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FREQUENCY DOMAIN SUBSPACE IDENTIFICATION OF A TAPE SERVO SYSTEM

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Introduction

As track density in tape recording systems increases, the need for high performance control becomes crucial due to significant high frequency disturbances such as lateral tape motion, inner track variability, servo track variability, noise and input saturation of the servo actuator. An essential step in designing high performance model based controllers is the identification of low order, control-oriented models. The objective is to identify not only the plant, but also the sensitivity function which is a measure of the closed-loop disturbance rejection.

The contributions of this work are to analyze the use of weighting functions in frequency domain subspace algorithms [3] for control-oriented identification. The control-relevant weighting is achieved through frequency weighted balanced truncation techniques [3]. Two subspace identification algorithms are developed for equidistantly and arbitrarily spaced frequency response data with implementation in a frequency domain identification toolbox for Matlab, FREQID [1]. The focus of this paper is the algorithm for which the frequency response data is equidistantly spaced along the frequency axis. The subspace identification techniques are applied to a modified tape drive where experimental results demonstrate the usefulness of the method in control relevant estimation.

Subspace Identification

In discrete-time any system S can be written in terms of its impulse response as

$$S : y(t) = \sum_{k=0}^{\infty} g_k u(t-k)$$

where $u(t) \in \Re^m$, $y(t) \in \Re^p$ denote the input/output signals and $g_k \in \Re^{p\times m}$ are the impulse response coefficients. If the system is of finite order n, the system can be described via state space realization

$$x(t+1) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

where the frequency response can be written as a function of the state matrices as follows

$$G\left(e^{j\omega}\right) = \sum_{k=0}^{\infty} g_k e^{-j\omega k} = C\left(e^{j\omega}I - A\right)^{-1} B + D .$$

Note that the state space realization (A,B,C,D) is not unique and all realizations are related via similarity transformation. Given frequency response samples $G_k\left(e^{j\omega_k}\right)$ at frequencies ω_k ($k = 0,...,M$), the objectives of the identification algorithm are as follows.

- Determine the McMillan degree n of the system
- Estimate a realization (*A*, *B*, *C*, *D*) of the discrete time space model

Frequency domain subspace identification algorithms are based on linear algebra results and provide non-iterative methods for directly realizing state-space models. Depending on the spacing of frequency data, either equidistant or arbitrarily spaced frequency data, the subspace identification requires slightly different algorithms. In case frequency data is equidistantly spaced, the algorithm is strongly consistent under weak noise assumptions.

The main linear algebra tool used in subspace identification methods is the singular value decomposition (SVD) for which efficient algorithms exist. Frequency response measurements $G\left(e^{j\omega}\right)$ sampled at equidistant frequencies ω_k are used to construct the system Hankel matrix, denoted by $\hat{H}$, by constructing the impulse response via the inverse discrete fourier transform (IDFT).

$$\hat{H} = \begin{bmatrix} \hat{h}_1 & \hat{h}_2 & \cdots & \hat{h}_r \\ \hat{h}_2 & \hat{h}_3 & \cdots & \hat{h}_{r+1} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{h}_q & \hat{h}_{q+1} & \cdots & \hat{h}_{r+q-1} \end{bmatrix} \in \mathbb{R}^{qpxrm} \quad n < q \le M,\ \ n \le r \le M$$

Where: $\hat{h}_i = \dfrac{1}{2M} \sum_{k=0}^{2M-1} G_k e^{j2\pi ik/2M}$

In the noise-free case, the system Hankel matrix has rank n, the dimension of the state space realization. Performing an SVD operation on the Hankel matrix, $\hat{H} = \hat{U}\hat{\Sigma}\hat{V}^T$ the order of the

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system can be determined by inspecting the resulting non-zero singular values. Under noisy frequency response measurements all singular values will be non-zero and a decision for the order of the system is made by looking at the singular value spectrum and separating the large and small values

$$\hat{H} = \begin{bmatrix} \hat{U}_s & \hat{U}_o \end{bmatrix} \begin{bmatrix} \hat{\Sigma}_s & 0 \\ 0 & \hat{\Sigma}_o \end{bmatrix} \begin{bmatrix} \hat{V}_s^T \\ \hat{V}_o^T \end{bmatrix} \cong \hat{U}_s \hat{\Sigma}_s \hat{V}_s^T$$

where Σ_s contains the n largest singular values, which is used for approximating $\hat{H}$ with a lower dimensional matrix. It can be shown that the matrix $\hat{U}_s$ has the same range space as the Hankel matrix $\hat{H}$ and thus provides a realization for an extended observability matrix O_q [3] defined as

$$O_q = \begin{pmatrix} C \\ CA \\ \vdots \\ CA^{q-1} \end{pmatrix} \in \mathbb{R}^{qp \times n}$$

where $q > n$, the order of the system. The state space matrices (A,C) are extracted from the matrix U_s available from the SVD operation. Matrices (B,D) can be determined by solving the least squares problem:

$$\hat{B}, \hat{D} = \arg \min_{\substack{B \in \mathbb{R}^{n \times m} \\ D \in \mathbb{R}^{p \times m}}} \sum_{k=0}^{M} \left\| G_k - D - \hat{C}\left(e^{j\omega_k} I - \hat{A}\right)^{-1} B \right\|_F^2$$

Where $\|X\|_F^2 = \sum_k \sum_s |x_{ks}|^2$ denotes the Frobenius norm

Since the state space realization is not unique any other realization can be obtained via a coordinate transformation.

