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Brief paper

Control relevant estimation of plant and disturbance dynamics☆

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Abstract

Estimating models for both plant and disturbance dynamics is important in control design applications that focus on disturbance rejection. Several methods for low-order approximate model estimation on the basis of closed-loop data exist in the literature, but fail to address the simultaneous estimation of low-order approximate models of both plant and disturbance dynamics. In this paper a new extended two-stage methodology is proposed that allows for low-order approximate disturbance model estimation. In the proposed extended two-stage method the first stage is used to estimate high-order models for filtering purposes. In the second stage, filtered signals are used to provide the means for low-order approximate model estimation of both plant and disturbance dynamics.

Keywords: Identification; Closed-loop; Disturbance rejection; Parameter estimation

1. Introduction

The need for control relevant modeling has resulted in several methodologies that aim at iteratively improving closed-loop plant behavior on the basis of closed-loop experiments (Abe & Ichihara, 2000; Ichichara & Abe, 2002; Lee, Anderson, Mareels, & Kosut, 1995; Ochs & Engell, 2000). In most of the existing methods, the emphasis is placed on the control relevant approximation of plant dynamics only and ignore the approximate modeling of the disturbance dynamics that is relevant in disturbance control. For minimum variance and LQG control, successful modeling and control performance improvements have been shown in Gevers and Ljung (1986) and Hjalmarsson, Gevers, De Bruyne, and Leblond (1994), but these results assume consistent estimation of plant and disturbance dynamics.

In control relevant modeling, closed-loop experiments are inevitable to evaluate the closed-loop behavior of a plant. The correlation of the disturbance with any of the signals in the closed-loop is one of the problems in dealing with closed-loop data. Possible ways to overcome this problem is by assuming low disturbance correlation condition (Åström, 1993; Zang, Bitmead, & Gevers, 1995) or consistent estimation of the noise filter (Forssell & Ljung, 1999) to reduce bias on the plant model estimate. Alternatively, an external reference signal uncorrelated with the noise can be used to reparametrize the closed-loop identification problem by a direct parametrization of the closed-loop transfer function as done in van Donkelaar and Van den Hof (1996) or in the recursive algorithms for closed-loop identification of Landau and Karimi (1997, 1999). Although powerful for estimating control relevant plant dynamics, bias approximation results similar to a direct identification Ljung (1999) are obtained in case an approximate disturbance model is estimated (Karimi & Landau, 1998).

An alternative parametrization of the closed-loop identification problem is built on the dual-Youla parametrization (Lee et al., 1995), coprime factor identification (de Callafon, Van den Hof, & de Vries, 1994) or a two-stage identification (Van den Hof & Schrama, 1993). In these methods, an auxiliary or previously estimated model is used for filtering purposes to recast the closed-loop identification problem in a standard open-loop estimation problem (Van den Hof & Schrama, 1995). These methods have shown promising results for approximate and control relevant plant modeling, but do not address the approximate model estimation of the disturbance dynamics.

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This paper proposes an estimation method that allows for a control relevant estimation of both plant and disturbance dynamics on the basis of closed-loop data. The proposed methodology is an extension of the two-stage method (Van den Hof & Schrama, 1993), where the first stage is used to estimate high-order models for filtering purposes. In the second stage, filtered signals are used for lower order approximate model estimation. An analysis of the method is presented in the form of the bias distribution of the approximate plant and disturbance model estimates. It is illustrated how control relevant models for both the plant and the disturbance dynamics can be obtained using the bias distribution expressions.

2. Problem formulation and case study

2.1. Problem formulation

Consider a stabilizing feedback connection of an unknown (linear discrete time) plant $P_0(q)$ and feedback controller $C(q)$. For analysis purposes, both $P_0(q)$ and $C(q)$ are assumed to be stable and the output $y(t)$ of the plant $P_0(q)$ is fed back to the input $u(t)$ using a negative feedback

$$u(t) = r(t) - C(q)y(t) \tag{1}$$

and $r(t)$ is an external reference signal for closed-loop excitation purposes. Additionally, an additive disturbance $v(t)$ acts on the output of the plant

$$y(t) = P_0(q)u(t) + v(t), \quad v(t) = H_0(q)e(t) \tag{2}$$

which is modeled as a monic stable and stably invertible disturbance filter $H_0(q)$ having a white noise input $e(t)$ with variance λ.

