a small constant and  $q = 2$  then we get the Leaky LMS algorithm. Thus, it should be clear that the work we considering here is more general.

In general the gradient will have the form of

$$\nabla V_i(k) = \mathbb{E}\left\{(S(q)^T \otimes X(k))\bar{\varepsilon}(k) + \overline{\alpha \text{vec}(\Theta)}\right\} \quad (23)$$

with  $\bar{\varepsilon}(k) = \|\varepsilon(k)\|_p^{1-p}\tilde{\varepsilon}(k)$  for  $V_1(k, \Theta)$  and

$\bar{\varepsilon}(k) = p\tilde{\varepsilon}(k)$  for  $V_2(k, \Theta)$ ;  $\overline{\text{vec}(\Theta)} = \|\text{vec}(\Theta)\|_q^{1-q}\widetilde{\text{vec}(\Theta)}$

for  $V_1(k, \Theta)$  and  $\overline{\text{vec}(\Theta)} = q\widetilde{\text{vec}(\Theta)}$  for  $V_2(k, \Theta)$ . Each term with a tilde is defined similarly to lemma 3.

In general the Hessian will have the form

$$\mathcal{H}_i(k) = \mathbb{E}\left\{(S(q)^T \otimes X(k))W_i(k)(S(q)^T \otimes X(k))^T + \alpha \frac{\partial \overline{\text{vec}(\Theta)}}{\partial \text{vec}(\Theta)^T}\right\} \quad (24)$$

where

$$W_1(k) = [\text{diag}(|\varepsilon(k)|^{p-2}) - \|\varepsilon(k)\|^{-p}\tilde{\varepsilon}(k)\tilde{\varepsilon}(k)^T] (p-1)\|\varepsilon(k)\|_p^{1-p}$$

$$\frac{\partial \overline{\text{vec}(\Theta)}}{\partial \text{vec}(\Theta)^T} = (q-1)\|\text{vec}(\Theta)\|_q^{1-q}(\text{diag}(|\text{vec}(\Theta)|^{q-2})$$

$$- \|\text{vec}(\Theta)\|_q^{-q}\widetilde{\text{vec}(\Theta)}\widetilde{\text{vec}(\Theta)}^T)$$

for  $V_1(k, \Theta)$  and

$$W_2(k) = p((p-1)\text{diag}(|\varepsilon(k)|^{p-2}))$$

$$\frac{\partial \overline{\text{vec}(\Theta)}}{\partial \text{vec}(\Theta)^T} = q(q-1)\text{diag}(|\text{vec}(\Theta)|^{q-2})$$

for  $V_2(k, \Theta)$ . Again, each term with a tilde is defined similarly to lemma 3.

The expressions for the gradient and hessian contain the expectation operator and since the distribution of the arguments in not known a priori we need a method of estimating them online. If we knew the gradient and hessian explicitly then we could calculate the optimal solution without the need of adaptation and therefore we will approximate them based upon data.

#### 5. DATA-BASED APPROXIMATIONS

In this section we seek data based approximations to the gradient and hessian calculated in the previous section. This will enable us to implement the algorithm online and the choice of the approximation will play a central role in the convergence, robustness, and computational complexity of the resulting algorithm. The following approximation(s) are similar the the familiar RLS (Haykin [2002]) and in more general terms the Recursive Prediction-Error Methods (Ljung [1999]).