Subspace identification methods have some important advantages over more traditional identification techniques such as optimization algorithms [2], where the system is determined via parametric optimization of some criterion. Subspace identification provides the following advantages.

- Provides minimal realization, i.e. the model is both controllable and observable.
- Directly estimates state-space model compatible with modern control design techniques.
- Straight-forward MIMO estimation since there is no need for explicit parameterization.
- Based on linear algebra results avoiding non-linear parametric optimization problems

However as a disadvantage, subspace algorithms do not minimize any criterion, which makes control-relevant estimation challenging. Frequency weighted balanced truncation can be implemented in subspace algorithms as a means for introducing control-relevant weighting into the identification

Frequency Weighted Balanced Truncation

Frequency weighting in identification algorithms places emphasis on modelling some frequency ranges more accurately than others. In subspace identification the frequency weighted balanced truncation technique, as analyzed in [4] and further developed in [5], can be used to achieve control-relevant weighting of the identified models.

The problem is finding a state space realization of the system so that the error $\left\| F_y\left(e^{jw}\right)\left(\hat{G} - G\right) F_u\left(e^{jw}\right) \right\|_\infty$ is "small", where $F_y\left(e^{jw}\right)$ and $F_u\left(e^{jw}\right)$ are the (user specified) output and input weighting matrix, $\hat{G}$ the estimated model of the frequency response and G the measurements. The first step of the technique is constructing full rank block toeplitz matrices $\boldsymbol{F_u}, \boldsymbol{F_y}$ that contain the weighting information using state space realizations of $F_u\left(e^{jw}\right)$ and $F_y\left(e^{jw}\right)$.

When the SVD of $\boldsymbol{F_y}\hat{H}\boldsymbol{F_u}$ is taken instead of $\hat{H}$, there is no loss of information since

$$rank(F_y \hat{H} F_u) = rank(\hat{H}) = rank(O_q)$$

It can be shown that multiplying the Hankel matrix by weightings influences the state space basis of the observability range space and that the observability matrix of the sought realization can be written as $\boldsymbol{F_y}^{-1}\hat{U}_s\hat{\Sigma}_s^{1/2}$, where the SVD has once again been divided into a term with large and a term with small singular values:

$$\boldsymbol{F_y}\hat{H}\boldsymbol{F_u} = \hat{U}_s\hat{\Sigma}_s\hat{V}_s^T + \hat{U}_o\hat{\Sigma}_o\hat{V}_o^T \,.$$

The rest of the algorithm remains more or less the same, but instead of using $\hat{U}_s$ in the formulas of the subspace technique, one proceeds with $\boldsymbol{F_y}^{-1}\hat{U}_s\hat{\Sigma}_s^{1/2}$.

Applying the frequency weighted balanced truncation technique will result in a realization of the system, which has the diagonal matrix of weighted Hankel singular values as both controllability and observability grammian

The efficiency of the method is best shown by an example. Figure 1 shows the results of the estimation of a 4th order system by a second order model. In the plot on the left a low pass butterworth filter was used as weighting filter while in the plot on the right a high pass butterworth filter was used. The estimated model certainly reflects the frequency range of importance.

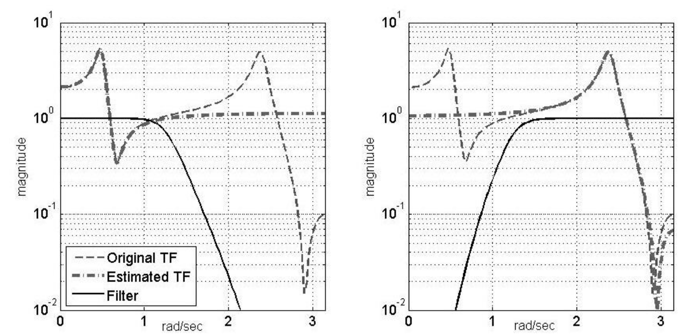

Fig.1 Estimation of a 4th order system by a 2nd order model

Tape drive setup

The test-setup consists of a modified tape drive to allow for injection of a reference signal (R) into the closed loop system. Measurements of this reference signal, the controller input (PES) and servo actuator input (U), are then used to retrieve both the servo actuator's dynamics (P) and the sensitivity function (S). If the coherence between the reference signal and the actuator input is high enough, the influence of the other disturbances on the PES will be negligible and the transfer function is:

$$\frac{\phi_U(\omega)}{\phi_R(\omega)} = \frac{1}{1+P(e^{i\omega})C(e^{i\omega})} = S(e^{i\omega})$$