Since $v(t)$ acts on the closed-loop system, both the input $u(t)$ and the output $y(t)$ are correlated with the disturbance, which makes the closed-loop identification problem challenging. The input/output data of the plant $P_0(q)$ subjected to the external reference signal $r(t)$ and an additive disturbance $v(t)$ can be described by

$$y(t) = P_0(q)S_0(q)r(t) + S_0(q)H_0(q)e(t), \tag{3}$$

$$u(t) = S_0(q)r(t) - C(q)S_0H_0(q)e(t), \tag{4}$$

where $S_0(q)$ is the sensitivity function defined by $S_0(q) = (1 + C(q)P_0(q))^{-1}$. Given input/output data measurements $\{u(t), y(t)\}$, $t = 1, \ldots, N$ obtained under closed-loop conditions, the identification problem posed in this paper is to estimate approximate models $P_\theta(q)$ and $H_\theta(q)$ of *both* the plant $P_0(q)$ and disturbance $H_0(q)$ dynamics.

The approximation of plant P_0 and disturbance dynamics H_0 is made control relevant by estimating models P_θ and H_θ that minimize

$$\|(P_0(q) - P_\theta(q))S_0(q)\|_2 + \delta\|(H_0(q) - H_\theta(q))S_0(q)\|_2, \tag{5}$$

where the additive differences $P_0(q) - P_\theta(q)$ and $H_0 - H_\theta(q)$ are weighted by the closed-loop sensitivity function $S_0(q)$ and measured by a two-norm. In addition, the value of δ can be used to specify the relative weighting of the two terms in (5). Instead of minimizing the additive open-loop differences $P_0(q) - P_\theta(q)$ and $H_0(q) - H_\theta(q)$, in (5) we emphasize the approximation of closed-loop behavior by weighting with the (unknown) sensitivity function $S_0(q)$. Obviously, a consistent estimate $P_\theta(q) = P_0(q)$ and $H_\theta(q) = H_0(q)$ will minimize both terms in (5), however, we stress the low-order approximation of $P_\theta(q)$ and $H_\theta(q)$ as low-order models are manageable and appealing in subsequent control design for disturbance rejection.

2.2. Case study

A case study will be used to illustrate the identification concepts proposed and analyzed in this paper. The case study consists of a discrete-time sixth-order plant $P_0(q)$ and monic stable and stably invertible disturbance filter $H_0(q)$. The dynamics of the plant and disturbance filter are given by a [6,6,6,1]-ARMAX structure (Ljung, 1999). An amplitude Bode plot of the sixth-order plant P_0 and disturbance H_0 is given in Fig. 1 on the left, where it can be observed that the plant P_0 exhibits a large resonance frequency at approximately 1 rad/s and small resonance modes on either side. The common dynamics of P_0 and H_0 is characteristic for a flexible mechanical system subjected to external disturbances that excite the same resonance modes of the system.

A simple second-order discrete-time feedback controller $C(q)$ is used for the reduction of the main resonance mode in the open-loop disturbance dynamics $H_0(q)$ in (2) and a comparison of the open-loop $H_0(q)$ and closed-loop $S_0(q)H_0(q)$ disturbance filter is given in Fig. 1 on the right. For identification purposes, closed-loop input $u(t)$ and output $y(t)$ signals in (3) and (4) consist of $N = 4096$ points, where the reference signal $r(t)$ and the noise $e(t)$ are chosen as independent Gaussian distributed white noise signals. The reference signal $r(t)$ has a unit variance, whereas the noise $e(t)$ is set to have a variance of $\mathrm{E}\{e(t)^2\} = \lambda = 0.04$.

