LINEAR REGRESSION METHOD FOR ESTIMATING APPROXIMATE NORMALIZED COPRIME PLANT FACTORS

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Abstract: Studies on iterative identification and model based control design have shown the necessity for identifying models on the basis of closed-loop data. Estimating models on the basis of closed-loop data requires special attention due to cross correlation of noise and input signals and the possibility to estimate unstable systems operating under a stabilizing closed-loop controller. This paper provides a method to perform an approximate identification of normalized coprime factorization from closed-loop data. During the identification, a constrained linear regression parametrization is used to estimate the normalized coprime factors. A servomechanism case study illustrates the effectiveness of the proposed algorithm.

Keywords: Closed-loop identification, coprime factorization, least squares identification, servomechanisms

1. INTRODUCTION

The interconnection between system identification and model-based control design has motivated contributions in the area of "identification for control," whereby identification algorithms are tuned toward the intended purpose of the resulting model i.e. control design. Typically models useful for control design are of low-order, capturing essential closed-loop dynamic behavior. In addition to safety and production requirements common in industrial applications, closed-loop experimental data supports the identification of models accurate in the frequency region relevant for control design (Hjalmarsson *et al.* 1996).

Difficulties in closed-loop identification, mainly due to the correlation between disturbances and control signals in the feedback loop, have inspired numerous methods which can be classified into direct, indirect and joint input-output approaches (Ljung 1999). Particular to control relevant identification are two-stage, dual-Youla and coprime factor methods where model quality depends upon the compensator used during the experiment. This suggests that control design on the basis of identified models requires an iterative procedure (Skelton 1989, Schrama 1992*a*), for which several schemes have been studied, see (de Callafon and Van Den Hof 1997, Kosut 2001, Date and Lanzon 2004) and (Hjalmarsson 2005) for an overview.

The concentration of this paper is on the identification step in such iterative schemes which utilize the coprime factor representation for systems modeling and control design. Additionally under certain circumstances, for example in highly complex systems, employing simple linear (regression) identification algorithms may be computationally attractive. This paper presents a control relevant coprime factor identification algorithm that relies on a linear regression form, whereby the result-

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ing coprime factors are restricted to be normalized. The algorithm used for estimating approximately normalized coprime factors was introduced in (Van Den Hof *et al.* 1995). Here the proposed linear (regression) identification algorithm is an extension of that work.

Preliminaries and a general framework for comprime factor identification are discussed in Section 2 and Section 3. Presentation of an algorithm for the estimation of approximately normalized coprime factors is in Section 4 along with the proposed linear regression identification method in Section 4.1. A servomechanism case study presents the effectiveness the proposed identification method in Section 5.

2. PRELIMINARIES

The closed-loop system considered in this study is shown in Figure 1 where C is a feedback controller that stabilizes the (possibly unstable) LTI plant P_0, u is the plant input, y is the plant output, v is the disturbance, r_1 and r_2 are the possible reference signals available. For convenience define the general reference signal $r(t) := r_1(t) + C(q)r_2(t)$ as the overall reference to the system. Note that r_1 and r_2 may be considered as unmeasurable disturbances assuming measurability of u and y and knowledge of the controller C since $r(t) = u(t) + C(q)y(t)$. Then the system equations can be written as

$$y(t) = P_0(q)S_0(q)r(t) + W_0(q)H_0(q)e_0(t) \quad (1)$$

$$u(t) = S_0(q)r(t) - C(q)W_0(q)H_0(q)e_0(t) \quad (2)$$

where $S_0 = [1 + CP_0]^{-1}$ and $W_0 = [1 + P_0C]^{-1}$. For brevity the dependency on the delay operator q will be dropped whenever it is clear.

