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Coprime Factor Based Closed-Loop Model Validation Applied to a Flexible Structure

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***Abstract*— This paper addresses the problem of checking the consistency of experimental closed-loop frequency-domain data with uncertainty models that are structured using coprime factorizations. The uncertainty models presented in this paper use the knowledge of a stabilizing feedback controller to structure and formulate the uncertainty on a model. Subsequently, the controller dependent coprime factor uncertainty model can be used to formulate model (in)validation tests on the basis of closed-loop data. The model (in)validation is performed on sample data from a flexible structure to illustrate the presented model validation results. An open-loop based uncertainty model is also used to demonstrate the benefits of closed-loop uncertainty modeling over open-loop uncertainty modeling.**

I. INTRODUCTION

Model validation is a critical procedure to establish whether or not a model can reliably predict the output of a system. In model (in)validation a distinction must be made between validating models on their open-loop or closed-loop behavior. A model validated and suitable to predict open-loop data may be different from a model that validates data obtained under closed-loop or feedback controlled conditions.

In the last few years there has been much attention directed towards various techniques of performing uncertainty model validation. Specifically, the model validation of a general Linear Fractional Transformation (LFT) of discrete and continuous uncertain systems are studied in [1] and [2]. Model validation techniques using LFT's are applied to the frequency domain in [3] where the validation tests were illustrated to have a low level of computational complexity by formulating the model validation problem as a convex optimization.

In this paper a fractional representation approach is presented to address the control oriented identification and model validation problem. The work on fractional model identification was initiated by [4] and further developed in the work by [5] [6] and [7]. This approach allows for a formulation of a unified method to estimate models for stable, marginally stable or unstable systems via the estimation of stable coprime factorizations on the basis of closed-loop data. Moreover, the fractional representation approach preserves convexity of the model validation problem by using the knowledge of the controller.

The fractional approach forms an excellent framework to address the identification of systems on the basis of closed-loop data [8] and control oriented model validation [9]. A model validation problem using open-loop frequency-response data in a coprime factor framework was presented in [10]. The results of [10] are specialized to the open-loop case and cover the noisy and noise-free conditions. However, in this paper the coprime factorizations of the uncertainty model depend on the knowledge of a stabilizing feedback controller to facilitate the closed-loop (in)validation of the uncertainty model.

The model validation tests presented in this paper involve the computation of a structured singular value $\mu(\cdot)$ over a finite frequency grid. Model validation techniques using (inverse) μ have also been studied in [11] with the application towards aero-servoelastic systems. Model validation results using μ for SISO and MISO systems were also studied in [12]. Unfortunately, most of these results were applied to open-loop model validation and this paper extends these results to address the closed-loop model validation problem. It should be noted that this application paper relies heavily on the closed-loop model validation results developed in [13].

II. EXPERIMENTAL SETUP AND FREQUENCY DOMAIN DATA

For the illustration of the model validation results presented in the subsequent section, the experimental data of a flexible structure at the System Identification and Control Laboratory at UCSD is used. The structure is shown in Figure 1 and resembles a two-story building where the bottom floor can be perturbed by a vibration disturbance and the top floor has a spring/damper compensator that is used to counteract the effects of the vibration disturbance.

The applied lateral force on the top floor is used to control the vibration of the structure and dampen out the major resonance modes. Implementing a controller between the second floor lateral acceleration response and the top-level input force reduces the vibration of the highly flexible structure. The controller is used to actively reduce the motion of the structure that is induced by the bottom floor disturbance.

Open-loop data is collected from the structure by measuring the second floor lateral acceleration in response to the applied force at the top of the structure. An amplitude Bode plot of open-loop frequency response data of the plant Φ_{ol}

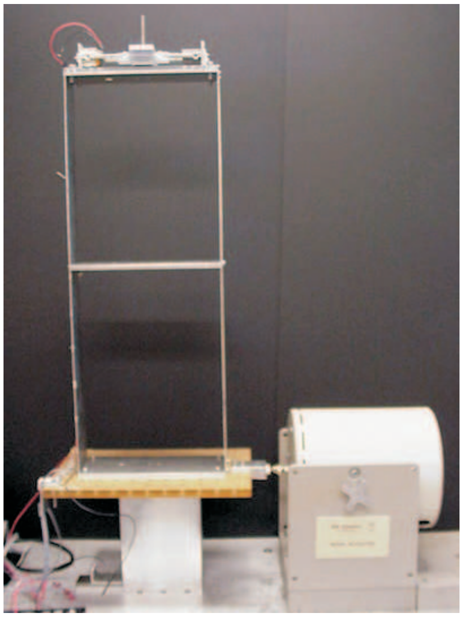

Fig. 1. Flexible test structure used for model validation comparison

and a nominal sixth-order model $\hat{P}$ is given in Figure 2. The sixth-order model is used to describe the major resonance modes of the structure.