With the knowledge of the common dynamics in P_0 and H_0, the low-order plant model P_θ and noise model H_θ could be parametrized using an ARMAX model structure (Ljung, 1999). However, to eliminate the effects of bias due to a joint parametrization of plant and noise dynamics, an independent parametrization is used. An approximate (low order) plant model P_θ and noise model H_θ are parametrized using a BJ model structure:

$$P(q,\theta) = \frac{b_2q^{-2} + b_1q^{-1} + b_0}{q^{-2} + a_1q^{-1} + a_0},$$

$$H(q,\theta) = \frac{q^{-2} + c_1q^{-1} + c_0}{q^{-2} + d_1q^{-1} + d_0},$$

$$\theta = [a_1 \quad a_0 \quad b_2 \quad b_1 \quad b_0 \quad c_1 \quad c_0 \quad d_1 \quad d_0] \in \mathbb{R}^{1\times 9}, \tag{6}$$

where the second-order models are required to capture the main resonance mode of P_0 and H_0. The estimated low-order models P_θ and H_θ can then be used to design a low-order controller for disturbance suppression of the main resonance mode.

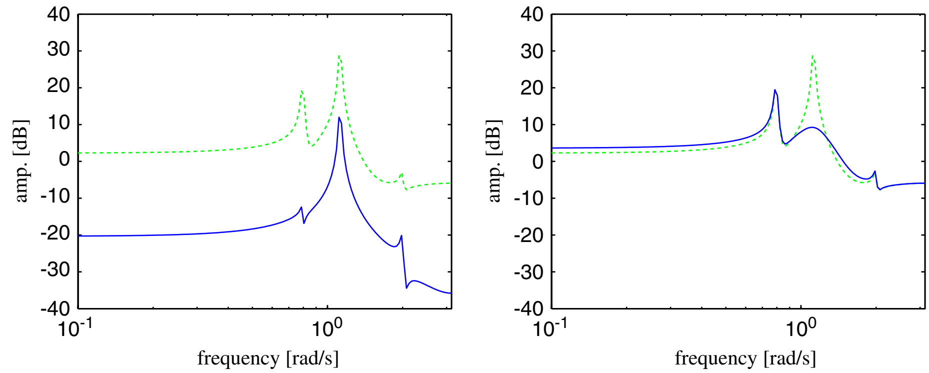

Fig. 1. Left: amplitude Bode plot of plant dynamics P_0 (solid) and disturbance dynamics H_0 (dashed). Right: amplitude Bode plot of open-loop H_0 (dashed) and closed-loop S_0H_0 disturbance filter (solid).

3. Two-stage identification

3.1. Method description

In the two-stage method, identification of the plant model and disturbance model in closed loop is performed in two separate steps. The two steps are used to eliminate the correlation between the input $u(t)$ and the noise $e(t)$ in case of closed-loop data. The two-stage method does not require the knowledge of the controller $C(q)$. If we assume the knowledge of controller $C(q)$ is unknown, then the knowledge of the reference signal $r(t)$, the input signal $u(t)$ and the output signal $y(t)$ are needed. The two-stage method can be summarized as follows (Van den Hof & Schrama, 1993).

In the first step, a model S_β of the sensitivity function S_0 is estimated by considering the map from reference signal $r(t)$ to the plant input $u(t)$ in (4). Estimation is done by minimizing the two-norm of the prediction error

$$\varepsilon_1(t,\beta) = u(t) - S_\beta(q)r(t)$$

using a high-order model S_β for the sensitivity function. The model S_β is used only for filtering purposes in the second step of the method and no specific restrictions on the order of S_β are imposed.

In the second step of the two-stage method, the estimate S_β is used to simulate a disturbance free input signal $u_r(t)$ via

$$u_r(t) = S_\beta(q)r(t)$$

that will be uncorrelated with noise $e(t)$ on the closed-loop data. In case a consistent estimate $S_\beta = S_0$ is obtained in the first step, (3) rewrites into

$$y(t) = P_0u_r(t) + S_0H_0e(t).$$

Subsequently, in the second step of this method a plant model P_θ (and possibly a disturbance model H_θ) can be estimated by minimizing the two-norm of the prediction error

$$\varepsilon_2(t,\theta) = H_\theta^{-1}[P_0u(t) - P_\theta u_r(t) + (H_0 - H_\theta)e(t)] + e(t), \tag{7}$$

where P_θ and H_θ are again the desirable low-order approximation of the plant P_0 and disturbance filter H_0. In general, the two-stage method is used only to estimate (low order) approximate model P_θ of P_0 in the second step and the estimation of disturbance filters is omitted.

