Proceedings of IMECE2005
2005 ASME International Mechanical Engineering Congress and Exposition
November 5-11, 2005, Orlando, Florida USA

IMECE2005-82372

FILTERS PARAMETRIZED BY ORTHONORMAL BASIS FUNCTIONS FOR ACTIVE NOISE CONTROL

Jie Zeng
Dynamics Systems and Control
Department of Mechanical and Aerospace Engineering
University of California, San Diego
La Jolla, California 92037
Email: jzeng@ucsd.edu

Raymond de Callafon*
Dynamics Systems and Control
Department of Mechanical and Aerospace Engineering
University of California, San Diego
La Jolla, California 92037
Email: callafon@ucsd.edu

ABSTRACT

Parametrization of filters on the basis of orthonormal basis functions have been widely used in system identification and adaptive signal processing. The main advantage of using orthonormal basis functions for a filter parametrization lies in the possibility of incorporating prior knowledge of the system dynamics into the identification process and adaptive signal process. As a result, a more accurate and simplified filter with less parameters can be obtained. In this paper, several construction methods of orthonormal basis function are discussed and analyzed. An application of active noise control based on these orthonormal basis constructions is presented.

INTRODUCTION

Many researchers have contributed to the use of orthonormal basis functions in the area of system identification and model approximation [1–7]. In the constructions of the orthonormal basis functions, Laguerre and Kautz basis have been used successfully in system identification and signal processing [8, 9]. A unifying construction in [2] generalized both the Laguerre and Kautz basis in the context of system identification. Laguerre basis can be used for the identification of well-damped dynamical systems with one dominant first-order [8], whereas a Kautz basis is suitable for the identification of dynamical systems with second order resonant modes [9]. A further generalization of these results for arbitrary dynamical systems was reported in [1] and is called generalized (orthonormal) basis functions. The generalized orthonormal basis and unifying construction can be used for systems with wide range of dominant modes, i.e, both high frequency and low frequency behavior.

It has been shown that [1] if the dynamics of the orthonormal basis functions can approach the dynamics of the dynamical system to be estimated, the convergence rate of the parameter estimation will be very fast, and also the number of the parameters to be determined to accurately approximate the system is much smaller. Therefore, the choice of the orthonormal basis becomes an important issue in order to obtain accurate models.

In this paper, we will focus on different constructions of the orthonormal basis functions. Different constructions of the orthonormal basis will be analyzed and compared. An application of generalized FIR filter based on different set of orthonormal basis to the active noise control is implemented. The performance of active noise control with different generalized FIR filter structure are calculated and compared to illustrate the characteristics of constructions of orthonormal basis.

SETS OF ORTHONORMAL BASIS FUNCTIONS

Consider a linear time invariant stable discrete time system $P(z)$ written as

$$P(z) = \sum_{k=0}^{\infty} M_k z^{-k} \tag{1}$$

*Address all correspondence to this author.

where $\{M_k\}_{k=0,1,2\cdots}$ are the sequence of Markov parameters. In general, the system $P(z)$ can be approximated by $P(z,\theta)$ using a finite number of expansion coefficients $\theta = \{M_k\}_{k=0,1,\cdots,N-1}$ through

$$P(z,\theta) = \sum_{k=0}^{N-1} M_k z^{-k} \tag{2}$$

The model $P(z,\theta)$ represented in (2) is a Finite Impulse Response (FIR) model and has some favorable properties. Firstly, it is linearly parameterized. Secondly, least square estimation of the parameters θ on the basis of input/output measurements of $P(z)$ is robust against colored noise on the output measurements, which is one of the main features exploited in recursive filter estimation.

However, it is known that for a dynamical system including both high and low frequency dynamics, a large number of coefficients N are required in order to capture the most important dynamics of the system $P(z)$ into the model $P(z,\theta)$. Therefore, FIR model structure in general is too simple to capture a system with a broad-band dynamics.

Suppose $\{V_k(z)\}_{k=0,1,2,\cdots}$ is an orthonormal basis sequence for the set of systems in $\mathcal{H}_2$. Then there exists a unique expansion

$$P(z) = D_0 + \sum_{k=0}^{\infty} L_k V_k(z) \tag{3}$$

where $\{L_k\}_{k=0,1,2,\cdots}$ are the generalized orthonormal expansion coefficients for the basis $\{V_k(z)\}$, and D_0 is a constant feedthrough term. Based on this rationale, a model of the dynamical system $P(z)$ can be represented by an finite length N series expansion

$$P(z,\theta) = D_0 + \sum_{k=0}^{N-1} L_k V_k(z),\ \theta = [D_0, L_0^T, \cdots, L_{N-1}^T]. \tag{4}$$