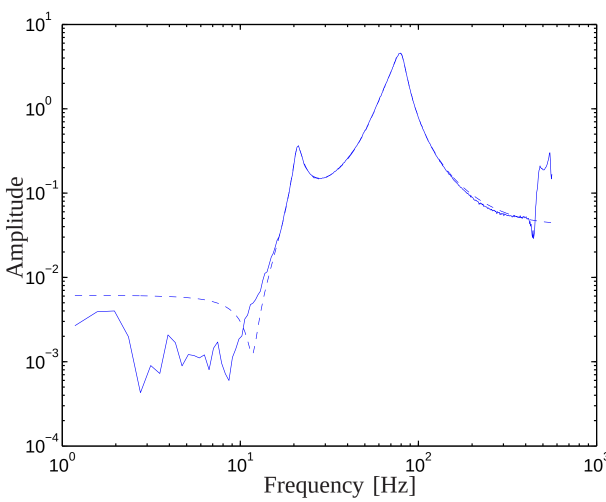

Fig. 2. Amplitude Bode plot of open-loop frequency domain data (solid) and 6th order nominal open-loop model (dashed)

Once the open-loop data is measured and the nominal model is known, the information can be used to develop a controller and measure a closed-loop response from the structure. On the basis of the sixth-order model $\hat{P}$, a simple discrete time lead-lag compensator

$$C(q) = \frac{-1.878q + 1.484}{q - 0.2201}$$

is designed to reduce the second resonance mode of the system. The controller is implemented on the structure at a sampling frequency of 1kHz and a closed-loop frequency response is measured for model validation purposes.

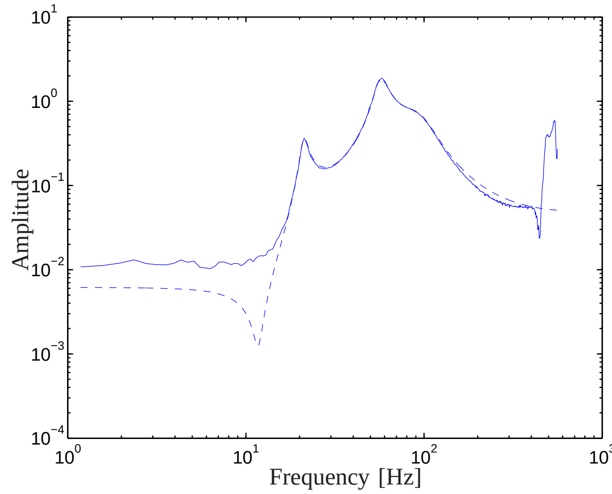

Fig. 3. Amplitude bode plot of closed-loop frequency domain data (solid) and nominal closed-loop model $\frac{\hat{P}}{1+C\hat{P}}$ (dashed)

The closed-loop data and the computed closed-loop model are shown in Figure 3 where it can be seen that a small reduction of the second resonance mode has been achieved. Given the (noisy) experimental frequency domain data, the sixth-order nominal model and the first-order lead-lag compensator, a model validation is performed on the basis of the closed-loop data that will confirm the validity of the model. This is done by validating the sixth-order model using a closed-loop relevant coprime factor based uncertainty structure and a standard open-loop multiplicative uncertainty model. It is shown in this paper that for this example only the coprime factor based uncertainty structure is able to validate the model successfully on the basis of closed-loop data. More details on the coprime factor uncertainty structure and the model validation technique are given in the following section.

III. PROBLEM FORMULATION AND MODEL VALIDATION TECHNIQUE

A nominal model $\hat{P}$ is augmented with a perturbation or uncertainty Δ that is used to capture bounded, but unknown errors due to inaccurate or approximate modeling of the actual plant P_0. The nominal model $\hat{P}$ along with the perturbation Δ constitutes an uncertainty model $\mathcal{P}$ for which model validation techniques can be used to verify if $P_o \in \mathcal{P}$ is not invalidated by a set of measurements. Given a nominal model $\hat{P}$ of a system P_0, an uncertainty structure Δ, and a set of input and output measurements (u, y) acting on the actual system P_0, the model (in)validation problem is to determine whether the measurements (u, y) could have been reproduced by the model $\hat{P}$ with the uncertainty Δ.