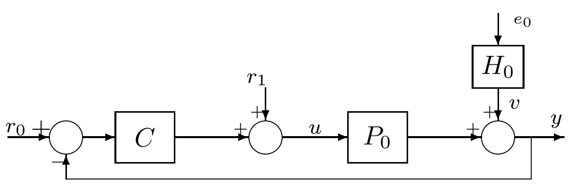

Fig. 1. Feedback configuration

Any system P has a *right coprime factorization* (r.c.f) (N, D) over $\mathcal{RH}_\infty$ if there exist $X, Y, N, D \in \mathcal{RH}_\infty$ such that (Vidyasagar 1985)

$$P(z) = N(z)D^{-1}(z); \quad XN + YD = I \quad (3)$$

Dual definitions exist for left coprime factorizations and are denoted by $(\tilde{N}, \tilde{D})$. Normalized coprime factors are defined such that

$$N^T(z^{-1})N(z) + D^T(z^{-1})D(z) = I. \quad (4)$$

Numerically efficient algorithms for computing continuous-time and discrete-time normalized coprime plant factors can be found in (Varga 1998), but the problem basically involves solving an appropriate Riccati equation.

3. ACCESS TO COPRIME FACTORS

The general framework for identification of coprime factors from closed-loop data is well established and allows the flexibility of consistently estimating models from possibly unstable and/or non-minimum phase plants. Consider a stable filter F that generates an auxiliary signal

$$x(t) = F(q)r(t) = F(q)[u(t) + C(q)y(t)] \quad (5)$$

According to Figure 2 then (1),(2) can be written as

$$y(t) = N_0(q)x(t) + W_0(q)H_0(q)e_0(t) \quad (6)$$

$$u(t) = D_0(q)x(t) - C(q)W_0(q)H_0(q)e_0(t) \quad (7)$$

where $N_0 = P_0S_0F^{-1}$ and $D_0 = S_0F^{-1}$.

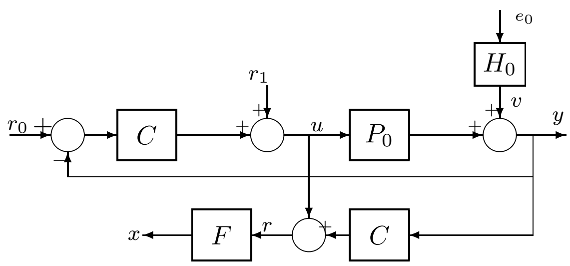

Fig. 2. Construction of auxiliary signal x from closed-loop data for coprime factor identification

The signal x is uncorrelated with the noise e_0 provided that r_1, r_2 are uncorrelated with e_0 thus the identification from x to $(y, u)^T$ is an open-loop identification of the factors (N_0, D_0) where the plant is constructed as $P_0 = N_0D_0^{-1}$. For stable (N_0, D_0) and bounded x, the limited freedom in choosing F is summarized by the following proposition.

Proposition 1. (Van Den Hof *et al.* 1995). Consider the filter

$$F = (D_x + CN_x)^{-1} \quad (8)$$

where (N_x, D_x) are *r.c.f.* of an auxiliary system P_x, then F provides a stable mapping $(y, u)^T \to x$ and $x \to (y, u)^T$ if and only if the auxiliary system P_x is stabilized by C. For all such F the plant factors induced from closed-loop data satisfy

$$\begin{bmatrix} N_0 \\ D_0 \end{bmatrix} = \begin{bmatrix} P_0 (I + CP_0)^{-1} (I + CP_x) D_x \\ (I + CP_0)^{-1} (I + CP_x) D_x \end{bmatrix} \quad (9)$$

where $P_0 = N_0 D_0^{-1}$ is also a right coprime factorization.

The above proposition shows an obvious connection to the well-known dual-Youla parametrization. Since the feedback connection of the auxiliary model P_x with *r.c.f.* (N_x, D_x) and a controller C with *r.c.f.* (N_c, D_c) is stable then a system P_0 with *r.c.f.* (N_0, D_0) that is stabilized in feedback with C can be described by (Schrama 1992*b*)

$$\begin{bmatrix} N_0 \\ D_0 \end{bmatrix} = \begin{bmatrix} N_x + D_c R_0 \\ D_x - N_c R_0 \end{bmatrix} \tag{10}$$

if and only if there exists a stable transfer matrix R_0. Additionally the R_0 that satisfies (10) is uniquely determined by

$$R_0 = D_c^{-1}(I + CP_0)^{-1}(P_0 - P_x)D_x. \tag{11}$$

In fact the dual-Youla parametrization provides all auxiliary models P_x stabilized by C. The freedom in choosing the filter F outlined in Proposition 1 suggests that access to the dual-Youla parameter may also be viewed as a natural method for tuning the auxiliary model P_x such that desired coprime factor representations in (9) are attained. Before further discussing how this may be used to tune the estimation of coprime factors through exploiting the freedom in choosing F, a brief overview of dual-Youla parameter identification is provided.