When $V_k(z)$ are chosen as $V_k(z) = z^{-(k+1)}$, then (4) simplifies to (2). Therefore, (4) will be called a generalized FIR filter in this paper. Because for real-time implementation of the filter, the feedthrough term D_0 should be set to 0, in the remain part of this paper, D_0 will be neglected. The orthonormal basis sequence $\{V_k(z)\}_{k=0,1,2,\cdots}$ can incorporate the possible prior knowledge of the system to be approximated, and the model $P(z,\theta)$ can be more accurate for a finite number of coefficients N compared to a standard FIR model structure. It is obvious that the accuracy of the model $P(z,\theta)$ depends on the choice of the basis function $V_k(z)$.

A unifying construction of the orthonormal basis function was presented in [2] and given by

$$V_k(z) = \left(\frac{\sqrt{1-|\xi_k|^2}}{z-\xi_k}\right) \prod_{i=0}^{k-1} \left(\frac{1-\bar{\xi}_i z}{z-\xi_i}\right) \tag{5}$$

where $\{\xi_i\}_{i=0,1,2,\cdots,N-1}$ is the variety of the poles. A model structure using unifying construction is illustrated in Fig. 1. The advantage of using the unifying construction (5) lies in the possibility to include knowledge of multiple possible pole locations in the generalized FIR filter, while the orthonormality of basis $V_k(z)$ is still preserved. This property may lead to a more accurate model $P(z,\theta)$ of the system $P(z)$ to be approximated.

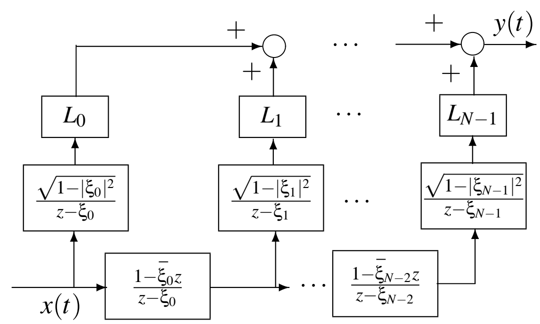

Figure 1. ILLUSTRATION OF MODEL STRUCTURE USING UNIFYING CONSTRUCTION OF THE BASIS FUNCTIONS.

The set of (generalized) orthonormal basis functions presented in [1] provides an alternative way to construct an orthonormal basis with all-pass functions. For details on the construction of the generalized basis functions $V_k(z)$ one is referred to [1]. The following result shows the existence and construction of the inner function which is crucial to create the orthonormal basis functions.

Proposition 1. *Let (A,B) be the state matrix and input matrix of an input balanced realization of a discrete time transfer function $H \in \mathcal{RH}_2^{p\times m}$ ($\mathcal{RH}_2^{p\times m}$ indicates the set of real rational $p\times m$ matrix functions) with a McMillan degree $n > 0$, and with rank$(B) = m$. Then*

(a) *There exist matrices C,D such that (A,B,C,D) is a minimal balanced realization of a square inner function P_a.*
(b) *A realization (A,B,C,D) has the property mentioned in (a) if and only if*

$$\begin{aligned} C &= B^T(I_n + A^T)^{-1}(I_n + A) \\ D &= [B^T(I_n + A^T)^{-1}B - I_m] \end{aligned} \tag{6}$$

where I_n is $n \times n$ identity matrix, and I_m is $m \times m$ identity matrix.

Proof. For the proof, one is referred to [10].

Proposition 1 yields a square $m \times m$ inner transfer function $P_a(z) = D + C(zI - A)^{-1}B$, where (A,B,C,D) is a minimal balanced realization. With the information obtained in Proposition 1, the orthonormal basis functions can be created with following proposition.

Proposition 2. *Let $P_a(z)$ be a square inner function with McMillan degree $n > 0$ and (A,B,C,D) is a minimal balanced realization of $P_a(z)$. Define the input to state transfer function $V_0(z) := (zI - A)^{-1}B$ and*

$$\begin{aligned} V_k(z) &= (zI-A)^{-1}BP_a(z)^k \\ &= V_0(z)P_a(z)^k \end{aligned} \tag{7}$$

then the set of functions $\{V_k(z)\}_{k=0,1,2,\dots}$ are orthonormal basis functions which have the following property

$$\frac{1}{2\pi j}\oint V_i(z)V_j^T(1/z)\frac{dz}{z} = \begin{cases} I & i = j \\ 0 & i \neq j \end{cases} \tag{8}$$

Proof. For the proof, one is referred to [10].

