Modeling and Low-Order Control of Hard Disk Drives With Considerations for Product Variability

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To maximize bandwidth for a hard disk servo system in the presence of modeling uncertainty, a tradeoff between performance and controller complexity is addressed by control-relevant modeling and robust control. Control-relevant modeling is accomplished by using the information of the appropriate weighting functions used in the control design to find a model via frequency domain curve fitting. This paper illustrates control relevant modeling using frequency domain data and design of a high bandwidth servo system in the presence of modeling uncertainty. Results using real-time implementation show agreement between the designed and achieved bandwidths.

***Index Terms*—Digital control, modeling, servosystems.**

I. INTRODUCTION

As STORAGE capacity of hard disk drives (HDDs) increases, the requirements for position control of the read/write head becomes more challenging to satisfy [1], especially for control firmware that is required to operate in the presence of product variability.

Performance considerations under product variability are well suited under the worst case $\mathcal{H}_\infty$-norm based optimal control which allows robustness issues to be incorporated into the servo design [2] in the form of uncertainty models. Uncertainty models, as well as restrictions on the complexity of the controller to be used, make the design of a feedback control law for disk drive servo systems a challenging task.

Recent methods for fixed-structure $\mathcal{H}_\infty$ control optimization have been developed [3] where the poles of the controller are specified *a priori* in a proportional integral derivative (PID) structure. Fixing the poles allows for convex optimization techniques in determining the zeros of the controller. As disk drive servo systems move toward more advanced structured controllers, the location of poles in the controller is no longer intuitive and optimization techniques are generally computationally intensive [4].

The contribution of this work is to incorporate product variability based on model uncertainty into low-order control design via control-relevant model estimation and closed-loop controller reduction. The control design is based on standard $\mathcal{H}_\infty$ optimization and motivates the control-relevant identification problem. The method requires an upper bound on the characterization of model uncertainty as well as specification of a performance weighting function that directly relates to the closed-loop transfer functions of interest. The approach is demonstrated via implementation on a production disk drive servo system.

II. ROBUST CONTROL DESIGN

For improved track following performance the bandwidth of the servo system should be increased, however this is limited by sampling frequency and mechanical resonances of the head/disk assembly. Bandwidth and disturbance rejection are characterized by the sensitivity function

$$S = (1 + PC)^{-1} \tag{1}$$

where P denotes a dynamical model of the voice coil motor (VCM) with flexibilities of the E-block and suspension and C denotes the VCM servo controller. The desired performance characterized by the shape of the sensitivity function is captured in the weighting function W_S, which is specified in terms of bandwidth ω_b and maximum disturbance amplification M_s requirements in the form

$$W_S = \left(\frac{s/\sqrt[k]{M}_s + \omega_b}{s + \omega_b\sqrt[k]{\epsilon}}\right)_k \qquad \text{for } \geq 1 \tag{2}$$

where ϵ determines the level of steady-state error rejection. It should be noted that the parametrization given above as a continuous-time transfer function can be converted to discrete-time via a bilinear transformation.

Control design such that $\|W_S S\|_\infty < 1$ guarantees that the sensitivity function is bounded above by the desired shape W_S^{-1}. Minimization of this cost function alone is not practical as it leads to infinite controller gains. In practice it is useful to also consider a bound on the transfer function between the disturbance and control signal $\|CS\|_\infty < 1$ as a mechanism for limiting the size and bandwidth of the controller by considering robust stability with respect to additive plant uncertainty [5]. An upper bound on model uncertainty can account for low-frequency effects such as friction as well as neglected high-frequency dynamics within product variation in flexible HDD suspensions. Consider a set of models $\mathcal{P}$ consisting of a nominal model P along with an upper bound allowable additive model perturbation W_A

$$\mathcal{P} = \{P | P + \Delta W_A, \|\Delta\|_\infty \leq 1\} \tag{3}$$

such that the real system, represented by P_0, is contained in the model set $P_0 \in \mathcal{P}$ Given an upper bound on the additive plant uncertainty W_A then stability robustness is satisfied when $\|W_A CS\|_\infty < 1$ yielding the desired bound on control energy. Combining stability robustness and performance, a performance robustness condition can be constructed. Consider a performance weight of the form (2) with M_s fixed, then a mixed

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sensitivity optimization problem for finding a stabilizing controller to achieve maximum bandwidth ω_b can be proposed as

$$\max |\omega_b| \text{ s.t.} \left\| \begin{matrix} W_S S \\ W_A CS \end{matrix} \right\|_\infty < 1. \tag{4}$$

The above optimization may be implemented as an outer loop around standard mixed-sensitivity $\mathcal{H}_\infty$ control design [6]. For the purpose of robust control design described in (4), control-relevant estimation of the set $\mathcal{P}$ is considered by means of system identification techniques such that the product variation is captured in the set of models (3).