It is important to note that the model (in)validation test can be formulated as either an open-loop or closed-loop problem [14]. The difference between open- and closed-loop data is not only determined by the data used for the model validation, but also depends on the way in which the uncertainty model $\mathcal{P}$ is structured. Knowing this, we present the model validation problem and the considerations behind the choice of the model uncertainty structure in the following.

A. Coprime Factor Uncertainty Structure

Following the developments of [13], an (upper) Linear Fractional Transformation (LFT)

$$\mathcal{F}_u(Q,\Delta) := Q_{22} + Q_{21}\Delta(I - Q_{11}\Delta)^{-1}Q_{12}$$

provides a general notation to represent all models $P \in \mathcal{P}$ as follows

$$\begin{array}{c}\mathcal{P} = \{P \mid P = \mathcal{F}_u(Q,\Delta) \\ \text{with } \Delta \in \mathbb{R}\mathcal{H}_\infty \text{ and } \|\Delta\|_\infty < 1\}\end{array} \quad (1)$$

where Δ indicates an unknown (but bounded) uncertainty. The entries of the coefficient matrix Q in (1) is formed by considering a model perturbation that is structured according to a Youla-Kucera parameterization as in Figure 4.

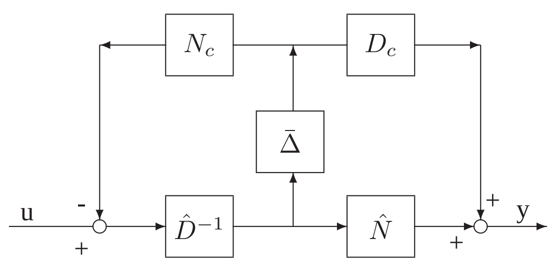

Fig. 4. Uncertainty Model Based on Perturbations on Coprime Factorizations

Following this general fractional formulation, the uncertainty model $\mathcal{P}$ will be characterized by employing a fractional approach. A fractional based uncertainty model $\mathcal{P}$ is characterized by specifically using the knowledge of a controller C that stabilizes the nominal model $\hat{P}$. More specifically, the uncertainty model $\mathcal{P}$ proposed in this paper is structured as follows

$$\begin{array}{l}\mathcal{P} = \{P \mid P = ND^{-1} \text{ with} \\ N = \hat{N} + D_c\bar{\Delta}\ ,\ D = \hat{D} - N_c\bar{\Delta} \ \text{ and} \\ \bar{\Delta} := V\Delta,\ \|\Delta\|_\infty < 1\}\end{array} \quad (2)$$

where (N_c, D_c) and $(\hat{N}, \hat{D})$ respectively denote a right coprime factorization (*rcf*) of the controller C and a nominal model $\hat{P}$. The weighting function V is used to normalize the unknown but bounded uncertainty. The reader is referred to [13] for a detailed analysis of the development of these techniques.

Note that the uncertainty model $\mathcal{P}$ in (2) is different from standard additive coprime factor perturbations as used in [10]. In the uncertainty model of (2), the perturbation $\bar{\Delta}$ is used to model a combined perturbation on the *rcf*(N, D) of the model P. It can be observed that $\hat{N}$ is perturbed by $\Delta_N = D_c\bar{\Delta}$ and $\hat{D}$ is perturbed by $\Delta_D = N_c\bar{\Delta}$ where the *rcf*(N_c, D_c) of the controller plays an important role in assigning the common perturbations in the *rcf*(N, D). From this representation, the coprime factors (N, D) can be expressed as

$$N = \hat{N} + \Delta_N \text{ and } D = \hat{D} - \Delta_D \quad (3)$$

where Δ_N and Δ_D are coupled and controller dependent additive perturbations on the coprime factorization $(\hat{N}, \hat{D})$ of the nominal model.