Proposition 1 and Proposition 2 show how to use an inner function to construct the orthonormal basis function $V_k(z)$. In summary, if an orthonormal basis with poles at $\{\xi_i\}_{i=0,1,2,\cdots,N-1}$ is desired, then from Proposition 1 an inner function $P_a(z)$ with these poles can be created. As a result, a balanced realization (A,B,C,D) of inner function $P_a(z)$ can be found to form the orthonormal basis function $V_k(z)$ as in (7). The construction of a generalized FIR filter on the basis of the (generalized) orthonormal basis function is shown in Fig. 2.

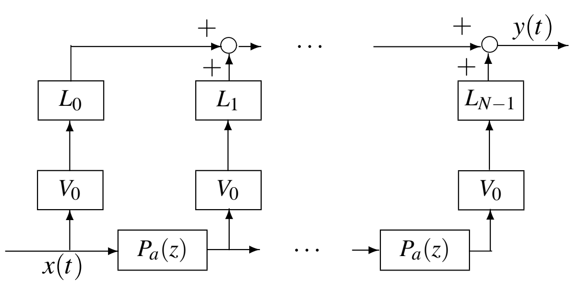

Figure 2. ILLUSTRATION OF MODEL STRUCTURE BASED ON (GENERALIZED) ORTHONORMAL BASIS FUNCTIONS.

The difference between the generalized basis functions in (7) and the unifying construction (5) is that in (7) the poles of $P(z,\theta)$ are restricted to a finite set $\{\xi_0, \cdots, \xi_{n-1}\}$. Using unifying construction (5), the poles of $P(z,\theta)$ can be extended to infinite. The reason the generalized basis functions can only incorporate finite modes is that the order of the inner function $P_a(z)$ in the orthonormal basis functions is finite. A more general orthonormal basis functions which can incorporate an infinite number of poles is summarized in the following proposition.

Proposition 3. *Consider a sequence of inner function $P_{ai}(z)$, $i = 0,1,2,\cdots$, where each $P_{ai}(z)$ has a corresponding balanced realization (A_i,B_i,C_i,D_i) and consider $\Phi_i(z) = (zI - A_i)^{-1}B_i$. Then the set of functions $\{V_i(z)\}_{i=0,1,2,\cdots}$ with*

$$\begin{gathered} V_0(z) = \Phi_0(z), \\ V_i(z) = \Phi_i(z)P_{a(i-1)}(z)\cdots P_{a1}(z)P_{a0}(z) \end{gathered} \tag{9}$$

is mutually orthonormal.

Proof. The proof is similar to the proof for Proposition 2.

Because the set of functions $\{V_i(z)\}_{i=0,1,2,\cdots}$ are mutually orthonormal, then $V_i(z)$ can constitute a set of orthonormal basis functions. The construction of orthonormal basis function described in Proposition 3 is named (generalized) mutual orthonormal basis functions. With $V_i(z)$ in place, the model $P(z,\theta)$ of a dynamical system $P(z)$ can be represented as

$$\begin{aligned} P(z,\theta) &= \sum_{k=0}^{N-1} L_k\Phi_k(z)P_{a(k-1)}(z)\cdots P_{a1}(z)P_{a0}(z) \\ &= \sum_{k=0}^{N-1} L_kV_k(z). \end{aligned} \tag{10}$$

If $P_a(z) = P_{a0}(z) = P_{a1}(z) = \cdots = P_{a(N-2)}(z)$, then (9) can be simplified to (7) and therefore the construction of the basis functions in (9) is the generalization of the construction of basis functions in (7).

ACTIVE NOISE CONTROL

The application of feedforward compensation in the area of active noise control has received attention in recent years [11–15]. The basic principle and idea behind ANC is to cancel sound by a controlled emission of a secondary opposite (out-of-phase) sound signal [16, 17]. In most applications, a linearly parametrized filter such as a finite impulse response (FIR) is used for the recursive estimation and adaptation of noise cancellation because of the linear parameter and linear phase shift properties. However, for a complex system with broadband dynamics such as an airduct with several resonance modes, a sufficient high order of FIR filter is needed to characterize the dynamics. As a result, this would cause a slow convergence rate and increased computational burden.

In this section we will discuss the design of an ANC algorithm for an ACTA air ventilation silencer utilizing the theory of orthonormal basis functions. Located in the System Identification and Control Laboratory at UCSD, a commercial ACTA silencer for sound control in air ventilation systems is used for the case study. A photograph of the experiment is given in Fig. 3 and a schematic representation of the experimental setup is depicted in Fig. 4.

