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Feedforward estimation for active noise cancellation in the presence of acoustic coupling

Jie Zeng and R.A. de Callafon

***Abstract*— Feedforward filter algorithm has been widely used in active noise cancellation for broad band noise reduction. The basic principle of this algorithm is based on that the feedforward signal, i.e, the external noise source is an independent (or known) signal which is no correlation to the actuator output. Failure to reach this requirement implies that the overall system contains acoustic coupling, and then closed loop stability and robustness become very crucial and need to be considered. In this paper, a framework to design a feedforward filter via a fractional approach is presented. The feedforward filter is constructed by coprime factorization with the possible perturbation estimated by doula-Youla parametrization to provide the feedforward compensation.**

I. Introduction

In application of active noise cancellation (ANC) a feedforward filter has been widely used for broad-band noise cancellation [10], [15], [9], [4]. In the case that the noise measurement in feedforward compensation is not influenced by the feedforward control signal, feedforward ANC provides an effective resource to create a controlled emission for sound attenuation. Algorithms based on recursive (filtered) Least Mean Squares (LMS) minimization [8] can be quite effective for the estimation and adaptation of feedforward based sound cancellation [3]. To facilitate an output-error based optimization of the feedforward compensation, a linearly parameterized finite impulse response (FIR) filter [15] and generalized FIR filter [17], [18] also have been used for the recursive estimation and adaptation.

However, in many cases strong coupling may be present in an ANC system, which implies that the noise measurement in feedforward compensation is influenced by the ANC signal. From a control point of view, the ANC system will no longer be a pure feedforward algorithm, because a positive feedback (also called acoustic coupling) tends to destabilize an ANC system.

Therefore, modifications to the control algorithm have to be made to stabilize the feedforward based ANC system. Some methods to solve the problem of acoustic coupling can be found in [11], [9] which include directional microphones and loudspeakers, motional feedback loudspeakers, neutralization filter, dual-microphone reference sensing, filter-u LMS method, distributed parameter model, and so on. In addition, H_∞ synthesis technique can also be used to design feedforward filter because H_∞ theory can automatically incorporate the acoustic coupling during the design process [1].

In this paper, we adopt a new approach for the estimation of a feedforward filter in a ANC system, where the feedforward filter is parametrized and estimated using a dual-Youla parametrization [7], [12], [5]. This parametrization is able to take into account the acoustic coupling in the ANC system, by providing a framework to parametrize all stabilizing feedforward filters given the acoustic coupling in the ANC system. For the parametrization of the feedforward filter only a initial (low order) feedforward filter is needed that is known to be stable in the presence of the acoustic coupling. Subsequently, the feedforward filter is estimated via the optimization of a dual-Youla perturbation that aims at minimizing the error signal in the ANC system. The optimization of the perturbation is written as a system identification problem, which can be solved by standard open loop identification techniques [13].

The paper is outlined as follows. Following the analysis of the feedforward control design in Section II, the framework for dual-Youla parametrization is presented in Section III. Section IV illustrates the identification results of the design of a feedforward ANC system, parametrized in a dual-Youla parametrization. It is shown that a stabilizing feedforward filter can be estimated in the presence of a strong acoustic coupling and the ANC system demonstrates the effect of noise cancellation over a broad frequency range from 30 till 400 Hz.

II. Analysis of feedforward compensation

In order to analyze the design of the feedforward filter F, consider the schematic representation of a linear airduct is depicted in Figure 1. Sound waves from an external noise source are predominantly traveling from right to left and can be measured by the pick-up microphone at the inlet and the error microphone at the outlet.

