

**CONTROL RELATED TOPICS IN IDENTIFICATION -  
CLOSED LOOP EXPERIMENTS AND IDENTIFICATION  
FOR CONTROL**

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**Abstract:** This paper summarizes the organization of a graduate course on control related topics in identification. The course is taught at the department of Mechanical and Aerospace Engineering at the University of California, San Diego and analyzes the problems of estimation on closed-loop data and control relevant approximation of systems. As part of the course, several case studies are reviewed and include the well documented examples of a sugar cane crushing mill and the identification of the marginally stable electromechanical system found in a CD-ROM player.

**Keywords:** identification; education; closed-loop; electromechanical systems; process control

## 1. INTRODUCTION

Experimental data and system identification techniques can be used to estimate dynamical models that are useful in control applications. Models useful for control design are typically of low order and capture the essential closed-loop dynamical behavior needed for control design purposes. Instead of optimizing models for standard prediction or simulation objectives, control relevant models and control related identification are optimized for closed-loop control objectives.

The theory of Prediction Error methods can be extended to include the estimation of linear dynamical models from time domain observations obtained under closed-loop or feedback controlled conditions. Items such as bias, variance, experiment design, and closed-loop optimality are of concern during the experiments and estimation of control-relevant models. The expertise in the area on control relevant and closed-loop

system identification can be combined in a graduate course on system identification.

The graduate course *MAE283B Approximate Identification and Control* is a second course in System Identification offered in the Mechanical & Aerospace Engineering Department at the University of California, San Diego. Its place in the course sequence follows *MAE283A Parametric Identification: theory and methods* and, for many students, *MAE284 Robust and Multivariable Control*. The prescribed text is Lennart Ljung's textbook (Ljung 1999), although some material is also taken from the first edition (Ljung 1987). Next to this standard material, various research papers are included in the course and the recently published text book on *Iterative Identification and Control* (Albertos and Sala 2002) which contains many theoretical results and relevant applications for this course.

The primary focus of the material in this course is to provide students with understanding of the interconnections between modeling and control with a concentration on approximate models derived and validated using experimental data. Recently emerging

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techniques of iterative (closed-loop) system identification and (model-based) control design were a feature using sophisticated practical examples as a central pedagogical tool. Because of the emphasis on approximation in modeling, the thrust of the material is towards understanding compromises (bias and variance versus excitation) of model fitting versus questions of consistency or asymptotic normality.

## 2. PRESENTATION OF BIAS ANALYSIS

### 2.1 Frequency domain expressions

Although prediction errors methods provide a framework for model estimation, challenging problems in the field of system identification lie in the area of approximation of complex systems for control design purposes. Models intended for control design, may require a good approximation of the critical closed-loop behavior of the system to design reliable robust controllers. The objective of finding approximate models becomes even more challenging when closed-loop observations need to be used for identification purposes.

The frequency-domain formulation of Linear System Identification in a quasi-stationary setting is used. For the filtered prediction error

$$\varepsilon_f(t, \theta) = L(q)\varepsilon(t, \theta), \quad \varepsilon(t, \theta) := y(t) - y(t|t-1, \theta)$$

where  $y(t)$  is a measured output signal and  $y(t|t-1, \theta)$  denotes the one-step ahead prediction, the quasi-stationary setting allows variance properties of the prediction error to be represented in the frequency domain

$$\bar{E}\{\varepsilon_f^2(t, \theta)\} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \Phi(\omega, \theta) d\omega \quad (1)$$

where the frequency domain integral expression relies on the asymptotic expressions for the variance of the prediction error and application of Parseval's formula.