Figure 3. ACTA AIRDUCT SILENCER LOCATED IN THE SYSTEM IDENTIFICATION AND CONTROL LABORATORY AT UCSD

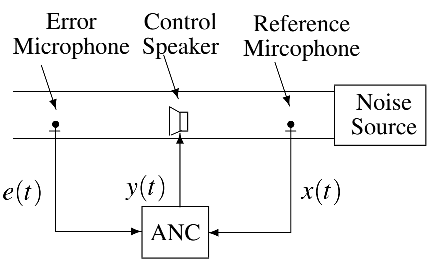

Figure 4. SCHEMATICS OF ACTIVE NOISE CONTROL SYSTEM IN AN AIRDUCT

As indicated in Fig. 4, sound waves from an external noise source are predominantly travelling from right to left (primary path) and can be measured by the reference microphone at the inlet and the error microphone at the outlet. The (amplified) signal $x(t)$ from the reference microphone is fed into an adaptive feedforward filter through a secondary path that controls the signal $y(t)$ to the control speaker for sound compensation. The signal $e(t)$ from the error microphone is used for evaluation of the effectiveness of the ANC system. The objective of feedforward filter is to minimize the measured sound noise by creating a secondary path antinoise to cancel the primary path noise.

Analysis of Feedforward Compensation

In order to analyze the design of the feedforward compensator F, consider the block diagram depicted in Fig. 5. Following this block diagram, dynamical relationships between signals in the ANC system are characterized by discrete time transfer functions, with $zx(t) = x(t+1)$ indicating a unit step time delay. The spectrum of noise disturbance $x(t)$ at the input microphone is characterized by filtered white noise signal $n(t)$ where $W(z)$ is a (unknown) stable and stable invertible noise filter [18]. The dynamic relationship between the input $x(t)$ and the error microphone signals $e(t)$ is characterized by primary path $H(z)$ whereas $G(z)$ characterizes the relationship between control speaker signal $y(t)$ and error microphone signal $e(t)$. Finally, $G_c(z)$ is used to indicate the acoustic coupling from control speaker signal back to the input microphone signal $x(t)$ that creates a positive feedback loop with the feedforward $F(z)$.

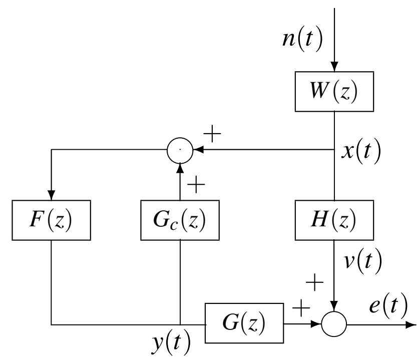

Figure 5. BLOCK DIAGRAM OF ANC SYSTEM WITH FEEDFORWARD

For the analysis we assume in this section that all transfer functions in Fig. 5 are stable and known. The error microphone signal $e(t)$ can be described by

$$e(t) = W(z)\left[H(z) + \frac{G(z)F(z)}{1 - G_c(q)F(z)}\right]n(t) \qquad (11)$$

and is bounded if the positive feedback connection of $F(z)$ and $G_c(z)$ is stable. In case the transfer functions in Fig. 5 are known, perfect feedforward noise cancellation can be obtained in case

$$\begin{aligned} F(z) &= -\frac{H(z)}{G(z) - H(z)G_c(z)} \\ &= \frac{\tilde{F}(z)}{1 + \tilde{F}(z)G_c(z)},\ \tilde{F}(z) := -\frac{H(z)}{G(z)} \end{aligned} \qquad (12)$$

and can be implemented as a feedforward compensator in case $F(z)$ is a stable and causal transfer function. The expression in (12) can be simplified for the situation where the effect of acoustic coupling G_c can be neglected. In that case, the feedforward compensator F can be approximated by

$$F(z) \approx \tilde{F}(z) = -\frac{H(z)}{G(z)} \qquad (13)$$

and for implementation purposes it would be required that $F(z)$ is a causal and stable filter.

In general, the filter $F(z)$ in (12) or (13) is not a causal or stable filter due to the dynamics of $G(z)$ and $H(z)$ that dictate the solution of the feedforward compensator. Therefore, an optimal approximation has to be made to find the best causal and stable feedforward compensator.