3.2. Bias distribution for two-stage method

The minimization of the two-norm of the prediction error in (7) during the second step of the two-stage method yields the asymptotic expression

$$\hat{\theta} = \arg\min_\theta \frac{1}{2\pi}\int_{-\pi}^{\pi} |H_\theta^{-1}|^2[|(P_0 - P_\theta)S_0 + P_\theta(S_0 - S_\beta)|^2\Phi_r + |H_0S_0 - H_\theta|^2\Phi_e]\,\mathrm{d}w \tag{8}$$

for $N \to \infty$ (Van den Hof & Schrama, 1993). It can be observed that the estimation of the plant model P_θ depends on the estimate S_β of the sensitivity function S_0 in the first step. In case $S_\beta \neq S_0$ the term $P_\theta(S_0 - S_\beta)$ influences the estimation of the model P_θ, but this term can be made small by estimating an accurate model S_β of the sensitivity in the first step of the method (Van den Hof & Schrama, 1993).

An explicit tunable expression for the bias of the plant model can be obtained by using an independent parametrization of the plant model P_ξ and the disturbance model H_η. For example, for an OE-model with a fixed disturbance model $H_\eta = 1$, the asymptotic expression of (8) can be simplified to

$$\hat{\xi} = \arg\min_\xi \frac{1}{2\pi}\int_{-\pi}^{\pi} |P_0 - P_\xi|^2|S_0|^2\Phi_r\,\mathrm{d}w$$

in case $S_\beta = S_0$ and clearly indicates the tunable bias expression of the plant model estimate. Moreover, a consistent estimate of the plant dynamics P_0 can be obtained, even though H_η is fixed. Such consistency results of the plant model estimate for $H_\eta \neq H_0$ are, for example, not shared by a direct identification method Ljung (1999).

Unfortunately, the favorable properties of consistency and tunable bias expression for the plant model do not carry over to the disturbance model estimate. It can be observed from (8) that

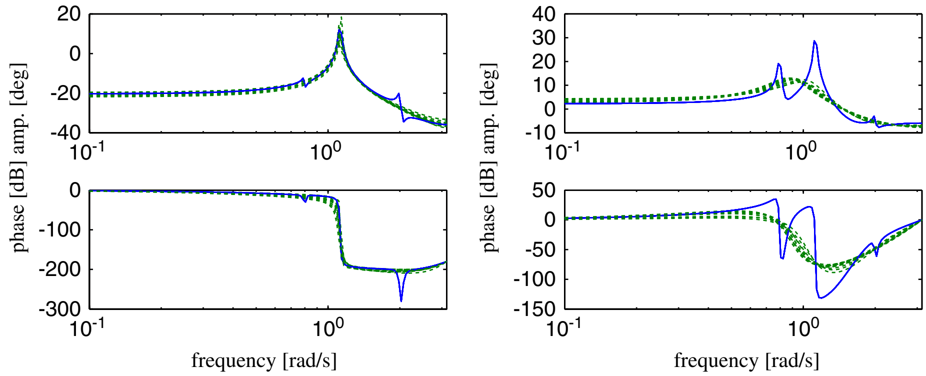

Fig. 2. Application of two-stage method using Monte-Carlo simulations. Left: Bode plot of plant P_0 (solid) and 10 estimated second-order models P_θ (dashed). Right: Bode plot of H_0 (solid) and 10 estimated second-order disturbance models H_θ (dashed).

the estimation of an independently parametrized disturbance model H_η will always be biased, as it aims at the approximation of the closed-loop disturbance model H_0S_0. As a result, the estimation of the disturbance models in the two-stage identification method does not share the consistency and tunable bias expressions found for the plant model estimate.

3.3. Illustration of two-stage method

The approximation results of the plant model estimate and the bias effects of the disturbance filter can be illustrated with the case study. Second-order models P_θ and H_θ parametrized according to (6) are used to estimate the main resonance mode in P_0 and H_0 on the basis of closed-loop data.