In order to deal with closed-loop data, we consider a feedback connection of a system, denoted by P_o, and a feedback controller $\bar{C}$, with $y = P_o u + v$ and $u = r - \bar{C}y$. Note that C was used in the construction of Q in (1) and that $\bar{C}$ denotes the controller used in the closed-loop experiments. Following this, application of the feedback law $u = r - \bar{C}y$ to all models $P \in \mathcal{P}$ in (1) yields a set of closed-loop models $\mathcal{S}$ that is structured as follows.

$$\mathcal{S} = \{S \mid S = \mathcal{F}_u(M,\Delta) \text{ with M given by}$$

$$\begin{array}{rcl} M_{11} & = & V(\hat{D} + \bar{C}\hat{N})^{-1}(\bar{C} - C)D_c \\ M_{12} & = & V(\hat{D} + \bar{C}\hat{N})^{-1} \\ M_{21} & = & (I + \hat{P}\bar{C})^{-1}(I + \hat{P}C)D_c \\ M_{22} & = & (I + \hat{P}\bar{C})^{-1}\hat{P} \end{array} \quad (4)$$

$$\text{and } \Delta \in \mathbb{R}\mathcal{H}_\infty,\ \|\Delta\|_\infty < 1\}$$

Using the definition of the coprime factor uncertainty model given above, the model validation problem can be summarized next.

B. Model Validation Problem

To facilitate brevity of the results, only the model validation results for the case of an uncertainty due to undermodeling are mentioned here. In the application we will combine the effects of noise and undermodeling into a single uncertainty contribution. Results that separate undermodeling from noise perturbations on the frequency domain measurements can be achieved by extending the structure of the uncertainty model with an additional bounded but unknown perturbation that is used to model the effect of the noise [13].

Consider a closed-loop system where a reference signal r is applied and a noise-free system response y is measured. For model validation purposes, the frequency domain data of the closed-loop system can be described by

$$\mathcal{F}_u(\hat{M},\Delta) = 0,\ \omega \in \Omega \quad (5)$$

where the entries of $\hat{M}$ are given by

$$\begin{array}{rclrcl} \hat{M}_{11} & := & M_{11}(\omega) & \hat{M}_{12} & := & -M_{12}(\omega)R(\omega) \\ \hat{M}_{21} & := & M_{21}(\omega) & \hat{M}_{22} & := & Y(\omega) - M_{22}(\omega)R(\omega) \end{array} \quad (6)$$

In (6) the entries of $\hat{M}$ are frequency dependent functions where $\omega \in \Omega$ and $Y(\omega)$ and $R(\omega)$ are the respective Discrete Fourier Transforms of the signals $y(t)$ and $r(t)$. The uncertainty Δ models the effect of unknown but bounded errors due to model approximation and can possibly include unknown but bounded noise disturbances on the Discrete Fourier Transforms of the signals $y(t)$ and $r(t)$.

Using the above assumptions with the knowledge of the uncertainty model represented in M, the closed-loop model validation problem can be summarized as follows.

Closed-loop model validation problem: consider the closed-loop measurements $Y(\omega)$ and $R(\omega)$, $\omega \in \Omega$. The closed-loop uncertainty model is not invalidated by the data if there exists a Δ with $\|\Delta\|_\infty < 1$ such that (5) holds.

For the closed-loop model validation problem the objective is to determine whether there exists a stable perturbation Δ with $\|\Delta\|_\infty < 1$ such that (5) holds. In the next section, the main model validation result is presented.

C. Main Result

The reader is referred to [13] for a complete analysis of the closed-loop model validation results presented in this section. Restricting the results to the noise-free case, or more practically feasible, to the situation where the noise on the closed-loop data is modeled as part of the uncertainty Δ on the nominal model $\hat{P}$, the following result can be summarized.

Theorem 1: Model Validation
Let $Y(\omega)$ and $R(\omega)$ denote the frequency response measurements of the feedback controlled system P_0 and let the entries of $\hat{M}$ be defined as in (6), then the uncertainty model is not invalidated by $Y(\omega)$ and $R(\omega)$ iff $\mu_\Delta(\hat{M}_{11} - \hat{M}_{12}\hat{M}_{22}^{-1}\hat{M}_{21}) > 1$ where $\mu_\Delta(\cdot)$ is computed with respect to the uncertainty structure Δ.
The proof of this result is based on the fact that the inverse of an LFT is again an LFT. In order for the inverse not to exist, e.g. $\mathcal{F}(\hat{M}, \Delta) = 0$, $\hat{M}_{11} - \hat{M}_{12}\hat{M}_{22}^{-1}\hat{M}_{21}$ must be singular.