Estimation of Feedforward Compensation

In case the mechanical and geometrical properties of the silencer in Fig. 4 are fixed, the transfer functions $H(z)$, $G(z)$ and $G_c(z)$ are predetermined, but possibly unknown. It is important to make a distinction between varying dynamics and fixed dynamics in the ANC system for estimation and adaptation purposes. An off-line identification technique can be used to estimate these transfer functions to determine the essential dynamics of the feedforward controller. Subsequently, the spectral contents of the sound disturbance characterized by the (unknown) stable and stably invertible filter $W(z)$ is the only varying component for which adaptation of the feedforward control is required.

For the analysis of the direct estimation of the feedforward compensator we assume that the acoustic coupling G_c can be neglected to simplify the formulae. In that case, the error signal $e(t)$ is given by

$$e(t,\theta) = H(z)x(t) + F(z,\theta)G(z)x(t)$$

and definition of the signals

$$y(t) := H(z)x(t),\ x_f(t) := G(z)x(t) \tag{14}$$

leads to

$$e(t,\theta) = y(t) + F(z,\theta)x_f(t)$$

for which the minimization

$$\min_{\theta} \frac{1}{N}\sum_{t=1}^{N} e(t,\theta) \tag{15}$$

to compute the optimal feedforward filter $F(z,\theta)$ is a standard output error (OE) minimization problem in a prediction error framework [18]. Using the fact that the input signal $x(t)$ satisfies $\|x\|_2 = |W(z)|^2\lambda$, the minimization of (15) for $\lim_{N\to\infty}$ can be rewritten into the frequency domain expression

$$\min_{\theta} \int_{\pi}^{-\pi} |W(\mathrm{e}^{j\omega})|^2 |H(\mathrm{e}^{j\omega}) + G(\mathrm{e}^{j\omega})F(\mathrm{e}^{j\omega},\theta)|^2 \mathrm{d}\omega \tag{16}$$

using Parseval's theorem [18].

It should be noted that the signals in (14) are easily obtained by performing a series of two experiments. The first experiment is done without a feedforward compensator, making $e(t) = H(z)x(t) \triangleq y(t)$ and $e(t)$ is the signal measured at error microphone. The input signal $x_f(t)$ can be obtained by applying the measured input microphone signal $x(t)$ from this experiment to the control speaker in a second experiment that is done without a sound disturbance. In that situation $e(t) = G(z)x(t) \triangleq x_f(t)$.

Feedforward Design with Generalized FIR Filter

To facilitate the use of the generalized FIR filter shown in (4), the basis functions $V_k(z)$ in (9) can be selected. A relative low order model for the basis functions will suffice, as the generalized FIR model will be expanded on the basis of $V_k(z)$ to improve the accuracy of the feedforward compensator.

With no feedforward compensator in place, the signal $y(t)$ is readily available via

$$y(t) := H(z)x(t) \tag{17}$$

and an initial low order IIR model $\hat{F}(z)$ of the feedforward filter $F(z)$ can be estimated using the OE-minimization

$$\hat{F}(z) = F(q,\hat{\theta}),\ \hat{\theta} = \min_{\theta} \frac{1}{N}\sum_{t=0}^{N} \varepsilon^2(t,\theta) \tag{18}$$

of the prediction error

$$\varepsilon(t,\theta) = y(t) + F(z,\theta)x_f(t)$$

where $x_f(t)$ is given as

$$x_f(t) = \hat{G}x(t) \tag{19}$$

The initial low order IIR model $\hat{F}(z)$ can be used to generate the basis functions $V_k(z)$ of the generalized FIR filer of the feedforward compensator $F(z)$. An input balanced state space realization of the low order model $\hat{F}(z)$ is used to construct the basis function $V_k(z)$ in (9).

With a known (initial) feedforward $F(q,\hat{\theta})$ and the basis function $V_k(z)$ in place, the signal $y(t)$ can be generated via

$$\begin{aligned} y(t) :&= H(z)x(t) \\ &= e(t) + \left[\sum_{k=0}^{N-1} L_k V_k(z)\right] x_f(t) \end{aligned} \tag{20}$$

and requires measurement of the error microphone signal $e(t)$, and the filtered input signal $x_f(t) = \hat{G}(z)x(t)$. Since the feedforward filter is based on the generalized FIR model, the input $x_f(t)$ is also filtered by the tapped delay line of basis functions. A new filtered input signal $\overline{x}_k(t)$ can be defined as

$$\overline{x}_k(t) = V_k(z)\hat{G}(z)x(t) \tag{21}$$

With the signal $y(t)$ in (20), $x_f(t)$ in (19), $\overline{x}_k(t)$ in (21) and the basis function $V_k(z)$ in (9) from the initial low order model in (18), (20) can be rewritten as a linear regression form

$$y(t) = \phi^T(t)\theta, \quad \theta = [L_0, L_1, ..., L_{N-1}]^T \tag{22}$$

where $\phi^T(t) = [\overline{x}_0^T(t), ..., \overline{x}_{N-1}^T(t)]$ is the available input data vector and θ is the parameter vector to be estimated of the generalized FIR feedforward compensator. Therefore, the parameter vector θ can be estimated with recursive least square (RLS) estimation [19].