Using a 20th-order ARX model to estimate a model S_β of the sensitivity function in the first step of the method, the minimization of the two-norm of the prediction error (7) leads to the second-order model estimates depicted in Fig. 2. It can be seen that an accurate approximation of the main resonance mode can be obtained with the second-order model P_θ due to the use of a filtered and noise free input signal $u_r(t)$ in the second step of the method. However, the estimation of H_θ is always biased as H_θ approximates the closed-loop noise model H_0S_0 as depicted in Fig. 1. The knowledge of sensitivity function S_0 available in the form of the model S_β obtained in the first step of the two-stage method can be used to compute an estimate of the open-loop noise model via $H_\theta S_\beta^{-1}$. Unfortunately, this increases the order of the disturbance model.

4. Extended two-stage method

4.1. Method description

For explanation of the proposed extended two-stage method, define $\bar{P} = P_0S_0$ and $\bar{H} = H_0S_0$. With this definition, (3) can be rewritten into

$$y(t) = \bar{P}r(t) + \bar{H}e(t), \tag{9}$$

$$y(t) = P_0(1 - C\bar{P})r(t) + H_0(1 - C\bar{P})e(t), \tag{10}$$

where the knowledge of the controller C is exploited. From (10) it can be observed that with knowledge of $\bar{P}$, the controller C and a time realization of $e(t)$, the estimation of P_0 and H_0 on the basis of closed-loop data becomes a standard open-loop identification problem. A time realization of $e(t)$ can be obtained via an accurate estimation of $\bar{P}$, $\bar{H}$ on the basis of the closed-loop data in (9).

Consistent estimation of the closed-loop disturbance filter $\bar{H}$ is possible in the prediction error framework if $\bar{H}$ is a stable and stably invertible filter. If the controller C internally stabilizes the plant P_0, and the open-loop disturbance filter H_0 is stable and stably invertible, then it is straightforward to see that $\bar{H}$ is stable and stably invertible provided both C and P_0 are stable. Under these conditions, the consistent estimation of $\bar{P}$, $\bar{H}$ on the basis of the closed-loop data in (9) also becomes a standard open-loop identification problem. From these observations, the extended two-stage method can be summarized by the following two steps:

(1) In the first step, a standard open-loop identification of $\bar{P}$ and $\bar{H}$ is performed on the basis of the closed-loop reference $r(t)$ and output $y(t)$ signal in (9). Using the estimated models $\bar{P}_*$ and $\bar{H}_*$, the closed-loop prediction error

$$\varepsilon_{cl}(t) = \bar{H}_*^{-1}(y(t) - \bar{P}_*r(t)) \tag{11}$$

is computed to obtain a realization of the (unfiltered white) noise present on the closed-loop data.

(2) In the second step of the method, the estimated model $\bar{P}_*$ is used to create a filtered input $u_f(t)$ and a filtered prediction error $\varepsilon_f(t)$:

$$u_f(t) := (1 - C\bar{P}_*)r(t), \tag{12}$$

$$\varepsilon_f(t) := (1 - C\bar{P}_*)\varepsilon_{cl}(t) \tag{13}$$

using the knowledge of the feedback controller C. Subsequently, the signals $u_f(t)$ and $\varepsilon_f(t)$ according to (10) are used to estimate low-order models P_θ and H_θ by minimizing the two-norm of the output error

$$\varepsilon(t,\theta) = y(t) - [P_\theta(q) H_\theta(q)] \begin{bmatrix} u_{\mathrm{f}}(t) \\ \varepsilon_{\mathrm{f}}(t) \end{bmatrix} \tag{14}$$

that allows for a low-order approximation of the open-loop plant P_0 and disturbance filter H_0.

During the open-loop identification of $\bar{P}$ and $\bar{H}$ in the first step of this method, a stable plant model $\bar{P}$ and a stable and stably invertible disturbance model $\bar{H}$ are estimated. The reason for the construction of the closed-loop residuals in (11) in the first step of the method is to allow control over the order of the estimated disturbance model in the second step. Similar to the standard two-stage method (Van den Hof & Schrama, 1993), the models in the first step are only used for filtering purposes and no restriction on the order of these models is required.

4.2. Bias distribution for the extended two-stage method

The asymptotic frequency domain expression for the minimization of the two-norm of the prediction error in (14) depends on the estimation results of $\bar{P}_*$ and $\bar{H}_*$ in the first step of the method. In case modeling errors are made in the first step, i.e. $\bar{P}_* \neq P_0 S_0$, $\bar{H}_* \neq H_0 S_0$, then the following asymptotic bias expression is obtained.