### 2.2 Open-loop analysis

To present the bias results by means of the standard integral expressions, first a slightly modified version of the standard and intuitively appealing open-loop bias expression from (Ljung 1987) is presented. In the standard open-loop framework the data generating system is represented by

$$y(t) = G_0(q)u(t) + v(t), \quad v(t) = H_0(q)e(t)$$

where, for the moment, the additive noise  $v(t) = H_0(q)e(t)$  on the output  $y(t)$  is assumed to be uncorrelated with the input  $u(t)$ . The prediction error  $\varepsilon(t, \theta)$  can be written as

$$\varepsilon(t, \theta) = H(q, \theta)^{-1}((G_0(q) - G(q, \theta))u(t) + (H_0(q) - H(q, \theta))e(t) + e(t)) \quad (2)$$

where  $e(t)$  is a white noise with variance  $\lambda_0$ . As both  $H_0(q)$  and  $H(q, \theta)$  are monic noise filters and  $e(t)$

is a white noise,  $\bar{E}\{e(t)\tilde{e}(t)\} = 0$  where  $\tilde{e}(t) := (H_0(q) - H(q, \theta))e(t)$ . Due to the open-loop experiments,  $\bar{E}\{e(t)u(t-\tau)^T\} = 0 \forall \tau$  and as a result, the spectrum  $\Phi(\omega, \theta)$  of the (filtered) prediction error in (2) is given by

$$\begin{aligned} & |G_0 - G_\theta|^2 \Phi_u \frac{|L|^2}{|H_\theta|^2} + \\ & |H_0 - H_\theta|^2 \lambda_0 \frac{|L|^2}{|H_\theta|^2} + \lambda_0 \end{aligned} \quad (3)$$

where the arguments of  $e^{j\omega}$  and  $\theta$  have been dropped for brevity and clarity of the bias formula. The result in (3) can be used to explain the tradeoff in approximate open-loop identification:

- Optimization of  $\theta$  aims at 'whitening' the prediction error, as  $G(q, \theta) = G_0(q)$  and  $H(q, \theta) = H_0(q)$  (consistent estimation) makes  $\Phi(\omega, \theta) = \lambda_0$ .
- Choice of a fixed (stable and stably invertible) noise filter  $H_*(q)$  is equivalent to the choice of a prediction error filter  $L(q) = H_*^{-1}(q)$ .
- In case joint parameters occur in the parametrization of  $G(q, \theta)$  and  $H(q, \theta)$ , there is a tradeoff between modeling  $H_0$  versus  $G_0$ . It can be seen from (3) that this tradeoff is highly determined by the signal to noise ratio  $\Phi_u(\omega)/\lambda_0$ .

For control applications, the modeling and approximation of  $G_0$  is often considered of more importance than the noise dynamics  $H_0$ . Certainly, this is true for stability as the estimated (low order) model  $G(q, \hat{\theta})$  is used for control design purposes and closed-loop stability solely depends on the properties of  $G_0$ . The tradeoff between modeling  $H_0$  versus  $G_0$  in approximate identification can be eliminated by either choosing a fixed noise filter (Output Error model structure) or an independent parametrization (Box-Jenkins model structure). Both come with the price of an unavoidable non-linear optimization, but allow an explicit bias tuning.

### 2.3 Closed-loop analysis

One of the problems in approximate closed-loop identification of plant and noise dynamics on the basis of closed-loop data, is the correlation of the noise  $e(t)$  with the signals  $\{u(t), y(t)\}$  in the closed-loop. Due to the noise correlation, an approximate identification method that directly uses the input and output of the plant  $G_0$  and ignores the feedback, will lead to biased approximation results for the system dynamics.

The spectrum  $\Phi(\omega, \theta)$  of the (filtered) prediction error in (2) in case  $u(t)$  is correlated with  $e(t)$  has been summarized in (Ljung 1999) as follows

$$\frac{L}{|H_\theta|^2} [\bar{G} \quad \bar{H}] \begin{bmatrix} \Phi_u & \Phi_{ue} \\ \Phi_{eu} & \lambda_0 \end{bmatrix} \begin{bmatrix} \bar{G}^* \\ \bar{H}^* \end{bmatrix} + \lambda_0$$

where the arguments of  $e^{j\omega}$  and  $\theta$  have been dropped for brevity and  $\bar{G} := (G_0 - G_\theta)$ ,  $\bar{H} := (H_0 - H_\theta)$

and  $\Phi_{ue}$  indicates the correlation between  $u$  and  $e$ . By using Schur's complement