R.A. de Callafon is with Faculty of Department of Mechanical and Aerospace Engineering, University of California, San Diego, 9500 Gilman Drive, La Jolla, CA 92093-0411, U.S.A callafon@ucsd.edu

J. Zeng is with the Department of Mechanical and Aerospace Engineering, University of California, San Diego, 9500 Gilman Drive, La Jolla, CA 92093-0411, U.S.A jzeng@ucsd.edu

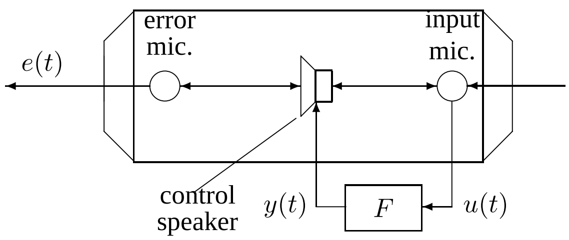

Fig. 1. Schematics of ANC system

The (amplified) signal $u(t)$ from the input microphone is fed into a feedforward filter F that controls the signal $y(t)$ to the control speaker for sound compensation. The signal $e(t)$ from the error microphone is used for evaluation of the effectiveness of the ANC system.

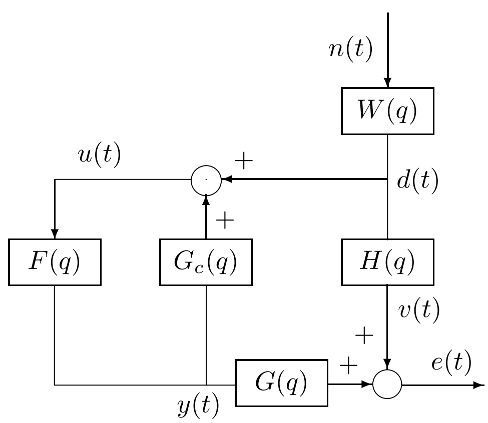

Fig. 2. Block diagram of ANC system with feedforward

The block diagram that models the dynamical relationships between the signals in the ANC is given in Figure 2. Following this block diagram, dynamical relationship between signals in the ANC system are characterized by discrete time transfer functions, with $qd(t) = d(t+1)$ indicating a unit step time delay, and q is a shift operator. For notational convenience, the shift operator q will be dropped in most of the remaining part of the paper.

The spectrum of noise disturbance $d(t)$ at the input microphone is characterized by filtered white noise signal $n(t)$ where W is a (unknown) stable and stable invertible noise filter [13]. The dynamic relationship between the input $d(t)$ and the error $e(t)$ microphone signals is characterized by H whereas G characterizes the relationship between control speaker signal and error microphone signal $e(t)$. Finally, G_c is used to indicate the acoustic coupling from the control speaker signal back to the input microphone signal $d(t)$.

It can be seen that the acoustic coupling G_c creates a positive feedback loop with the feedforward filter F. The presence of the acoustic coupling G_c might lead to an undesirable or unstable feedforward compensation if G_c is not taken into account in the design of the feedforward filter F for active noise cancellation. Henceforth, to guarantee stability of the feedback connection of F and G_c, the presence of acoustic coupling must be taken into account during the design of feedforward filter F.

The error microphone signal $e(t)$ can be described by

$$\begin{aligned} e(t) &= \left[H + \frac{GF}{1-G_cF}\right] d(t) \\ &= Hd(t) + \frac{F}{1-G_cF} \cdot Gd(t) \end{aligned} \tag{1}$$

and definition of the signals

$$v(t) := Hd(t),\ r(t) := Gd(t) \tag{2}$$

leads to

$$\begin{aligned} e(t) &= v(t) + \frac{F}{1-G_cF} r(t) \\ &= v(t) + Lr(t) \quad L := \frac{F}{1-G_cF} \end{aligned} \tag{3}$$

From (3) it can be observed that certain signals can be used for estimation purposes of the dynamics of the various transfer functions in the ANC system. In case the signals $v(t)$ can be measured and the signal $r(t)$ can be created by filtering the measured signal $d(t)$ through a filter that models the dynamics of G, the estimation of the feedforward filter F can be considered as a closed loop identification problem, where the error $e(t,\theta)$

$$e(t,\theta) = v(t) + L(\theta)r(t)$$

is minimized according to

$$\hat{\theta} = \min_{\theta} \|e(t,\theta)\|_2 \tag{4}$$

Minimization of $\|e(\theta)\|_2$ in (4) is a non-trivial output error based identification and the closed-loop identification of the feedforward filter F can be done in several ways.