III. Control-Relevant Modeling

A. Estimation of a Nominal Model

Disk drive actuator dynamics are commonly modeled from an intuitive modeling of a flexible structure as

$$P(s) = K_{VCM} \frac{\omega_l^2}{s^2 + \zeta_l \omega_l + \omega_l^2} \cdot \frac{\omega_h^2}{s^2 + \zeta h \omega_h + \omega_h^2} \tag{5}$$

where low-frequency parameters, $(K_{VCM}, \zeta_l, \omega_l)$ modeling effects of the VCM motor, and high-frequency parameters, (ζ_h, ω_h) modeling flexibility in the suspension, are tuned to provide reasonable representation of the HDD dynamics. What is not clear, however, is whether this model (5) is relevant for control design.

The link between modeling and control is established by considering the difference between the designed performance (4) and the achieved performance of the controller implemented on the real system. In order to design an enhanced performing robust controller, it is preferable to estimate a set of models $\mathcal{P}$ for which this difference is minimized with respect to the nominal model P and uncertainty weight W_A. Consider minimizing additive uncertainty, i.e., making $W_A = 0$, then a control-relevant identification criteria for the mixed-sensitivity objective in (4) is given by [7]

$$\|W_S(1 + P_0 C)^{-1} - W_S(1 + PC)^{-1}\|_\infty \tag{6}$$

such that the achieved closed-loop performance of the plant P_0 with the controller C is close to the designed closed-loop performance of the model P with controller C. Identification of a model P from the control-relevant criteria (6) can be written into a weighted additive difference between the actual VCM dynamics P_0 and the nominal model P via

$$\|W_S(l + P_0 C)^{-1}(P_o - P)C(l + PC)^{-1}\|_2 \tag{7}$$

motivated by the fact that $\mathcal{L}_2$ approximation tends to $\mathcal{L}_\infty$ approximation [8].

The weighted additive difference between P_0 and the nominal model P can be written in terms of frequency domain criterion. When using experimental frequency domain data $P_0(\omega_k)$, where $\omega_k, k = 1, 2, \ldots, N$ refers to a frequency domain grid, the two-norm criterion in (7) can be approximated by

$$\sum_{k=1}^{N} |(P_0(\omega_k) - P(\omega_k))W(\omega_k)|^2$$

where the frequency weighting $W(\omega k)$ given by

$$W(\omega_k) = \frac{W_S(\omega_k)}{1 + C(\omega_k)P_0(\omega_k)} \cdot \frac{C(\omega_k)}{1 + C(\omega_k)P(\omega_k)}. \tag{8}$$

From (8), it can be observed that the frequency weighting requires information of the actual sensitivity function $(1+CP)^{-1}$ and the sensitivity function $(1 + CP)^{-1}$ based on the model to be estimated. Data of the actual sensitivity $(1+CP_0)^{-1}$ is easily constructed from closed-loop experiments using the controller C or can be computed from the open-loop frequency response data $P_0(\omega_k)$ and the controller frequency response $C(\omega_k)$. The weighting $(1+CP)^{-1}$ can only be computed once the frequency response of a model $P(\omega_k)$ is available. However, an iterative update of the frequency weighting can be used to update the weighting $(1 + CP_i)^{-1}$ on the basis of a model P_i.

As a result, a discrete-time model $P(q, \theta)$ can be found by a standard least squares minimization [9]

$$\hat{\theta} = \min_{\theta} \sum_{k=1}^{N} E_i(\omega_k, \theta_i)^* E_{i-1}(\omega_k, \theta_i)$$

in which the curve fit error

$$E_{i-1}(\omega_k, \theta_i) = (P_0(\omega_k) - P(\omega_k, \theta_i))W_{i-1}(\omega_k),$$
$$W_{i-1}(\omega_k) = \frac{W_S(\omega_k)}{1 + C(\omega_k)P_0(\omega_k)} \cdot \frac{1}{1 + C(\omega_k)P_0(\omega_k, \hat{\theta}_{i-1})}$$

is being minimized. Computation of a finite order discrete-time model $\hat{P}(q) = P(q, \hat{\theta}_i)$ using the above weighting function iteration is done with the (nonlinear) least squares optimization techniques available in [9].

B. Estimation of Additive Uncertainty Bounds

The open-loop frequency domain data $P_0(\omega_k)$ can be used to account for modeling and approximation errors made by the nominal model $\hat{P}(q)$ obtained by the frequency weighted least squares identification. To facilitate the design of a robust performing feedback controller, such modeling errors are considered as model perturbations to account for possible variations in the nominal response modeled by $\hat{P}(q)$.

Estimation of an allowable model perturbation involves the characterization of an upper bound on Δ in (3) via W_A such that $\|W_S S\|_\infty$ is minimized and $P_0 \in \mathcal{P}$. A frequency dependent upper bound is available via a model error model identification between the control signal and prediction error residual [10]. A control-relevant frequency dependent upper bound can be obtained such that

$$\|\Delta\|_\infty \leq \delta(\omega) \text{ with prob. } \geq \alpha. \tag{9}$$

Subsequently, spectral over bounding routines [11] can be used to construct a limited complexity stable and stably invertible weighting filter W_A that over bounds $\delta(\omega)$.