ACTIVE NOISE CONTROL PERFORMANCE

Modeling of ANC System Dynamics

In order to initializate and calibrate the feedforward controller, an 18th order ARX model $\hat{G}(z)$ of $G(z)$ was estimated in order to be able to create the filtered input signal $x_f(t)$ in (19). The filtered input signal $x_f(t)$ and the observed error microphone signal $y(t)$ sampled at 2.56kHz were used to estimate a low order OE model $\hat{F}(z)$ to create the basis functions $V_k(z)$ in (9) for the generalized FIR filter parametrization of the feedforward controller. During the estimation of the model $\hat{F}(z)$ also an estimate of the expected time delay n_k was performed and was found to be $n_k = 16$.

To limit the complexity of the orthonormal basis functions for real-time filter implementation, it is assumed that only two complex conjugate pole pairs of $-H(z)/G(z)$ are available for the construction of basis function $V_k(z)$. In addition, the order of the feedforward filter expressed in terms of its orthonormal basis functions is limited to 20 for a fair comparison between the usage of different basis functions for feedforward based active noise control.

The 4th order IIR model $F_f(z,\theta)$ used for all pass function generation is shown in Fig. 6. From this figure it can be observed that the 4th order model $F_f(z,\theta)$ only approximately models two resonance modes of the ideal, and possibly unstable, feedforward filter $-\frac{H(z)}{G(z)}$.

In order to illustrate the effect of the orthonormal basis functions on the performance of the feedforward filter for active noise control, different orthonormal basis functions are chosen for comparison in the next section.

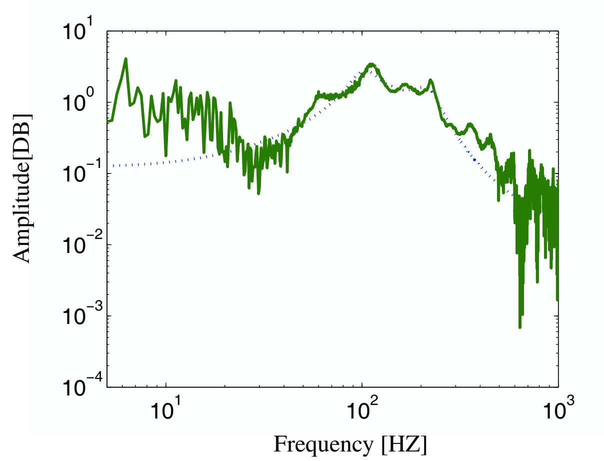

Figure 6. AMPLITUDE OF SPECTRAL ESTIMATE OF $-\frac{H(z)}{G(z)}$ (SOLID) AND 4TH ORDER PARAMETRIC MODEL $F_f(z,\theta)$ (DOTTED)

Construction of Orthonormal Basis

With the model $F_f(z,\theta)$ shown in Fig. 6, 2 complex conjugate pole pairs $z_1, \overline{z}_1 = 0.8045 \pm 0.4751i$, $z_2, \overline{z}_2 = 0.9225 \pm 0.2358i$ are available for the construction of basis functions. Using this information, the performance of feedforward active noise cancellation is compared for different sets of basis functions for the feedforward filter.

The first set of basis functions considered here uses the knowledge of the two complex conjugate pole pairs to create a single all pass function

$$P_a(z) = |z_1|^2|z_2|^2 \frac{(z-z_1^{-1})(z-\overline{z}_1^{-1})(z-z_2^{-1})(z-\overline{z}_2^{-1})}{(z-z_1)(z-\overline{z}_1)(z-z_2)(z-\overline{z}_2)} \tag{23}$$

for the standard orthonormal FIR expansion $V_k(z)$ as presented in (7). Since $P_a(z)$ is a simple 4th order all-pass function, from Proposition 1 and Proposition 2, the parametrization of the first orthonormal FIR expansion $F_5(z,\theta)$ is given by

$$F_5(z,\theta) = z^{-d}\sum_{k=1}^{5} L_{k-1}V_{k-1}(z),\ V_{k-1}(z) = \Phi_0(z)P_a(z)^{k-1} \tag{24}$$

where $d = 16$ to account for the observed time delays and $\Phi_0(z) = (zI - A)^{-1}B$ in which (A,B) are computed from an input balanced state-space realization of $P_a(z)$. Furthermore, n is limited to $n = 5$ to ensure that $F_1(z,\theta)$ has a McMillan degree less than or equal to 20 for a fair comparison between the usage of different basis functions for feedforward based active noise control.