The first possibility is an indirect identification method, where the closed loop transfer function $\hat{L} = L(\hat{\theta})$ is estimated. Subsequently, then the feedforward filter $\hat{F}$ is computed with

$$\hat{F} = \frac{\hat{L}}{1+\hat{L}G_c}$$

using the knowledge of the acoustic coupling G_c present in the estimated closed loop transfer function. But the model $\hat{F}$ can be computed only in the case that the inverse of $(1+\hat{L}G_c)$ is well-defined and stable, so the stability of the feedback connection $\mathcal{T}(F,G_c)$ of the feedforward filter F and acoustic coupling G_c is guaranteed.

A second possibility is to use a tailor-made parametrization of the closed-loop transfer function [6], [2]. In that case

$L(\theta)$ is parametrized via

$$L(\theta) := \frac{F(\theta)}{1 - G_c F(\theta)}$$

and the minimization in (4) would require a non-linear optimization over a intricate restricted model structure. Although gradient expression for the non-linear optimization are available [2], such an optimization will be hard to implement in real-time adaptive filter estimation of the feedforward filter.

Closely related to the use of a tailor-made parametrization is the the third possibility exploited in this paper, which is an estimation based on a so-called dual-Youla parametrization. This will be discussed in Section III and opens a possibility to guarantee the internal stability of the $\mathcal{T}(F, G_c)$ by constructing a feedforward filter F via estimation of dual-Youla parameter.

III. Dual-Youla parametrization

A. Structure of feedforward filter via coprime factorization

Using the theory of fractional representations, a feedforward filter F can be expressed by $F = ND^{-1}$ where N and D are two stable mapping. Refer to [16], the following definitions are used in this paper.

Definition 1: let $N, D \in \mathcal{RH}_\infty$, the pair (N, D) is called a right coprime factorization (rcf), if there exist $X, Y \in \mathcal{RH}_\infty$ such that $XN + YD = I$
where $\mathcal{RH}_\infty$ indicates the set of all rational stable transfer functions.

Definition 2: let N, D be a right coprime factorization (rcf), the pair (N, D) is a rcf of a filter F if $det\{D\} \neq 0$ and $F = ND^{-1}$
Using the definition 1 and definition 2, a characterization of the set of feedforward filters $F = ND^{-1}$ that yields an internally stable feedback connection $\mathcal{T}(F, G_c)$ of the feedforward filter F and the acoustic coupling G_c can be expressed via a well-known dual-Youla parametrization [7], [12], [5] and is given in the following lemma.

Lemma 1: Let (N_x, D_x) be a rcf of any arbitrary auxiliary filter $F_x = N_x D_x^{-1}$, and (N_c, D_c) be a rcf of the acoustic coupling $G_c = N_c D_c^{-1}$ such that $\mathcal{T}(F_x, G_c) \in \mathcal{RH}_\infty$, then a feedforward filter F with a $rcf(N, D)$ satisfies $\mathcal{T}(F, G_c) \in \mathcal{RH}_\infty$ if and only if there exists $R_0 \in \mathcal{RH}_\infty$ such that

$$\begin{aligned} N &= N_x + D_c R_0 \\ D &= D_x - N_c R_0 \end{aligned} \tag{5}$$

Proof: For a proof, please refer to [14]. ■
From (5) it is obtained that R_0 can vary over all possible transfer functions in $\mathcal{RH}_\infty$ such that $det\{D_x - D_c R_0\} \neq 0$, which characterizes a set of filters F that are internally stabilized by the acoustic coupling G_c.

For the estimation of the feedforward filter we assume the following information. First we assume the availability of an initially stabilizing feedforward controller F_x that creates a feedback connection $\mathcal{T}(F_x, G_c)$ in the presence of the acoustic coupling G_c. Secondly, if a model for the acoustic coupling G_c is available, then a set of feedforward filters can be parameterized that is know to be stabilized by the acoustic coupling. The optimal feedforward filter F to minimize $\|e(t,\theta)\|_2$ in (4) can be constructed by means of the nominal filter F_x plus a possible filter perturbation R_0 given by (5). Because R_0 is the only unknown parameter, the estimation of a stable model $\hat{R}$ of R_0 will yield an estimate $(\hat{N}, \hat{D})$ of a rcf of feedforward filter $\hat{F}$ described by

$$\begin{aligned} \hat{N} &= N_x + D_c \hat{R} \\ \hat{D} &= D_x - N_c \hat{R} \end{aligned} \tag{6}$$

If the estimate $\hat{R}$ is stable, then the model $\hat{F} = \hat{N}\hat{D}^{-1}$ estimated in (6) is guaranteed to be stabilized by the acoustic coupling G_c to avoid instabilities of the feedforward ANC.