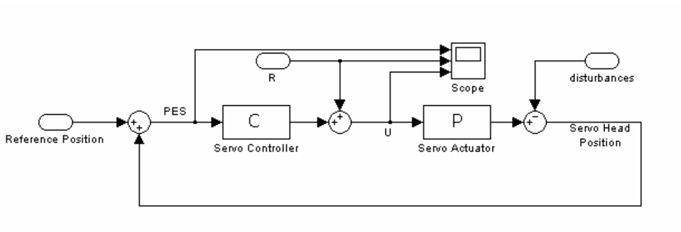

Fig.2 Schematic of the servo system and relevant signals

Using knowledge of the controller and measurements of the sensitivity function, the plant frequency response can be extracted. By using an appropriate weighting, an 11th order subspace model can be found for the plant, shown in Figure 3.

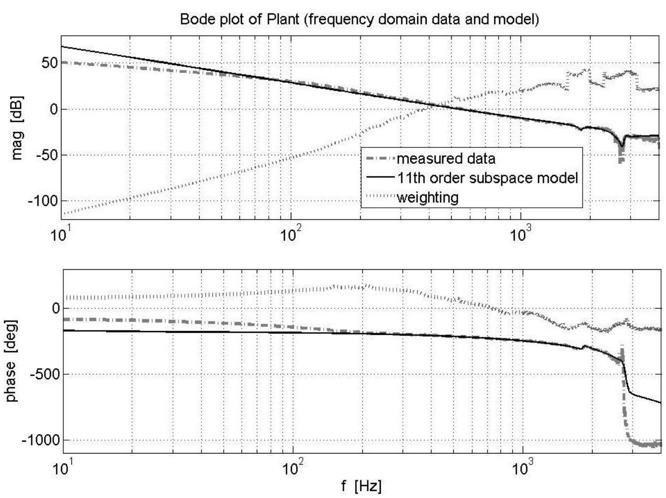

Fig.3 11th order subspace model of the plant

Figure 4 shows in detail the high-frequency fit of the plant model to the frequency response data.

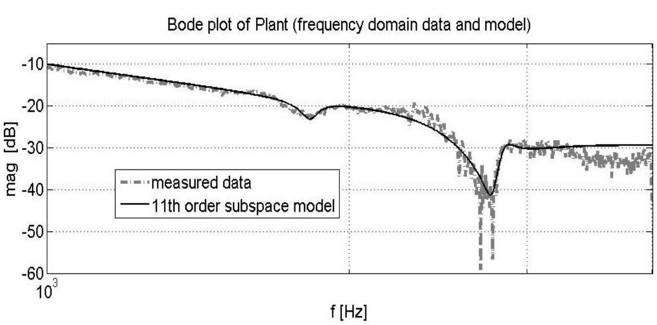

Fig.4 High frequency response of the plant

An advantage of subspace identification techniques is the selection of the model order. From the singular values of the Hankel matrix one can determine the model order such that the truncated model captures most of the relevant dynamics. Observe from Figure 5 that a decrease in magnitude occurs after the 11th singular value, indicating that an 11th order model sufficiently captures the dynamics represented in the frequency response. Truncating the Hankel matrix assumes that the non-zero singular values after the 11th are due to noise.

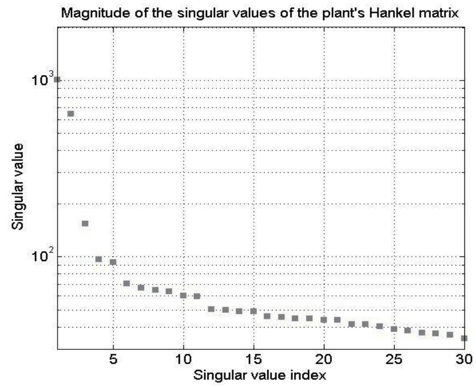

Fig.5 Magnitude of the singular values of the plant Hankel matrix

Conclusions and Future Work

Although subspace identification methods are primarily used for consistent estimation, they are also suitable for control relevant estimation, as long as an appropriate weighting filter is used. The latter can be accomplished by the "frequency weighted balanced truncation" technique. The algorithm is successfully applied to a tape drive and allows to use the servo actuator's model for control design in a later stage.

One of the drawbacks of the implemented subspace algorithm is the need for equidistant data. Indeed, the impulse responses of the system are approximated through an IDFT. Luckily this can be circumvented by using a different approach for finding a matrix with the same range space as O_q, based on projections.

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