Note, evaluation of $\mu_\Delta(\hat{M}_{11} - \hat{M}_{12}\hat{M}_{22}^{-1}\hat{M}_{21}) > 1$ is done frequency point-wise over $\omega \in \Omega$. In case Δ is unstructured $\mu_\Delta(\hat{M}_{11} - \hat{M}_{12}\hat{M}_{22}^{-1}\hat{M}_{21}) > 1$ can be replaced with the maximum singular value $\overline{\sigma}(\hat{M}_{11} - \hat{M}_{12}\hat{M}_{22}^{-1}\hat{M}_{21}) > 1$ or the minimum singular value $\underline{\sigma}(\hat{M}_{11} - \hat{M}_{12}\hat{M}_{22}^{-1}\hat{M}_{21}) < 1$. Since the validation problem is performed frequency point-wise, the model validation is decomposed into consistency problems evaluated over the frequency grid Ω. The consistency problems check the existence of $\Delta(w)$ with $\overline{\sigma}(\Delta(w)) < 1$ for $w \in \Omega$. In order to guarantee the existence of a $\Delta \in \mathbb{R}\mathcal{H}_\infty$ with $\|\Delta\|_\infty < 1$, a boundary interpolation result [3], [10] can be used where the result is summarized in the following lemma.

Lemma 1: Let $\overline{\sigma}(\Delta(w)) < 1 \; \forall \; w \in \Omega$, then $\exists \; \Delta \in \mathbb{R}\mathcal{H}_\infty$ with $\|\Delta\|_\infty < 1$.

This boundary interpolation result is used in the model validation problem addressed in this paper and a complete proof of Lemma 1 is given in [10].

Note that the continuity property of Lemma 1 applies to Theorem 1 and illustrates that if $\mu_\Delta(\hat{M}_{11} - \hat{M}_{12}\hat{M}_{22}^{-1}\hat{M}_{21}) > 1$, $\exists \; \Delta(w)$ with $\overline{\sigma}(\Delta(w)) < 1 \forall \; w \in \Omega$ and with Lemma 1, $\exists \; \Delta \in \mathbb{R}\mathcal{H}_\infty$ with $\|\Delta\|_\infty < 1$. Hence, it is sufficient to evaluate $\mu_\Delta(\cdot) > 1$ only at a specific frequency grid. When each frequency-wise evaluation of $\overline{\sigma}(\Delta(w)) < 1$ holds, we correctly conclude that the model cannot be invalidated by the data.

IV. APPLICATION AND COMPARISON OF CLOSED-LOOP MODEL VALIDATION

The model validation technique presented in the previous section is tested on sample data obtained from the flexible structure to illustrate the model validation results for closed-loop based uncertainty models. For comparison, both the coprime factor based uncertainty model and an open-loop based uncertainty model is used. The open-loop uncertainty model does not use the knowledge of the feedback controller and employs a multiplicative perturbation to describe the modeling errors and noise on the data.

A. Uncertainty Modeling

For comparison, we create two uncertainty models: one based on a multiplicative perturbation and one based on the coprime factor perturbation model discussed in this paper. The multiplicative uncertainty model is described by

$$\mathcal{P}_m = \{P \mid P = \hat{P}(1 + V_m\Delta) \text{ with } \|\Delta\|_\infty < 1\}$$

and the coprime factor uncertainty model $\mathcal{P}_{cf}$ has been described in (2). Combining modeling errors and noise on the open-loop frequency domain data in the uncertainty model, the multiplicative uncertainty perturbation is found by

$$\Delta_m = (\Phi_{ol} - \hat{P})/\hat{P}$$

using the open-loop frequency domain data $\Phi_{ol}(\omega)$. Similarly, the coprime factorization based uncertainty can be found relatively easily via

$$\Delta_{cf} = D_c^{-1}(I + \Phi_{ol}C)^{-1}(\Phi_{ol} - \hat{P})\hat{D}$$

using the open-loop frequency domain data $\Phi_{ol}(\omega)$. The amplitude bode plot of the resulting multiplicative and coprime factor perturbation Δ_m and Δ_{cf} are shown in Figure 5.