B. Dual-Youla parametrization

From equation (5), we know that the set of feedforward filters in Figure 2 can be replaced by the combination of (N_x, D_x), (N_c, D_c) and stable transfer function R_0, The representation of $\mathcal{T}(F, G_c)$ in Figure 2 can be found in Figure 3.

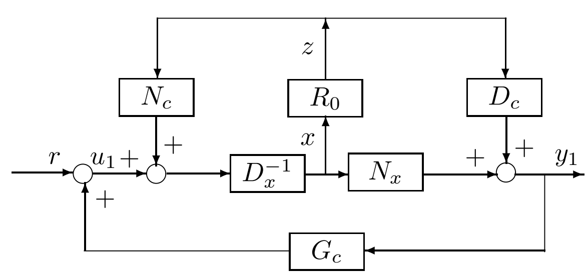

Fig. 3. Block diagram of rcf representation of the feedback connection $\mathcal{T}(G_c, F)$

The reference signal $r(t)$ in Figure 3 is given by

$$r = Gd(t)$$

and the output signal $y(t)$ is defined as

$$y_1 := -v(t) = -Hd(t) \tag{7}$$

where $d(t)$ is the external noise.

The intermediate signals $x(t)$ and $z(t)$ can be considered as an input signal and output signal

$$z = R_0 x \tag{8}$$

where x is defined by the filter operation

$$x := (D_x - G_c N_x)^{-1} \begin{bmatrix} G_c & I \end{bmatrix} \begin{bmatrix} y_1 \\ u_1 \end{bmatrix} \tag{9}$$

The so-called dual-Youla signal z is defined by the filter operation

$$z := (D_c - F_x N_c)^{-1} \begin{bmatrix} I & -F_x \end{bmatrix} \begin{bmatrix} y_1 \\ u_1 \end{bmatrix} \tag{10}$$

where u_1 can be obtained by $u_1 = r + G_c y_1$.

C. Summary of Feedforward Estimation

Since the intermediate signal x and the dual-Youla signal z can be created by (9) and (10), the estimation of the dual-Youla parameter R_0 from (8) is an open loop identification problem that can be computed by standard system identification techniques [13]. For more information about the dual-Youla parametrization, please refer [7], [12], [5] for more details.

As a result, the estimation of the feedforward filter F using the dual-Youla Parametrization can be summarized by the following steps.

1) First, a model $\hat{G}_c$ of the acoustic coupling G_c is needed to be used to design an initial nominal filter $F_x = N_x D_x^{-1}$ to stabilize the positive feedback loop. Furthermore, the model $\hat{G}_c$ along with the initially stabilizing F_x is used to parametrize the feedforward filter F to be estimated. The model $\hat{G}_c$ can be estimated via a standard open-loop identification problem by performing an experiment using the controller speaker signal as excitation signal and the input microphone signal signal as output signal.
2) A model $\hat{G}$ of G is also necessary for filtering purpose to create signal r. The model $\hat{G}$ can be estimated via a standard open-loop identification by performing an experiment using the controller speaker signal as excitation signal and the error microphone signal signal as output signal. Such a filtering is commonly used in filtered LMS algorithms to avoid bias of the estimate of the feedforward filter [8]. Using both models $\hat{G}$ and $\hat{G}_c$ for filtering purposes can be seen as a generalization of the filtering used in filtered LMS estimation of feedforward filters.
3) With the models $\hat{G}_c$, $\hat{G}$ and the initial feedforward filter F_x the reference signal r, input signal u_1 and output signal y_1 can be created. By means of the filtering in (9) and (10), the intermediate signal x and dual-Youla signal z can be created. With these signals, the optimal feedforward filter F can be estimated by minimizing $\|e(t,\theta)\|_2$ in (4) using the dual-Youla parametrization of the filter $F(\theta)$.

