Iterative Learning for Periodic Measurement Disturbance Cancellation with Application to Disk Drive Head Testing

M.R. Graham¹, R.A. de Callafon¹, L. Shrinkle²

¹University of California San Diego, Mechanical and Aerospace Engineering Dept.
9500 Gilman Drive, La Jolla, 92126, U.S.A.
mgraham@ucsd.edu, callafon@ucsd.edu

²Headway Technologies Inc.
678 South Hillview Drive, Milpitas, 95035, U.S.A.

Introduction
The challenges of compensating for periodic disturbances appears in various applications dealing with rotating machinery. For example, in hard disk drive (HDD) servomechanisms disturbances consist of both repeatable and non-repeatable nature appear in the position error signal (PES) of the head following a data track. The repeatable run-out (RRO) disturbance generally occurs at frequencies that are integer multiples of the frequency of rotation of the disk and is a considerable source PES with respect to the center of the data track [1].

Improved performance by the feedback system when the output measurement is perturbed by periodic disturbances leads to undesirable repeatable tracking errors. As an example, progress in HDD servo track writers has led to a two stage servo track writing process where a master servo disk, created in stage one, is used as a reference from which the servo tracks on the remaining disks in the stack are written in stage two [2]. Repeatable run-out either written-in during the servo track writing process or resulting from mechanical disk assembly can be eliminated by considering them as sources of periodic measurement noise and canceling them via a modified reference signal. Thus the objective of the servomechanism is not to follow the RRO error, but to follow a virtual perfectly circular track thereby reducing AC-squeeze of data track following. A learning algorithm designed for this purpose has been successfully applied to disk drive systems by [3], however knowledge of a nominal plant model and access to the control signal were required. The result was a reference signal called the zero acceleration path (ZAP) that canceled the RRO allowing the read/write head of the disk drive to follow a virtual circular track with zero actuator acceleration.

Compared to [3], the motivation for this study is to generalize periodic disturbance cancellation results for systems without knowledge of a nominal plant model or access to the control signal. This paper considers iterative learning control methods for determining disturbance canceling reference signals (DCRS) providing cancellation with periodic measurement disturbances based on closed-loop models.

ILC Configuration
Iterative learning control methods applied to systems have primarily focused on compensating for periodic disturbances via a modified control signal. Thus the control system has been redesigned with a feedforward component added to the system input [4,5]. Although this ILC structure is most commonly studied in the literature, modification of the reference signal in a cascaded ILC system can achieve improved control performance for systems where it is undesirable to reconfigure the control signal [6]. A block diagram representation for ILC with the cascaded structure is shown in Figure 2.

Fig. 2 Illustration of cascade ILC system.
Notice from Figure 2 that the ILC scheme is added outside the existing control loop through a modified reference signal, \( r_i(t) - z_i(t) \). The ILC update law is classified under previous cycle learning (PCL) whereby the current iteration of the adjunct reference signal \( z_{i+1}(t) \) is some form of signals from previous iterations:

\[
\begin{align*}
    z_{i+1}(t) &= z_i(t) + G_f(q) e_i(t) \\
    z_0(t) &= 0, \quad t \in [0, N]
\end{align*}
\]  

(1)

The learning convergence condition is derived by observing the feedback error evolution from one iteration to the next. Current iteration feedback error is given by:

\[
\begin{align*}
    e_{i+1}(t) &= \hat{r}_{i+1}(t) - \hat{y}_{i+1}(t) \\
    &= S(q)[r(t) - d(t) - z_{i+1}(t)] \\
    &= S(q)[r(t) - d(t) - z_i(t) - G_f(q) e_i(t)] \\
    e_{i+1}(t) &= \left[1 - S(q)G_f(q)\right]e_i(t). \\
\end{align*}
\]  

(2)

The desired convergence \( \lim_{i \to \infty} \| e_i(t) \| = 0 \) is obtained by designing the operator \( G_f(q) \) such that

\[
\| 1 - S(e^{j\omega})G_f(q) e^{j\omega} \|_\infty \leq \rho < 1
\]

(3)

where \( \| G(q) \|_\infty \) denotes the infinity norm of the discrete time transfer function \( G \). Designing \( G_f(q) \) such that condition (3) is satisfied, assures the amplitudes of all frequency components of the error up to the Nyquist frequency \( \omega_N \) decay monotonically at each iteration [7].