The second set of basis functions $V_k(z)$ used for comparison is based on mutually orthonormal basis functions created with the knowledge of two all-pass functions $P_{a1}(z)$ and $P_{a2}(z)$ that separate the knowledge of the two complex conjugate pole pairs

$z_1, \bar{z}_1$ and $z_2, \bar{z}_2$. The construction of $P_{a1}(z)$ and $P_{a2}(z)$ is similar to $P_a(z)$ in (23), but only one complex conjugate pole pair $z_1, \bar{z}_1$ or $z_2, \bar{z}_2$ is included to create $P_{a1}(z)$ and $P_{a2}(z)$, respectively. On the basis of the two all-pass functions $P_{a1}(z)$ and $P_{a2}(z)$, following the parametrization given in Proposition 3, the following feedforward filter $F_m(z,\theta)$ is considered:

$$
\begin{aligned}
&F_m(z,\theta) = z^{-d} \sum_{k=1}^{10} L_{k-1} V_{k-1}(z) \text{ where} \\
&V_{k-1}(z) = \begin{cases} \Phi_1(z) P_{a1}(z)^{k-1}, \; k = 1,\ldots,m \\ \Phi_2(z) P_{a2}(z)^{k-m-1} P_1(z)^m, \; k = 1+m,\ldots,10 \end{cases}
\end{aligned}
\tag{25}
$$

where $d = 16$ to account for the observed time delays and with $\Phi_1(z) = (zI - A_1)^{-1} B_1$ in which (A_1, B_1) are computed from an input balanced state-space realization of $P_{a1}(z)$ and $\Phi_2(z) = (zI - A_2)^{-1} B_2$ in which (A_2, B_2) are computed from an input balanced state-space realization of $P_{a2}(z)$.

In the above parametrization, the basis functions $V_{k-1}(z)$ are built up from a linear combination of the all-pass functions $P_{a1}(z)$ and $P_{a2}(z)$ such that $V_{k-1}(z)$ are mutually orthonormal, see also Proposition 3. The index m for $F_m(z,\theta)$ determines how many times the all-pass function $P_{a1}(z)$ is taken into account in the construction of the mutually orthonormal basis functions. E.g. $m = 0$ will only use the all-pass function $P_{a2}(z)$ whereas $m = 10$ will only use the all-pass function $P_{a1}(z)$. If m is chosen as $m = 5$, then $F_m(z,\theta)$ in (25) is equivalent to $F_5(z,\theta)$ in (24).

Comparison of ANC Performance

The variance of the error microphone signal $e(t)$ can be used to characterize the performance of active noise control using feedforward filters with different sets of basis functions. With the experimental setup described in Section , the variance of measured error microphone signal $e(t)$ in the case of no active noise control is found to be 0.6403. Application of a standard 20th order FIR feedforward filter (2) with 16 steps time delay in which the parameters are found by a Least Squares optimization reduces the variance of error microphone signal $e(t)$ to 0.2421. Modifying the FIR filter to the feedforward filter in (24), that uses a single 4th order all-pass function $P_a(z)$, reduces the variance of $e(t)$ to 0.0978 using the same Least Squares optimization. For this particular application of feedforward based active noise control a 60% performance improvement is obtained compared to the use of a standard FIR filter.

Different combinations m of basis functions in the mutual orthonormal basis functions in (25) to construct $F_m(z,\theta)$ will give variations on the active noise control performance. Using again a Least Squares optimization to compute the parameters L_{k-1} in (25), the variance of the error microphone signal $e(t)$ for combinations m of the all-pass functions $P_{a1}(z)$ and $P_{a2}(z)$ is shown in Fig. 7 as a final comparison for this case study.

From Fig. 7, the following observations can be made.