To approximate the gradient

$$\begin{aligned} \nabla V_i(k) &= \mathbb{E}\left\{(S(q)^T \otimes X(k))\bar{\varepsilon}(k) + \overline{\alpha \text{vec}(\Theta)}\right\} \\ &\approx \frac{1}{k} \sum_{n=m(k)}^k \left\{(S(q)^T \otimes X(n))\bar{\varepsilon}(n) + \overline{\alpha \text{vec}(\Theta)}\right\} \end{aligned} \quad (25)$$

where  $k-m(k)$  is the window size. If, in addition, we want to discount previous information we can include the term  $\lambda^{k-i}$ ,  $0 \leq \lambda \leq 1$  is the discount factor, in the estimate which is typically in the RLS cost function. However, in this case, it would be beneficial to change  $\frac{1}{k}$  to  $\frac{\lambda}{k+(\lambda-1)}$  to be consistent with the recursive prediction error methods in (Ljung [1999]) and thus properly scale our estimate. This gives

$$\begin{aligned} \nabla V_i(k) &\approx \frac{\lambda}{k+(\lambda-1)} \sum_{n=m(k)}^k \lambda^{k-n} \left\{(S(q)^T \otimes X(n))\bar{\varepsilon}(n) \right. \\ &\quad \left. + \overline{\alpha \text{vec}(\Theta)}\right\} \end{aligned} \quad (26)$$

If we set  $m = k$  and  $\lambda = 1$  then we get the standard instantaneous gradient approximation that is used in the LMS filter (Haykin [2002]).

To approximate the hessian

$$\begin{aligned} \mathcal{H}_i(k) &= \mathbb{E} \left\{ (S(q)^T \otimes X(n))W_i(n)(S(q)^T \otimes X(n))^T + \alpha \frac{\partial \text{vec}(\Theta)}{\partial \text{vec}(\Theta)^T} \right\} \\ &\approx \frac{\lambda}{k + (\lambda - 1)} \sum_{n=m(k)}^k \lambda^{k-n} \left\{ (S(q)^T \otimes X(n))W_i(n)(S(q)^T \otimes X(n))^T \right. \\ &\quad \left. + \alpha \frac{\partial \text{vec}(\Theta)}{\partial \text{vec}(\Theta)^T} \right\} \end{aligned} \quad (27)$$

where we will use a different  $m(k)$  and  $\lambda$  for the hessian and gradient. Notice that if  $\lambda \approx 1$  then  $\frac{1}{k}$  is a close approximation to  $\frac{\lambda}{k+(\lambda-1)}$ . If  $m = k$  and  $\lambda = 1$  then we get the standard Hessian approximation that, in certain situations, can be used in a recursive manner similar to RLS.

In addition to the aforementioned data-based approximations to the gradient and hessian, it is also beneficial (computationally) to implement the filter in block form. In this scenario, the update equations are the same. The difference is that the parameters are not updated every time sample, instead they are updated every  $N$  time samples.

## 6. APPLICATION

In this section we will apply a single channel version of the aforementioned algorithm to Hewlett-Packard's Blackbird 002 gaming PC. The gaming PC is shown in Fig. 4 and it should be noted that this application of ANC is in a small enclosure with an irregular geometry and high airflow. The distance between the reference and error microphones about 6" whereas the wavelength of sound that we are trying to cancel is around 23".



Fig. 4. Hewlett-Packard's Blackbird 002 gaming PC.

The noise that we are targeting is produced by the cooling fans mounted in various places throughout the PC. The over-clocked CPU is cooled via a liquid cooling system that is attached to a heat exchanger with two 120mm fans. The hard drive rack has a 120mm cooling fan, the video card has a fan, and other components inside the system have smaller fans or optional fan attachments. For this application we will concentrate on removing the noise from the hard drive fan since its location within the system allows for the attachment of speakers and microphones in the proper locations for noise control.

Additionally, for demonstration purposes, the stock 120mm fan was swapped with a very high speed 80mm fan so that