The weighting filters that over-bound the respective uncertainty Δ_m and Δ_{cf} for each uncertainty model are also shown in Figure 5. To complete the development of the uncertainty models $\mathcal{P}_m$ and $\mathcal{P}_{cf}$, it is necessary to determine appropriate parametric over-bounds of the estimated uncertainty Δ_m and Δ_{cf}. For that purpose, stable and stably invertible weighting filters are used to normalize the unknown but bounded uncertainty. In describing the multiplicative over

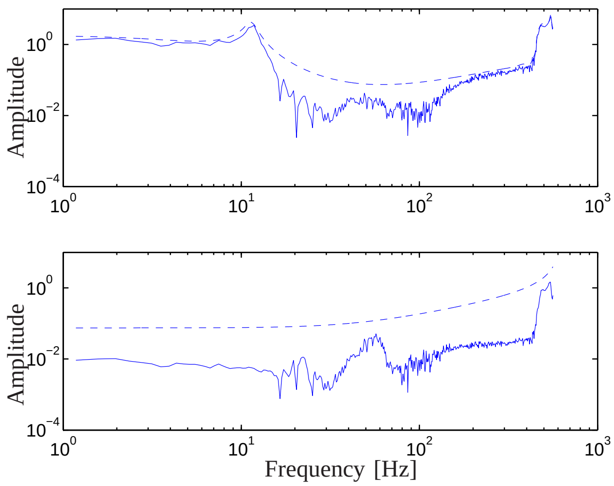

Fig. 5. Amplitude bode plot (solid) of perturbation and parametric upper bound (dashed) Δ_m (top) and Δ_{cf} (bottom)

bound V_m it is important that V_m maintains a tight bound of Δ_m in order to satisfy the standard robust stability test

$$\|V_m(I+\bar{C}\hat{P})^{-1}\bar{C}\hat{P}\|_\infty < 1 \quad (7)$$

for a multiplicative uncertainty description. Obviously, if the robust stability condition is not met, then $\mathcal{P}_m$ is not able to guarantee stability robustness and the uncertainty model $\mathcal{P}_m$ could be invalidated by closed-loop data.

The evaluation of the robust stability test (7), forces V_m to be a tight overbound of the multiplicative uncertainty Δ_m as indicated in Figure 5. As a result, a high (fourteenth-order) stable and stably invertible transfer function has to be used to over-bound the uncertainty Δ_m, without compromising the robust stability condition evaluated for V_m in Figure 6.

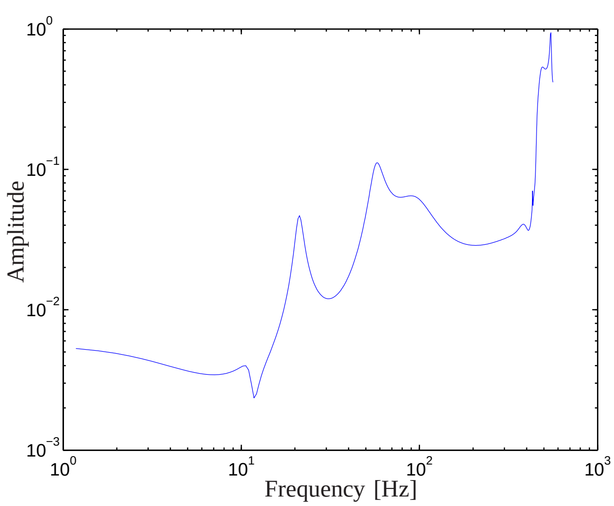

Fig. 6. Robust stability test for multiplicative uncertainty model $\mathcal{P}_m$

Comparatively, for the coprime factor uncertainty Δ_{cf} a low order weighting function V_{cf} can be used to over-bound the uncertainty data Δ_{cf}. Since the controller used in the development of the coprime factor uncertainty model is the same controller used to check the robust stability condition, robust stability is trivially satisfied. With the Youla-Kucera parameterization all models are initially parameterized so that they are stabilized by a known feedback controller C, irrespective of the size or shape of Δ. Since the magnitude of V_{cf} does not play a role in the trivially satisfied robust stability condition, it is acceptable to use an overly conservative weighting filter, as indicated in Figure 5.

Comparison of the two uncertainty models also highlights the increased complexity of the multiplicative uncertainty model $\mathcal{P}_m$ verses the coprime factor uncertainty model $\mathcal{P}_{cf}$. Although both the controller and weighting function V_{cf} are used in formulating $\mathcal{P}_{cf}$, the very high order weighting function V_m considerably increases the total complexity of the multiplicative model $\mathcal{P}_m$.