Theorem 1. *Consider the first step in the extended two-stage method where estimates $\bar{P}_*$ and $\bar{H}_*$ satisfy*

$$\bar{P}_* \neq \bar{P} = P_0 S_0, \quad \bar{H}_* \neq \bar{H} = H_0 S_0 \tag{15}$$

then for $N \to \infty$ the two-norm minimization of the output error in (14) *is equivalent to*

$$\begin{aligned} \min_\theta \int_{-\pi}^{\pi} [&|(P_0 - P_\theta) S_0 \\ &+ (\bar{P}_* - \bar{P})(P_\theta C + H_\theta (1 - C\bar{P}_*) \bar{H}_*^{-1})|^2 \Phi_r \\ &+ |(H_0 - H_\theta) S_0 + H_\theta (S_0 - (1 - C\bar{P}_*) \bar{H} \bar{H}_*^{-1})|^2 \Phi_e] \, \mathrm{d}w, \end{aligned} \tag{16}$$

where P_θ and H_θ denote the models estimated in the second step of the extended two-stage method.

Proof. Using (12), (13) and the fact that $\varepsilon_{\mathrm{cl}}(t)$ can be written as $\varepsilon_{\mathrm{cl}}(t) = \bar{H}_*^{-1}(\bar{P} - \bar{P}_*) r(t) + \bar{H}_*^{-1} \bar{H} e(t)$ the output error in (14) can be written as a filtered version of $r(t)$ and $e(t)$. Using the fact that $e(t)$ is uncorrelated with $r(t)$, autocorrelation of $\varepsilon(t,\theta)$ and application of Parseval's theorem leads to the bias distribution given in (16). □

More simplified expressions of the bias distribution can be obtained by assuming that consistent estimate of the closed-loop transfer functions $P_0 S_0$ and/or $H_0 S_0$ are obtained in the first step of the method. A bias distribution similar to the standard two-stage method is obtained by assuming the consistent estimation $\bar{P}_* = P_0 S_0$ and the result is summarized in the following.

Corollary 1. *Let $\bar{P}_* = \bar{P}$, $\bar{H}_* \neq \bar{H}$ in the first step in the extended two-stage method, then for $N \to \infty$ the two-norm minimization of the output error in* (14) *is equivalent to*

$$\begin{aligned} \min_\theta \int_{-\pi}^{\pi} &[|P_0 - P_\theta|^2 |S_0|^2 \Phi_r(w) \\ &+ |(H_0 - H_\theta) S_0 + H_\theta (1 - \bar{H} \bar{H}_*^{-1}) S_0|^2 \Phi_e(w)] \, \mathrm{d}w. \end{aligned} \tag{17}$$

Proof. Substitution of $\bar{P}_* = P_0 S_0$, $\bar{H}_* \neq \bar{H}$ into (16) yields the result of (17). □

The most simplified and intuitive result is obtained when consistent estimates of both the closed-loop transfer function $P_0 S_0$ and $H_0 S_0$ are obtained in the first step of the method. In that case, explicit tunable expressions for both the plant model P_θ and the disturbance model H_θ can be derived.

Corollary 2. *Let $\bar{P}_* = \bar{P}$, $\bar{H}_* = \bar{H}$ in the first step in the extended two-stage method, then for $N \to \infty$ the two-norm minimization of the output error in* (14) *is equivalent to*

$$\min_\theta \int_{-\pi}^{\pi} [|P_0 - P_\theta|^2 |S_0|^2 \Phi_r + |H_0 - H_\theta|^2 |S_0|^2 \Phi_e] \, \mathrm{d}w. \tag{18}$$

Proof. Substitute $\bar{P}_* = P_0 S_0$, $\bar{H}_* = \bar{H}$ into (16), then (18) is obtained. □