IV. Application of feedforward ANC

A. Modelling of the system dynamics

For the experimental verification of the proposed feedforward noise cancellation, the ACTA silencer depicted in Figure 4 was used. The system is an open-ended airduct located at the System Identification and Control Laboratory at UCSD that will be used as a case study for the ANC algorithm presented in this paper. Experimental data and real time digital control is implemented at a sampling frequency of 2.56kHz and experimental data of the error and input microphone were gathered for the initialization of the feedforward filter.

Fig. 4. ACTA airduct silencer located in the System Identification and Control Laboratory at UCSD

Once the mechanical and geometrical properties of the ANC system in Figure 4 are fixed, then G and G_c both are fixed. The models of G and G_c can be identified off-line. Estimation of a model $\hat{G}$ can be done by performing an experiment using the controller speaker signal as excitation signal and the error microphone signal signal as output signal. And at the same time we also can measure the input microphone signal as an another output to estimate the the model of acoustic coupling $\hat{G}_c$. Because these models G and G_c will be used to design nominal feedforward filter F_x and feedforward filter $\hat{F}$, the order of these models should be controlled. In order to estimate a low order feedforward filter $\hat{F}$, a 20th order ARX model $\hat{G}$ was estimated for filter purpose and a $17th$ order ARX model of $\hat{G}_c$ was estimated for feedforward filter design purposes. the identification results of $\hat{G}$ and $\hat{G}_c$ can be found in Figure 5 and Figure 6, respectively.

B. Estimation of dual-Youla parameter

In order to estimate the dual-Youla parameter, a simple 4th order nominal feedforward filter F_x is pre-computed to internally stabilize the positive feedback loop connection $\mathcal{T}(G_c, F)$ with standard control design technique [19] to create the intermediate signal x and dual-Youla signal z. The amplitude plot of the 4th order nominal feedforward filter F_x is shown in Figure 7.

The intermediate signals x and doul-Youla signal z can be obtained from (9) and (10), and they are plotted in Figure 8.

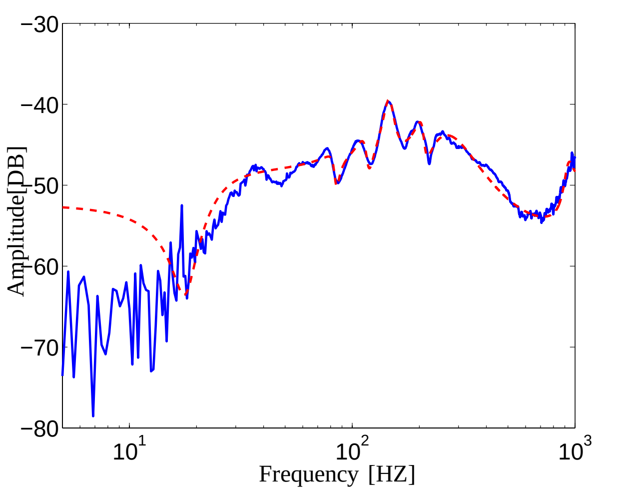

Fig. 5. Amplitude plot of spectral estimate of G (solid) and 20th order parametric model $\hat{G}$ (dashed)

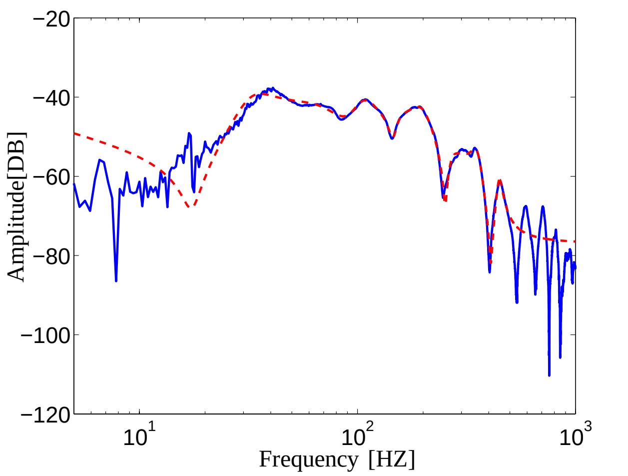

Fig. 6. Amplitude plot of spectral estimate of G_c (solid) and 17th order parametric model $\hat{G}_c$ (dashed)