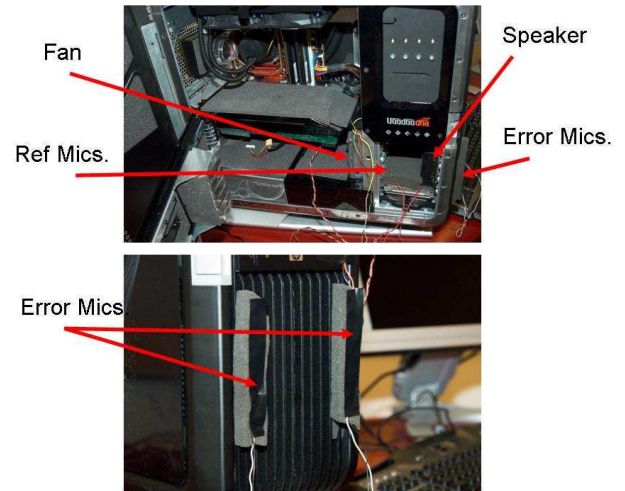


Fig. 5. Modification to PC for the purposes of active noise control.

this fan is by far the noisiest component. To allow for space inside of the PC for the ANC system, several hard drives were removed and the microphones and speaker shown in Fig. 5 were added. The error microphones are located directly outside the case and the reference microphones are located near the fan, both mounted in foam to reduce windage and vibrational noise. The error microphones signals and reference signals are each summed together (separately) to further reduce noise and focus on sound propagating in the downstream direction. To improve the acoustic properties of the system, acoustic foam was added so that the propagating sound will be more uniform and unidirectional.

The single channel Fx-LMS algorithm was applied to the system and to evaluate the performance of the system an external microphone, or listener microphone, was placed approximately 6" from the case directly in front of the PC. All of the reported experimental results were obtained from this microphone.

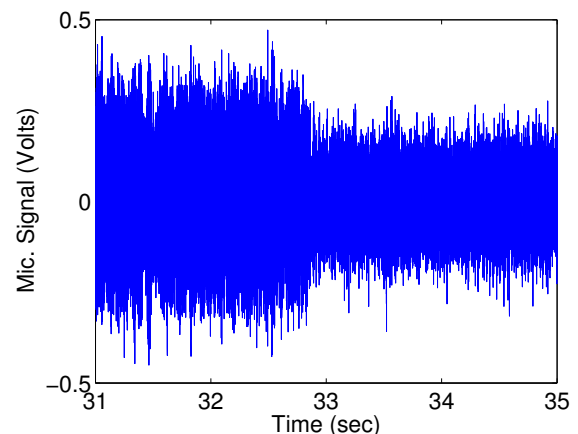


Fig. 6. Signal from listener microphone.

Figure 6 shows the signal obtained from the listener microphone before and after the control is applied. Between 32 and 33 seconds the control is switched on and the listener signal is reduced. The sample variance before control is 0.0235 and after control is 0.0056 giving a 76% reduction in

the sample variance. This reduction translates into slight more than a 6dB reduction in the sound pressure level as shown in Fig. 7. In this figure the sound pressure level is calculated by sliding a window over the signal and computing the rms value in dB scale. A simple calculation verifies that  $10 \log_{10}(0.0235/0.0056) \approx -6.2288$ , and timing in the drop of SPL given in Fig. 7 corresponds to the drop in signal size shown in Fig. 6.

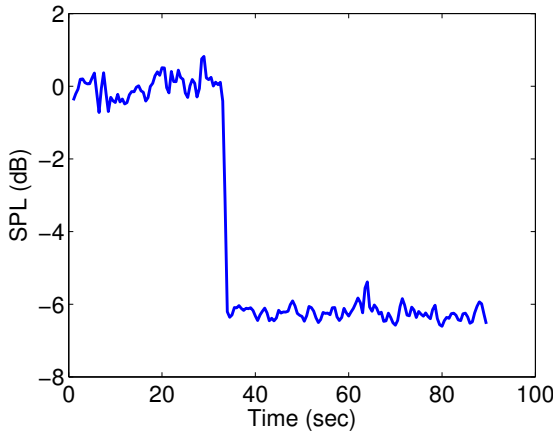


Fig. 7. Sound pressure level as a function of time.

In the frequency domain it can be seen that the tonal part of the fan noise is reduced significantly as shown in Fig. 8. Here, the power spectral density before and after control is shown. The dashed line is without control and the solid line is with control. Notice that the first and third harmonics are rejected by the controller.