It is easily observed that in the case $\bar{P}_* = P_0 S_0$ and $\bar{H}_* = H_0 S_0$, the difference $|P_0 - P_\theta|^2$ is weighted by the reference spectrum Φ_r and the difference $|H_0 - H_\theta|^2$ is weighted by noise spectrum Φ_e, while both are weighted by the sensitivity function S_0. As a result, explicit tunable bias expressions are obtained for both plant and noise model dynamics. It should be noted that this refers to our desire of estimating control relevant models as defined by the criterion in (5). With $e(t)$ white noise with variance λ, we have $\Phi_e(w) = \lambda$. By experiment design we can choose $r(t)$ white noise with variance α, for which $\Phi_r(w) = \alpha$. Then (18) can be reduced to

$$\min_\theta \int_{-\pi}^{\pi} [|P_0 - P_\theta|^2 |S_0|^2 \alpha + |H_0 - H_\theta|^2 |S_0|^2 \lambda] \, \mathrm{d}w \tag{19}$$

and from (19) it can be obtained that by the choice of the variance α of the reference signal, we can influence the relative contribution of both terms in the integral expression of (19), similar as in (5). Additional tuning of the approximation of plant and noise model dynamics can be obtained by filtering of the output error $\varepsilon(t)$ in (14). Furthermore, consistent models for plant or disturbance dynamics can be obtained if the model parametrization allows for the existence of a parameter θ such that $P_\theta = P_0$ and $H_\theta = H_0$.

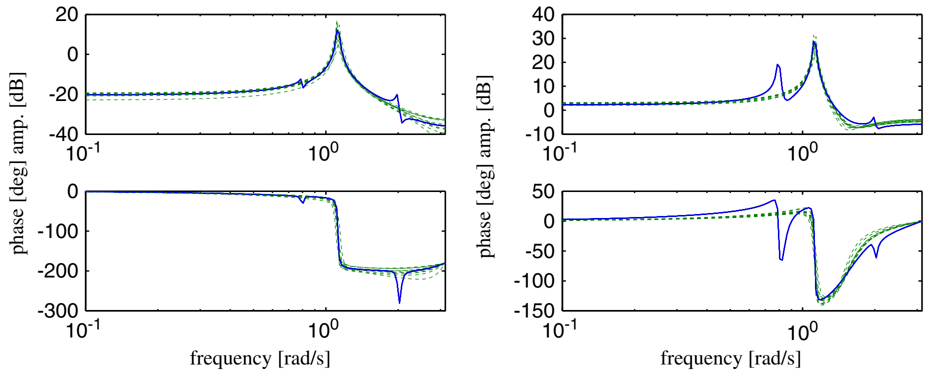

Fig. 3. Application of extended two-stage method using Monte-Carlo simulations. Left: Bode plot of plant P_0 (solid) and 10 estimated second order models P_θ (dashed). Right: Bode plot of H_0 (solid) and 10 estimated second-order disturbance models H_θ (dashed).

4.3. Illustration of extended two-stage method

Similar to case study example discussed for the standard two-stage method, a 20th-order ARX model is used to estimate model for $\bar{P} = P_0 S_0$ and $\bar{H} = H_0 S_0$ in the first step of the method. The models are used to construct a realization of the closed-loop noise $e(t)$ and subsequently the filtered (noise free) input signal and filtered prediction error in (12) and (13).

The minimization of the prediction error (14) leads to the second-order model estimates depicted in Fig. 3. It can be seen that a good approximation of the main resonance mode is obtained in plant model P_θ and noise model H_θ. Furthermore, it can be observed that the extended two-stage method gives a much better approximation of the closed-loop noise model due to implicit weighting of the sensitivity function S_0 in the approximation of both the plant and disturbance model.

5. Conclusions

An extension to an existing two-stage closed-loop identification method has been proposed to deal with the problem of simultaneous approximation of plant and noise dynamics. The estimate of a disturbance filter in the first step of the method is used to create filtered signals for the low-order model approximation of both plant and disturbance dynamics in the second step of the method. Bias distribution analysis of the proposed extended two-stage method indicates explicit tunable expressions for the bias distribution of plant model and disturbance model estimation. When consistent estimates of closed-loop transfer functions are obtained in the first step of the method, plant model and disturbance model estimates are weighted, respectively, by the reference spectrum and a constant white noise spectrum, while both are weighted by the sensitivity function. Due to the implicit weighting with the sensitivity function, control relevant models for both plant and disturbance models can be obtained via the proposed extended two-stage method.

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