Proposition 2. (Van Den Hof and Schrama 1995). Consider the data generating plant P_0 with *r.c.f.* (N_0, D_0) and an auxiliary model P_x with *r.c.f.* (N_x, D_x) both internally stabilized by controller C with *r.c.f.* (N_c, D_c). Define the intermediate signal x as given by (5),(8) and the dual-Youla signal ξ as

$$\xi(t) = (D_c(q) + P_x(q)N_c(q))^{-1}[I \quad -P_x(q)] \begin{bmatrix} y(t) \\ u(t) \end{bmatrix} \tag{12}$$

then the identification of the dual-Youla parameter R_0 is given by

$$\xi(t) = R_0(q)x(t) + \bar{H}(q)e(t) \tag{13}$$

where the signal x is uncorrelated with e since r in (5) is assumed uncorrelated with e and the transfer matrix R_0 is given by (11).

Thus the estimation of the dual-Youla parameter is an open-loop identification from the signals x and ξ, which can be constructed from known data filters and solved via standard identification techniques. The disadvantage in applying the dual-Youla parametrization, however, lies in the inability to directly control the order of the resulting model computed via (11) (Anderson 1998). A way to circumvent this problem is a direct estimation of the coprime factors.

4. IDENTIFICATION ALGORITHM

The estimation of approximately normalized coprime factors in (Van Den Hof *et al.* 1995) came from the observation that the factors available in (9) can be shaped according to the choice of auxiliary model P_x. Stability restrictions on the auxiliary model P_x suggest a dual-Youla parametrization, however instead the estimate R_0 can be used as an initialization step in a direct coprime factor estimation, providing control over the order of the model being estimated. For example, servomechanisms typically have double integrator which would naturally be incorporated into an initial auxiliary model. This leads to the following algorithm similar to (Van Den Hof *et al.* 1995) with the main difference of using a structured linear regression model to allow for an affine optimization during the estimation of the coprime factors.

(1) *Initialization:*
   (a) Start with an auxiliary model P_x stabilized by C with (normalized) coprime factors (N_x, D_x). Simulate auxiliary input x (5) with data filter (8) and dual-Youla signal ξ according to (12) then accurately identify (possibly high order) dual-Youla parameter $\hat{R}_0$ (13) using linear regression methods.
   (b) Update estimated (high order) coprime factors $(\hat{N}_0, \hat{D}_0)$ (10) and let $\hat{P} = \hat{N}_0 \hat{D}_0^{-1}$. Then compute a normalized coprime factorization (N_x, D_x) such that $\hat{P} = N_x D_x^{-1}$ and re-simulate auxiliary signal x (5) with updated filter (8).
(2) *Identification:*
   (a) Use signals $[y, u]^T$ and x in least squares multi-variable identification minimizing the prediction error

$$\varepsilon(t,\theta) = A(q,\theta) \begin{bmatrix} y(t) \\ u(t) \end{bmatrix} - B(q,\theta)x(t) \tag{14}$$

   where

$$\begin{aligned} A(q,\theta) &= I + A_1 q^{-1} + \ldots + A_{n_a} q^{-n_a} \\ B(q,\theta) &= B_0 + B_1 q^{-1} + \ldots + B_{n_b} q^{-n_b} \end{aligned} \tag{15}$$

   are *matrix polynomials* in q^{-1} and $B(q,\theta)$ decomposes to $B = \left[B_N^T, B_D^T\right]^T$.
   (b) Compute the coprime factor estimate via

$$\begin{bmatrix} N(\theta) \\ D(\theta) \end{bmatrix} = A^{-1}(q,\theta) \begin{bmatrix} B_N(q,\theta) \\ B_D(q,\theta) \end{bmatrix}. \tag{16}$$

Obviously a general matrix polynomial $A(q,\theta)$ in an ARX model structure does not provide a common left divisor in the coprime factorization, thus the McMillan degree of the constructed model $P(\theta) = N(\theta)D^{-1}(\theta)$ is not the same as the McMillan degree of $B_N(q,\theta)$ or $B_D(q,\theta)$. Additional structure on the matrix polynomial may be imposed such that $A(q,\theta)$ is a common left divisor preserving the McMillan degree of the individual coprime factors in the constructed model.