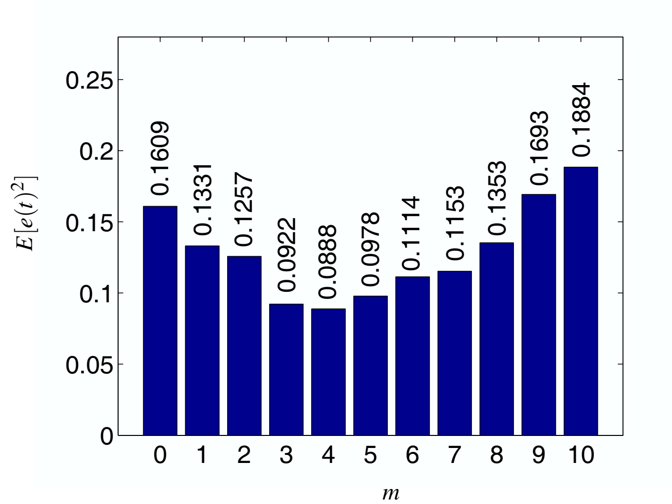

Figure 7. COMPARISON OF VARIANCE OF ERROR $e(t)$ USING 20TH ORDER ORTFIR FILTER $F_m(z,\theta)$ WITH DIFFERENT COMBINATIONS m OF MUTUAL ORTHONORMAL BASIS

Firstly, if only the 2nd order $P_{a1}(z)$, $m = 10$ or $P_{a2}(z)$, $m = 0$ all-pass functions are used to create orthonormal basis functions, the variance of $e(t)$ is worse compared to choosing 4th order basis function $P_a(z)$ or any linear combination of $P_{a1}(z)$ and $P_{a2}(z)$ as all-pass functions. Hence, higher order orthonormal basis functions which include more poles of the dynamic system to be approximated is preferable to reach an improvement in model approximation and also active noise control performance.

Secondly, the smallest variance of error signal $e(t)$ is obtained when $m = 4$. This implies that the quality of the approximation of feedforward controller (or active noise control performance) is not only related to the location of the poles of the basis function, but is also determined by the number of coefficients used for building the series expansion on the basis of a specific basis function.

Finally, it is also observed that the variance of $e(t)$ with orthonormal basis constructed by 4th order all pass function $P_a(z)$ is the same as that of two all pass functions $P_{a1}(z)$ and $P_{a2}(z)$ being equally weighted ($m = 5$). This phenomena confirms that $F_m(z,\theta)$ in (25) is exactly equal to $F_1(z,\theta)$ in (24) if $m = 5$.

The performance of feedforward ANC using generalized FIR filters with different construction of orthonormal basis functions is confirmed by estimate of the spectral content of the microphone error signal $e(t)$ plotted in Fig. 8. The spectral content of the error microphone signal has been reduced significantly by the generalized FIR feedforward compensator $\hat{F}$ in the frequency range from 30 till 400Hz.

CONCLUSIONS

The advantages of using generalized FIR filters in system identification / adaptive filtering are twofold. First, the general-

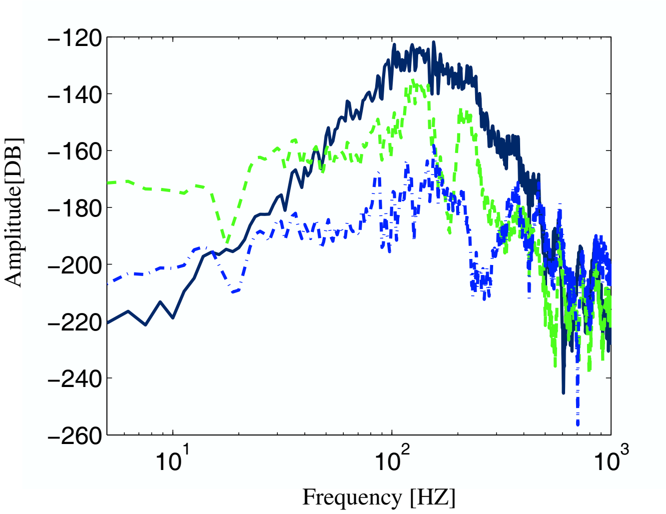

Figure 8. SPECTRAL ESTIMATE OF ERROR MICROPHONE SIGNAL $e(t)$ WITHOUT ANC (SOLID) AND WITH ANC : (DASHED) WITH 20TH FIR; (DASH-DOTTED) WITH 20TH ORTFIR (m=4).

ized FIR filter has the same linear parameter structure as a FIR filter that is favorable for recursive estimation and adaptation purposes. Second, the generalized FIR filter can incorporate possible prior knowledge of the system dynamics in the tapped delay line of the filter. This property can greatly reduce the number of parameters to be estimated during adaption and the tuning of the filter. In this paper different constructions of the orthonormal basis functions are discussed and analyzed. A comparison of these constructions in generalized FIR filter are made with the application to the active noise control in an airduct. The results show that during the construction of the orthonormal basis functions, a high order orthonormal basis functions with small number of parameters are preferable comparing with the low order orthonormal basis functions with relative larger number of parameters.

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