$$\begin{bmatrix} \Phi_u & \Phi_{ue} \\ \Phi_{eu} & \lambda_0 \end{bmatrix} = \begin{bmatrix} I & 0 \\ \frac{\Phi_{eu}}{\Phi_u} & I \end{bmatrix} \begin{bmatrix} \Phi_u & 0 \\ 0 & \lambda_0 - \frac{|\Phi_{eu}|^2}{\Phi_u} \end{bmatrix} \begin{bmatrix} I & \frac{\Phi_{ue}}{\Phi_u} \\ 0 & I \end{bmatrix}$$

with respect to  $\Phi_u$  and definition of

$$B(e^{j\omega}, \theta) = \frac{(H_0(e^{j\omega}) - H(e^{j\omega}, \theta))\Phi_{ue}(\omega)}{\Phi_u(\omega)}$$

allows the spectrum  $\Phi(\omega, \theta)$  of the (filtered) prediction error in (2) to be written as

$$\begin{aligned} & |G_0 + B_\theta - G_\theta|^2 \Phi_u \frac{|L|^2}{|H_\theta|^2} + \\ & |H_0 - H_\theta|^2 \left( \lambda_0 - \frac{|\Phi_{ue}|^2}{\Phi_u} \right) \frac{|L|^2}{|H_\theta|^2} + \lambda_0 \end{aligned} \quad (4)$$

and illustrates the bias effect in case  $\Phi_{ue}(\omega) \neq 0$ :

- Optimization of  $\theta$  still aims at 'whitening' the prediction error, as  $H_\theta = H_0$  yields  $B_\theta = 0$  and  $G_\theta = G_0$  (consistent estimation) makes  $\Phi(\omega, \theta) = \lambda_0$ .
- In an approximate identification where  $H_\theta \neq H_0$ , the model  $G_\theta$  will approximate  $G_0 + B_\theta$  even in the case where there exists a parameter  $\theta$  for which  $G_\theta = G_0$ .  $B_\theta$  clearly indicates an undesirable bias for the plant model  $G_\theta$ .
- Due to the ratio of  $L$  and  $H_\theta$  in (4), the choice of a fixed (stable and stably invertible) noise filter  $H_*$  remains equivalent to the choice of a prediction error filter  $L = H_*^{-1}$ .

In case joint parameters occur in the parametrization of  $G(q, \theta)$  and  $H(q, \theta)$ , there is again a tradeoff between modeling  $H_0$  versus  $G_0 + B_\theta$  in approximate identification. Unfortunately, the tradeoff cannot be eliminated by choosing a fixed noise filter (Output Error model structure) or an independent parametrization (Box-Jenkins model structure) to focus on the approximate identification of  $G_0$  only. The choice of a fixed noise filter  $H_* \neq H_0$  would be unable to eliminate the bias term

$$B_* = \frac{(H_0 - H_*)\Phi_{ue}}{\Phi_u}$$

resulting in a biased estimation of the plant dynamics  $G_0$  that highly depends on the chosen noise filter  $H_*$  and the (unknown) noise dynamics  $\Phi_{ue}$ .

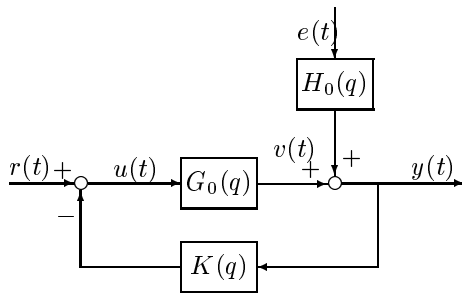


Fig. 1. Closed-loop system with reference signal

The bias effects can be illustrated to the students, by analyzing the correlation between input  $u(t)$  and noise