4.1 Structured linear regression parametrization for coprime factor identification

As a special case of prediction-error identification methods (PEM), the well-known least squares minimization criteria is a standard choice for its convenience in both computation and analysis (Ljung 1999). Consider the multi-variable ARX model structure (14). Then the system description is given by

$$\begin{bmatrix} y(t) \\ u(t) \end{bmatrix} = G(q,\theta)x(t) + H(q,\theta)e(t) \tag{17}$$

with

$$G(q,\theta) = A^{-1}(q,\theta)B(q,\theta), \quad H(q,\theta) = A^{-1}(q,\theta).$$

Recall that (14) can be parametrized by a linear regression with prediction error given by

$$\varepsilon(t,\theta) = \begin{bmatrix} y(t) \\ u(t) \end{bmatrix} - \varphi^T(t)\theta \tag{18}$$

where additional structure may be imposed on the parametrization such that a d-dimensional column vector θ and a corresponding $n_y + n_u \times d$ matrix $\varphi^T(t)$ containing past input, output and auxiliary signals are used in the least squares minimization. Employing the linear regression prediction error (18), the least squares criterion is given by

$$V_N(\theta, Z^N) = \frac{1}{N}\sum_{t=1}^{N} \varepsilon_f^T(t,\theta)\varepsilon_f(t,\theta) \tag{19}$$

where $\varepsilon_f(t,\theta) = L(q)\varepsilon(t,\theta)$ with $L = diag(L_y, L_u)$ and $L \in \mathcal{RH}_\infty$. Filtering the prediction error can be made equivalent to filtering the identification input-output data, however in the multivariable case all signals must be subject to the same filter, i.e. L_y and L_u must be multiples of the identity matrix (Ljung 1999). Different prefilters L_y and L_u account for the difference between the noise models made available from data (6), (7) where knowledge of the controller can be included into L_u. Prefiltering the input-output data, (19) remains quadratic in θ and can be minimized analytically giving

$$\hat{\theta}_N^{LS} = \left[\frac{1}{N}\sum_{t=1}^{N}\varphi(t)\varphi^T(t)\right]^{-1} \frac{1}{N}\sum_{t=1}^{N}\varphi(t)\begin{bmatrix} y(t) \\ u(t) \end{bmatrix}. \tag{20}$$

Additional structure imposed on the parametrization of the matrix polynomial $A(q,\theta)$ provides a common left divisor and preserves the McMillan degree of the constructed coprime factors in the constructed model.

Proposition 3. Consider minimizing the least squares identification criterion (19) with the prediction error $\varepsilon(t,\theta)$ in (14). Let the matrix polynomial $A(q,\theta)$ be parametrized by

$$A_i = a_i I_{n_y+n_u} \tag{21}$$

for $i = 1, ..., na$. Then the prediction error can be written into linear regression form (18) with

$$\theta^T = \left[a_1 ... a_{n_a} \; col(B_1)^T ... col(B_{n_b})^T\right] \tag{22}$$

$$\varphi^T(t) = \begin{bmatrix} -y(t-1) ... - y(t-n_a) \\ -u(t-1) ... - u(t-n_a) \end{bmatrix} \tag{23}$$
$$x^T(t-1) \otimes I_{n_y} ... x^T(t-n_b) \otimes I_{n_y}\Big]$$

where the *col* operator stacks the columns of a matrix and $\otimes$ denotes the Kronecker product.

Increasing the order of $A(q,\theta)$ does not sacrifice the order of the model being estimated. Thus estimating a high order $A(q,\theta)$ which incorporates the noise filter into the estimation and can improve the fit of the coprime factors ((Ljung 1999)). As a result the least squares solution (20) provides an estimate of the coprime factorization (16) such that $A(q,\theta)$ preserves the McMillan degree of the coprime factors in the constructed model $P(q,\theta) = N(\theta)D(\theta)^{-1}$. For SISO systems the above proposition results in a parametrization

$$\begin{aligned} N(\theta) &= a^{-1}(q)b_N(q) \\ D(\theta) &= a^{-1}(q)b_D(q) \end{aligned} \tag{24}$$

where a, b_N and b_D are (matrix) polynomials of specified degree with coefficients collected in the parameter vector θ. For MIMO the diagonal form of $A(q,\theta)$ is equivalent to a common denominator parametrization. By imposing the structure (21), the factorization $(N(\theta), D(\theta))$ has a common denominator which guarantees the McMillan degree of the coprime factorization is the same for the constructed model. Similar results are obtained when performing the least squares identification using an output error model structure (Van Den Hof *et al.* 1995), however one looses the computational benefits and unique analytic solution (20) provided by the linear regression.