IV. Application Disk Drive

The experimental system consists of a 2.5-in disk drive with 120 sectors and rotational speed at 4200 rpm giving it a servo sampling frequency f_s of 8.4 kHz. The disk drive servo processor is replaced by a DSP and host computer that allow access to parameters in a general control transfer function. The feedback control law for the existing drive is given by the discrete-time PID controller

$$u(t) = k_p e(t) + k_i \sum_{i-0}^{t} e(i) + k_d[e(t) - e(t-1)] \tag{10}$$

and has an initial bandwidth of 600 Hz. The objective is to increase the bandwidth but restrict the complexity of the feedback

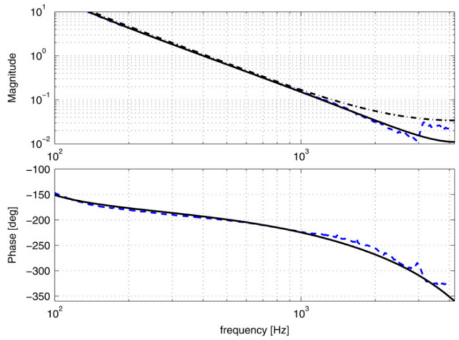

Fig. 1. Frequency response of the measured plant (dashed-line) and a nominal model (solid-line) with additive uncertainty (dash-dotted-line). (Color version available online at http://ieeexplore.ieee.org.)

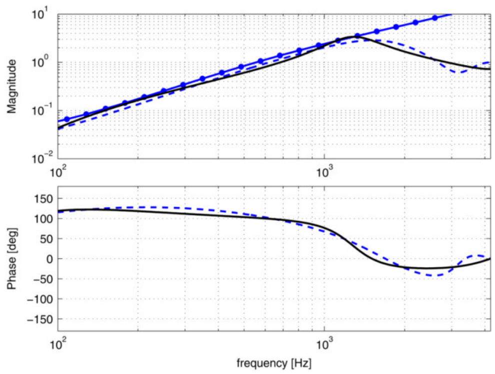

Fig. 2. Performance weight (solid-circles-line) and the sensitivity functions resulting from full-order $\mathcal{H}_\infty$ control synthesis (dashed-line) and reduced-order controller (solid-line). (Color version available online at http://ieeexplore.ieee.org.)

controller to a general second-order transfer function so that fair comparison can be made with the PID controller with the same order

$$C(q) = \frac{b_0(q+b_1)(q+b_2)}{(q+a_1)(q+a_2)} \tag{11}$$

where q denotes the time-shift operator $u(t+1) = qu(t)$.

A control-relevant third-order nominal model is obtained from an estimation with identification criteria (7). Using the nominal model, a second-order upper bound on the additive uncertainty is obtained with probability 99.9% by identifying a model error model and estimating a low-order spectral over bound. The nominal model and additive uncertainty over bound, which accounts for neglected high-frequency dynamics and low-frequency friction effects, are presented in Fig. 1.

The performance weight W_S with structure given by (2) was with fixed maximum allowable disturbance amplification M_s. The bandwidth of the performance weight was maximized in an outer loop around a standard $\mathcal{H}_\infty$ controller synthesis problem [5]. The design of a controller as mentioned in Section II generally leads to full order controllers, which in this case resulted in an eighth-order controller. Although the limited controller complexity has been addressed via low-order control-relevant model estimation, a reduction of the controller may be required where closed-loop balanced model reduction methods [12] can account for the performance objective function given by (4).

The full-order controller was reduced to a second-order controller via closed-loop balanced model reduction techniques. The designed sensitivity functions are shown in Fig. 2 for the full-order and reduced-order controllers. It can be seen that although the full-order controller satisfies performance requirements the reduced controller does not, however the lost bandwidth is considered minor and a new performance weight could be used to validate performance.

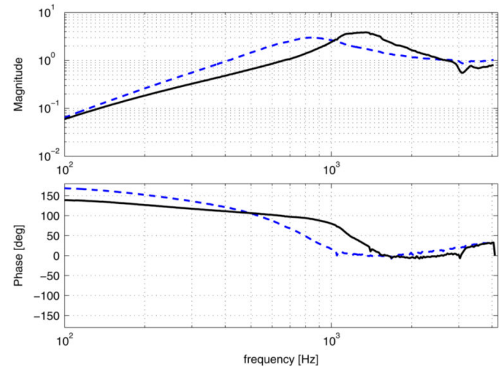

Fig. 3. Measured disk drive sensitivity functions resulting from servo controllers PID (dashed-line) and reduced-order Hoc (solid-line). (Color version available online at http://ieeexplore.ieee.org.)

Although the designed performance has been improved via $\mathcal{H}_\infty$ control techniques, the improvement in achieved performance should be the driving factor for the new controller. Frequency response measurements of the disk drive sensitivity functions operating under feedback with PID controller and $\mathcal{H}_\infty$ controller are compared in Fig. 3.

The achieved frequency responses are similar to the designed and demonstrate that larger bandwidths are possible with $\mathcal{H}_\infty$ control design techniques at the price of slightly more complex servo control law.

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