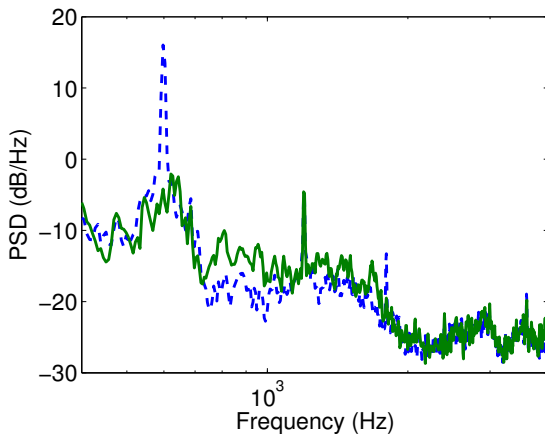


Fig. 8. Power spectral density of listener microphone before (dashed) and after (solid) control.

## 7. CONCLUSIONS AND FUTURE RESEARCH

In this paper, we presented a multichannel adaptive filter and applied the single channel version to HP's Blackbird

002 gaming system. It was shown that a 6dB reduction in sound pressure level of the PC cooling fan is possible with minor modifications to the PC. It is important to note that the results were obtained with an external microphone to reflect the experience of the listener. Based upon this application, we conclude that active noise control is a viable solution to reduce the tonal noise of cooling fans inside of PCs. With a re-design of hardware, it is expected that much more reduction is possible and that broadband noise can be reduced as well.

Future research topics that will be beneficial to this application are related to hardware design that will improve the control system. For example, the use of beamformers to improve the signal to noise ratio of the reference and error signals, additional reference and error signal for the application of a multichannel filter, and the design of the PC enclosure for beneficial acoustic properties.

## ACKNOWLEDGEMENTS

Special thanks to HP Labs and HP Lab employee Antonius Kalker.

## REFERENCES

- M. Bouchard and S. Quednau. Multichannel recursive-least-squares algorithms and fast-transversal-filter algorithms for active noise control and sound reproduction systems. *IEEE TRANSACTIONS ON SPEECH AND AUDIO PROCESSING*, 8(5):606–618, September 2000.
- S.J. Chen and J.S. Gibson. Feedforward adaptive noise control with multivariable gradient lattice filters. *IEEE TRANSACTIONS ON SIGNAL PROCESSING*, 49(3): 511–520, March 2001.
- L.C. Coradine, J.M.T. Romano, and M.G. Bellanger. Adaptive filtering in cascade for using a fast multichannel RLS algorithm. *IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS-II: ANALOG AND DIGITAL SIGNAL PROCESSING*, 44(9):757–762, September 1997.
- V.I. Djigan. Multichannel parallelizable sliding window RLS and fast RLS algorithms with linear constraints. *Signal Processing*, 86:776–791, April 2006.
- S.J. Elliott, I.M. Stothers, and P.A. Nelson. A multiple error LMS algorithm and its application to the active control of sound and vibration. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, ASSP-35(10): 1423–1434, October 1987.
- E. Esmailzadeh, A.R. Ohadi, and A. Alasty. Multi-channel adaptive feedforward control of noise in an acoustic duct. *Transactions of the ASME*, 126:406–415, June 2004.
- S. Haykin. *Adaptive Filter Theory*. Prentice-Hall, Inc, Upper Saddle River, New Jersey 07458, fourth edition, 2002.
- S.M. Kay. *Fundamentals of Statistical Signal Processing*. Prentice Hall, 1993.
- L. Ljung. *System Identification-Theory for the User*. Prentice-Hall, Englewood Cliffs, NJ USA, second edition, 1999.
- J. Zeng and R.A. de Callafon. Recursive filter estimation for feedforward noise cancellation with acoustic coupling. *Journal of Sound and Vibration*, 291:1061–1079, 2006.