4.2 Description of the limit model

Observe the filtered prediction error under the constrained ARX model structure

$$\varepsilon_f(t,\theta) = L(q)A(q,\theta)\begin{bmatrix} y(t) - N(q,\theta)x(t) \\ u(t) - D(q,\theta)x(t) \end{bmatrix}. \tag{25}$$

With fixed noise model (4.1) the asymptotic parameter estimate $\theta^* = \lim_{N\to\infty} \hat{\theta}_N^{LS}$ is characterized by

$$\theta^* = \arg\min_{\theta} \int_{-\pi}^{\pi} |L(e^{j\omega})|^2 |A(e^{j\omega})|^2 \times \tag{26}$$
$$\left\{ |N_0(e^{j\omega}) - N(e^{j\omega},\theta)|^2 + |D_0(e^{j\omega}) - D(e^{j\omega},\theta)|^2 \right\} \Phi_x(\omega) d\omega$$

where $\Phi_x(\omega)$ is the frequency spectrum of the auxiliary input signal (5). As a result of the ARX model structure chosen the asymptotic parameter estimate includes an implicit high-frequency weighting by $|A(e^{j\omega})|^2$. To enhance the model fit in the desired frequency range the prediction error is filtered through a low-pass filter $L(q)$. If a reasonable assumtion is that the measurement errors are white this may also suggest the Steiglitz-McBride method for improving the estimate with a θ-dependent prefilter (Ljung 1999).

5. RESULTS

5.1 Motivation

The development of high performance controllers of industrial servomechanical systems with significant product variations may require accurate modeling of each product on the basis of its own closed-loop experiment. Consider the frequency responses of several of the same servomechanism product presented in Figure 3.

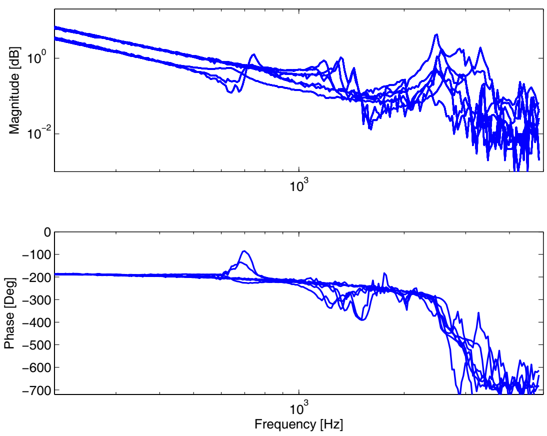

Fig. 3. Frequency response of 16 of the same servomechanism product

The product variation between the servomechanisms would require too conservative a controller satisfying stability over all plants. The results presented in this section illustrate the proposed identification method applied to one test bench, see Figure 4, with the intension of using the model in future work to design a high performance model based controller. Typically servomechanisms contains a double integrator, which makes it open-loop unstable. The current feedback loop is stabilized via a proportional integral derivative (PID) controller however the large resonance modes of the plant limit the bandwidth of the closed-loop system.

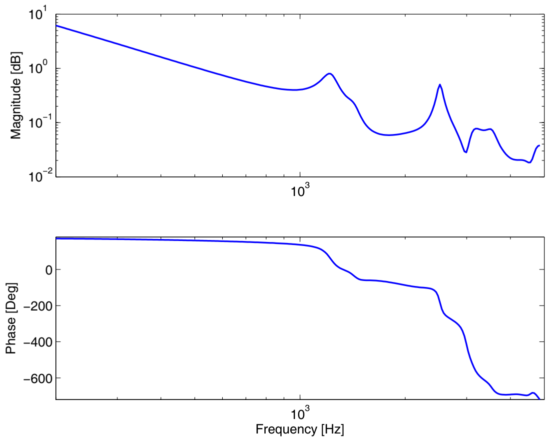

Fig. 4. Frequency response of a 20^{th} order case study model (solid-line).