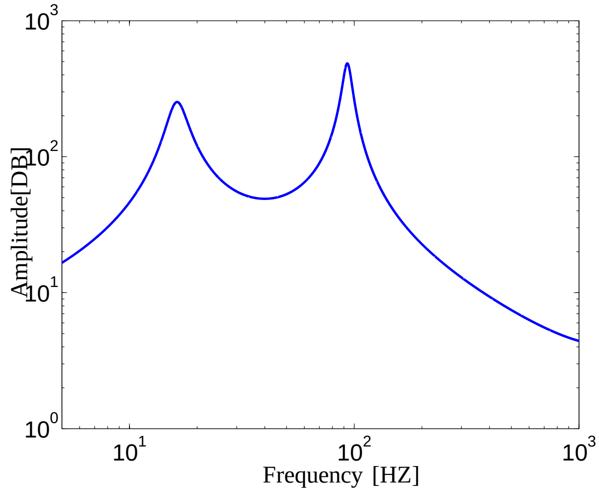

Fig. 7. Amplitude plot of 4th order nominal feedforward filter F_x

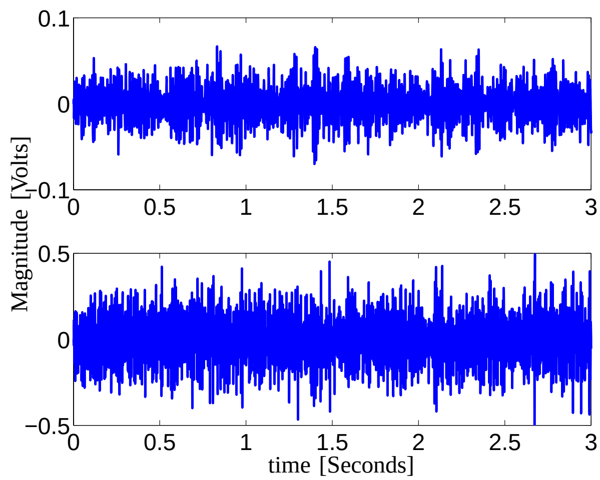

Fig. 8. Plots of intermediate signal x (top) and dual-Youla signal z (bottom)

The model $\hat{R}$ can be estimated from (8) by using standard open loop identification technique [13]. In order to control the order of the filter F, a $4th$ order OE model $\hat{R}$ is estimated which is shown in Figure 9.

From Figure 9, it is shown that the simple 4th order model $\hat{R}$ can not fit the spectral estimate of R very well, but it is accurate enough to be used in (6) to perform the feedforward compensation in the ANC system. Application of the estimated feedforward compensator to the ANC system are illustrated next.

C. Application of feedforward ANC

After the model $\hat{R}$ is obtained, the feedforward filter $\hat{F}$ can be computed via (6). Implement this feedforward filter $\hat{F}$ to the airduct system to validate the active noise compensation, the performance of the feedforward filter is confirmed by estimates of the spectral content of the microphone error signal $e(t)$ plotted in Figure 10. The spectral content of the error microphone signal has been reduced significantly by the 25th order feedforward filter $\hat{F}$ in the frequency range from 30 Hz till 400 Hz.

A final confirmation of the performance of the ANC has been depicted in Figure 11. The significant reduction of the error microphone signal observed in the time domain traces and the norm of the signal displayed on the right part of Figure 11 indicates the effectiveness of the feedforward filter $\hat{F}$ estimated via dual-Youla parametrization for feedforward sound compensation.