**Experimental Results**

The closed-loop servomechanism of disk drives operates by the feedback of PES perturbed by disturbances that are a combination of the repeatable and non-repeatable run-out. In certain HDD applications, such as two-stage servo track writing and read/write head testing, the objective servomechanism is to follow a virtual perfect circle track instead of following the center of the data track. This is equivalent to assuming periodic perturbed measurements of the PES and providing cancellation of the periodic disturbances via the DCRS.

The experimental system consists of a 2.5” form-factor magnetic disk drive spinning at 4200rpm with 120 data sectors and a track pitch of 8µm. The PCL learning system of Figure 2 generates the current iteration signal based on information from previous iterations, where the finite time interval \( t=0, \ldots, N \) corresponds to the number of sectors on the disk. Under the assumptions of ILC, at the end of each finite time interval the initial conditions of the system are resent, thus the information must be repeatable. The periodic disturbance \( d(t) \) includes RRO effects such as the eccentricity of the track that contribute to the PES, \( e(t) \), at integer multiples of the frequency of rotation of the disk.

The disk drive sensitivity function model \( \hat{S}(q) \) is not stably invertible and thus its inverse cannot be used directly as learning compensator design. The model is used to generate closed-loop frequency response \( \hat{S}(e^{j\omega}) \) at the frequencies of the periodic disturbance \( \omega_k \), where \( \omega_k = [70, 140, \ldots, 4200] \) Hz are the integer multiples of the frequency of rotation up to the Nyquist frequency. An FIR model is generated from the inverse of the closed-loop frequency response of \( \hat{S}^{-1}(e^{j\omega}) \). The FIR model \( G_f(q, \theta) \) describes a stable system that exactly intersects the frequency response of \( \hat{S}^{-1}(e^{j\omega}) \) at the frequencies \( \omega_k \), as shown in Figure 5. Since the learning compensator is designed from the frequency response of a model of the sensitivity function, bias error between the model and the true system could result in a violation the learning convergence condition.

![Fig. 5 Frequency response of FIR model \( G_f(q, \theta) \) approximation (solid line) to \( \hat{S}(e^{j\omega}) \) (dotted line) and the intersection of the FIR model with \( \hat{S}(e^{j\omega}) \) at \( \omega_k \) (circles).](image)

As a result of the estimated model \( \hat{S}(q) \) used in designing the learning compensator, the first DCRS \( z_1(t) \) achieves significant reduction in the original PES, \( e_0(t) \). Further iteration of the DCRS according to the ILC update law (1) demonstrates an improved performance with each iteration of the modified reference signal for canceling the periodic measurement disturbance in the PES. Figure 6 shows the DCRS through the fourth iteration and the resulting reductions in the periodic measurement disturbance on the PES.
A close look at the progression of the DCRS shows very little change from the first iteration to the fourth and yet these subtle changes have a significant impact on the resulting PES. As the error approaches zero in the iteration domain, $e_k(t) \rightarrow 0$, the DCRS also displays convergence, $z_{k+1}(t) \rightarrow z_k(t)$, to the adjunct reference signal that directly cancels the effect of the periodic measurement disturbance in the PES feedback with almost zero error.

The effectiveness of the DCRS for canceling periodic disturbances at multiple frequencies is observed in the spectral content of the PES. A comparison between the spectrum of the original PES and the PES with the fourth iteration of the DCRS applied is shown in Figure 8.

It is also important to mention that the non-repeatable disturbances in the disk drive system were neither amplified nor decreased by the application of the ILC algorithm. This results from the cascaded ILC structure where modification of the reference signal does not affect the properties of the closed-loop transfer function.

Conclusions
In this paper iterative learning control for the design of reference signals has been developed for the cancellation of periodic measurement disturbances. The method is illustrated with experimental results on a hard disk drive read/write head tester. As with most control design, with ILC one has to address the issues of complexity of computation and implementation. The contributions of this paper are to discuss the design of the learning compensator and demonstrate experimentally the effectiveness of ILC reference design methods for improving control performance. Although knowledge of the system is not required for the ILC algorithm to work, approximations of the system closed-loop frequency response are used to design learning compensators as FIR model approximations of the system inverse providing for fast rates of convergence in the learning system.

References