5.2 Case study

As a case study to illustrate the proposed identification method consider the frequency response presented in Figure 4. Time series data $u(t)$ and $y(t)$ were obtained via simulation of a 20^{th} order model fitted to the frequency response with input signals r_0 and r_1 chosen as zero mean white noise each with a variance of 1. The measurement noise that enters the system $v = H_0 e_0$ was modeled as zero mean white noise with variance 0.1.

The identification algorithm outlined in Section 4 is used to estimate normalized coprime plant factors. Results from the last step are presented in Figure 5, where approximately normalized coprime factor estimates $(N(\theta), D(\theta))$ are obtained from a constrained ARX least squares linear regression identification. Because of the implicit high frequency weighting which results from using an ARX model structure, the prediction error prefilter $L(q)$ is initially chosen as a 4^{th} order low-pass butterworth filter with a cut-off at around half the sampling frequency. To improve the quality of the estimated factors the linear regression identification is performed a second time with prefilter chosen according to the Steiglitz-McBride method, $L(q) = A(q,\theta)$, where the prefilter is applied to the original data set. The resulting model constructed from the approximately normalized coprime factors $P(\theta) = N(\theta)D^{-1}(\theta)$ is presented in Figure 6. The computational complexity involved in linear filtering and computing the linear regression identification is less than a non-linear optimization identification of the same order, yet these results show that for this experimental set-up and for sufficiently chosen order the resulting model is comparable. Additional benefits of using the constrained linear regression identification come from the vast body of research available, i.e. so-called "fast algorithms" and the ability to compute estimates of multiple (high) orders.

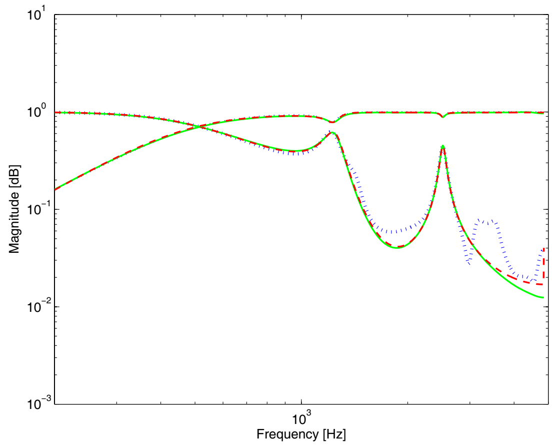

Fig. 5. Frequency response of coprime factors computed from case-study model (dotted-line) and from 7^{th} order: constrained ARX (solid-line), Prediction Error minimization with OE structure (dashed-line).

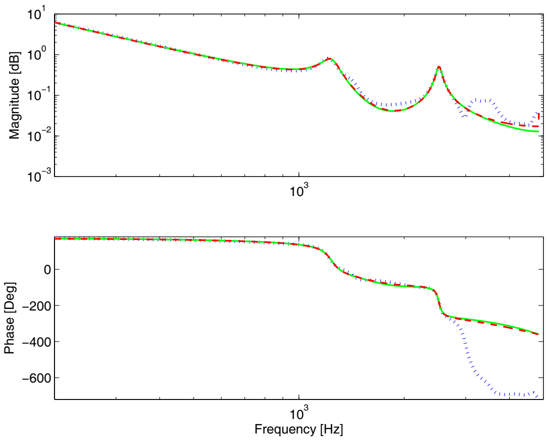

Fig. 6. Frequency response of case study model (dotted-line) and 7^{th} order constructed plant: $P(\theta) = N(\theta)D^{-1}(\theta)$ (solid-line) and Prediction Error minimization with OE structure (dashed-line).

6. CONCLUSION

This paper presented an extension of the work done in (Van Den Hof *et al.* 1995) for estimating approximately normalized coprime factors using linear (regression) identification methods. The proposed algorithm shows that a constrained ARX model structure maintains a linear regression form and preserves the McMillan degree of a constructed model from its coprime factors. The results demonstrate that the proposed identification algorithm effectively estimates coprime plant factors from closed-loop data. Future work will be to extend the proposed linear regression method further to include multivariable system identification.

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