V. Conclusions

In this paper a new methodology has been proposed for the active noise control in an airduct using dual-Youla parametrization. The dual-Youla parametrization can incorporate the acoustic coupling path which is usually an intricate problem to feedforward filter design. The experimental results of the dual-Youla parametrization exhibit an excellent performance for active noise cancellation,

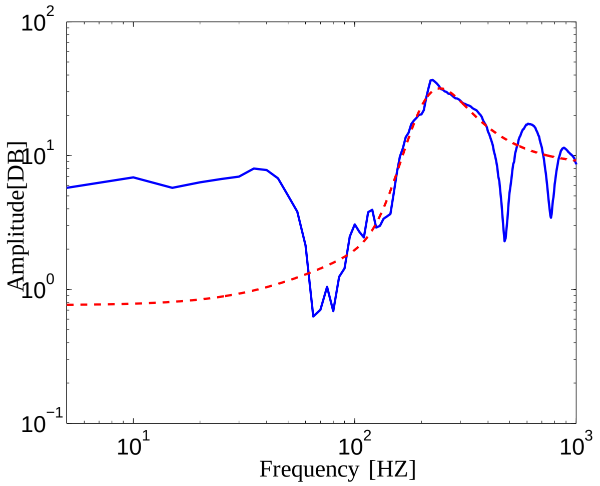

Fig. 9. Amplitude plot of spectral estimate of R (solid) and 4th order parametric model $\hat{R}$ (dashed)

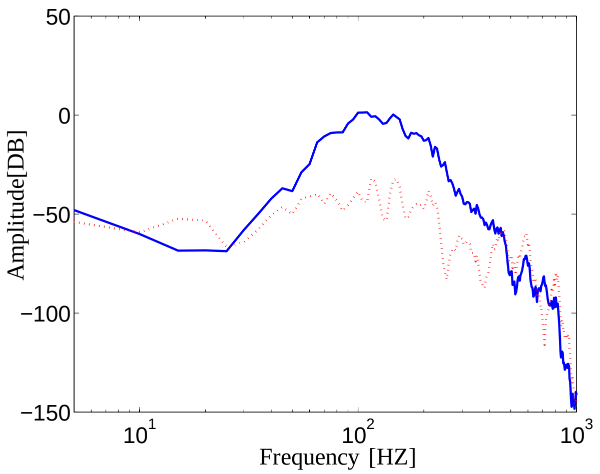

Fig. 10. Estimate of spectrum content of error microphone signal $e(t)$ without ANC (solid) with ANC using 25th order feedforward filter $\hat{F}$ (dashed)

even in the presence of acoustic coupling. Nevertheless, it should be mentioned that the feedforward filter via dual-Youla parametrization is a fixed filter that may not be able to accommodate the excessive plant disturbance. An on-line adaptive filter would make it possible to improve the robustness which will be explored in future studies.

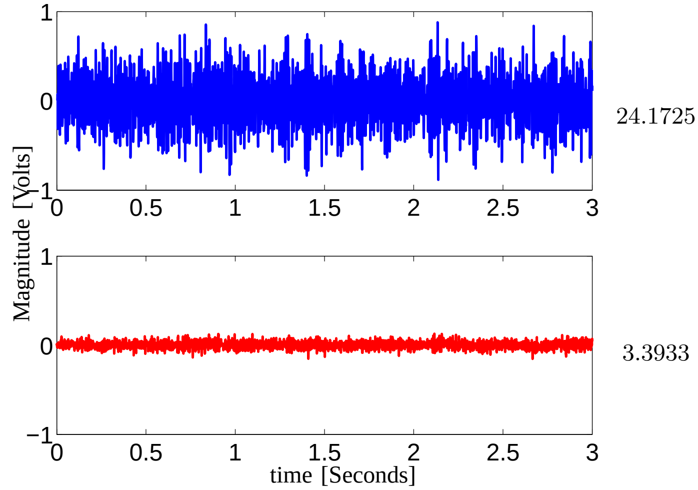

Fig. 11. Evaluation of error microphone signal $e(t)$ before ANC (top) and with ANC using 25th order feedforward filter $\hat{F}$ (bottom) parameterized with a dual-Youla parametrization

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