B. Comparison of Closed-Loop Model Validation

After noting that the stability robustness test was satisfied for both uncertainty models $\mathcal{P}_m$ and $\mathcal{P}_{cf}$, the models were then used to perform closed-loop model validation as described in Theorem 1. Following this, for the multiplicative model validation test consider the M matrix given by

$$\begin{aligned} M_{11} &= -V_m(I+\bar{C}\hat{P})^{-1}\bar{C}\hat{P} \\ M_{12} &= V_m(I+\bar{C}\hat{P})^{-1}\hat{P} \\ M_{21} &= (I+\hat{P}\bar{C})^{-1} \\ M_{22} &= (I+\hat{P}\bar{C})^{-1}\hat{P} \end{aligned} \quad (8)$$

where (8) describes how the nominal model is structured within a multiplicative uncertainty description. The model validation test for both uncertainty models determines whether $\bar{\sigma}(\hat{M}_{11} - \hat{M}_{12}\hat{M}_{22}^{-1}\hat{M}_{21}) > 1$ where $\hat{M}$ is given by (6). For the multiplicative uncertainty description consider M given by (8) and for the coprime factor uncertainty description consider M given in (4). Note that the uncertainties Δ_m and Δ_{cf} are unstructured so $\mu(\cdot)$ can be replaced by $\bar{\sigma}(\cdot)$. Note also that $Y(\omega)$ and $R(\omega)$ denotes the frequency response measurement of the closed-loop system Φ_{cl}.

By comparing the model validation results in Figure 7, it can be seen that the closed-loop data invalidates the multiplicative uncertainty model $\mathcal{P}_m$ for the entire frequency range. Since the coprime factor uncertainty model validation test holds for every frequency point, Φ_{cl} does not invalidate the coprime factor uncertainty model $\mathcal{P}_{cf}$.

C. Summary of Results

The model validation of the flexible structure illustrates the practical application and benefits of uncertainty modeling using coprime factorizations. It can be seen that the multiplicative uncertainty model is invalidated by the closed-loop data Φ_{cl}, whereas the coprime factor uncertainty model cannot be invalidated. This indicates that the coprime factor

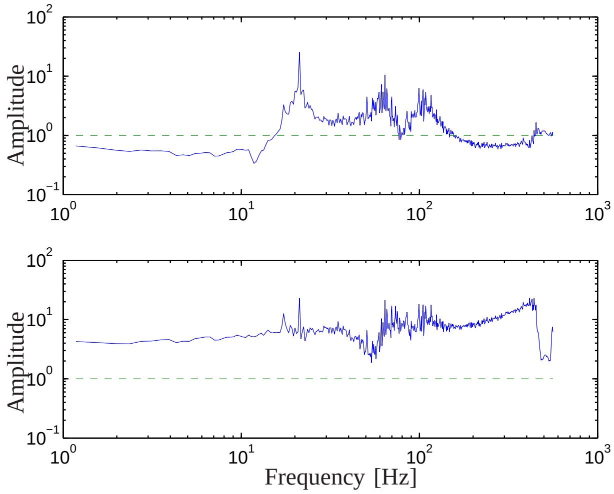

Fig. 7. Model Validation Test, multiplicative uncertainty description (top) and coprime factor uncertainty description (bottom)

uncertainty model is more suited for the closed-loop model validation problem presented in this paper.

Since the coprime factor based uncertainty set is parameterized using the knowledge of a controller, robust stability is trivially satisfied for this feedback controller. As a result, coprime factor uncertainty modeling allows more freedom in choosing a weighting filter that over-bounds the coprime factor uncertainty. It can be observed from the application that there is a significant discrepancy between the frequency domain data and the nominal model at high frequencies. Since high frequency modeling errors are not important for the lead/lag feedback controller used in the closed-loop experiments, it is not necessary to insure a close fit between the data and the model in that region. Note that this effect also carries over into the model validation results.

V. CONCLUSIONS

The model validation problem presented in this paper determines whether a model is capable of reproducing closed-loop measurement data and whether a model is appropriate for control design purposes. The closed-loop relevant model (in)validation problem is solved using uncertainty models with coprime factor perturbations and a frequency point-wise evaluation of the singularity of a Linear Fractional Transformation. Important in this formulation of the uncertainty model is the dependency on the controller, creating a closed-loop oriented model (in)validation of the uncertainty model.

The procedure is illustrated on the experimental data of a flexible structure, where both the coprime factor uncertainty model and a standard open-loop multiplicative uncertainty model are subject to a closed-loop validation problem. It is shown in the application example that only the coprime factor based uncertainty structure is able to validate the model successfully.

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