$e(t)$  for a closed-loop system as indicated in Figure 1. With Figure 1, the data coming from the plant  $G_0(q)$  and subjected to external reference signal  $r(t)$  and additive noise  $H_0(q)e(t)$  operating under closed-loop condition can be described as follows:

$$\begin{aligned} y(t) &= G_0(q)S_{in}(q)r(t) + S_{in}(q)H_0(q)e(t) \\ u(t) &= S_{in}(q)r(t) - K(q)S_{in}H_0(q)e(t) \end{aligned} \quad (5)$$

where  $S_{in}(q)$  is the input sensitivity function defined by

$$S_{in}(q) = \frac{1}{1 + K(q)G_0(q)}$$

With the reference signal  $r(t)$  uncorrelated with the noise  $e(t)$

$$\begin{aligned} B_\theta &= (H_0 - H_\theta) \frac{\Phi_{ue}}{\Phi_e} \frac{\Phi_e}{\Phi_u} \\ &= (H_0 - H_\theta) K S_{in} H_0 \frac{\lambda_0}{\Phi_u} \end{aligned}$$

which clearly indicates the (implicit) character of the bias  $B_\theta$  in case of closed-loop experiments. The bias  $B_\theta$  depends on the noise model  $H_\theta$  being estimated, the way in which the noise is present on the input ( $K S_{in} H_0$ ) and the signal to noise ratio  $\Phi_u/\lambda_0$ .

The effect of the bias can also be illustrated for the extreme situation where no reference  $r(t)$  is present on the closed-loop system to provide sufficient excitation. For that purpose, an alternative formulation is used: substitution of (5) in the formulation of the prediction error (2) yields the following prediction error

$$\begin{aligned} \varepsilon(t, \theta) &= H_\theta^{-1} S_{in} (G_0 - G_\theta) r(t) + \\ & H_\theta^{-1} S_{in} ((I + G_\theta K) H_0 - H_\theta) e(t) + S_{in} e(t) \end{aligned} \quad (6)$$

With at least one step delay in the product  $G_\theta K$ , both  $(1 + G_\theta K)H_0(q)$  and  $H_\theta$  are monic filters. As a result,  $\bar{E}\{e(t)\tilde{e}(t)\} = 0$ , as  $e(t)$  is a white noise signal, where  $\tilde{e}(t) := ((1 + G_\theta K)H_0 - H_\theta)e(t)$ . With the reference signal  $r(t)$  uncorrelated with  $e(t)$  we also find  $\bar{E}\{e(t)r(t - \tau)^T\} = 0 \forall \tau$  and the spectrum  $\Phi(\omega, \theta)$  of the (filtered) prediction error in (2) can be written as

$$\begin{aligned} & |G_0 - G_\theta|^2 \Phi_r \frac{|S_{in}|^2 |L|^2}{|H_\theta|^2} + \\ & |(1 + G_\theta K)H_0 - H_\theta|^2 \lambda_0 \frac{|S_{in}|^2 |L|^2}{|H_\theta|^2} + |S_{in}|^2 \lambda_0 \end{aligned} \quad (7)$$

which also gives clear insight in the bias effects. In case of lack of external excitation of the closed-loop system it can be seen that:

- The estimation of models  $G_\theta$  and  $H_\theta$  is done such that  $H_0 - H_\theta + H_0 G_\theta K$  is minimized. As a result, the actual plant  $G_0$  does not play a role in the estimation of  $G_\theta$ .
- In case a fixed noise model  $H_*$  is chosen (OE model structure), the optimal model  $G_\theta$  is given by  $G_\theta = -K^{-1}$ , and we would be estimating the inverse of the controller.

The analysis presented above demonstrates clearly to the students that the lack of consistency and a tunable

bias expression for the estimation of the model  $G_\theta$  and  $H_\theta$  in closed-loop identification.

### 3. PRESENTATION OF VARIANCE RESULTS

To complete the analysis and provide students with the concepts on the tradeoff between bias and variance in typical System Identification problems, the standard variance result in (Ljung 1999) are presented. The results are presented without going into the details of the technical conditions involved with the formulation of the variance results.

Consider estimate  $\hat{\theta}_N$  determined by minimizing the Least Squares criterion

$$V_N = \frac{1}{2N} \sum_{t=1}^N \varepsilon^2(t, \theta)$$

and define

$$\psi(t, \theta_*) = \left. \frac{\partial \varepsilon(t, \theta)}{\partial \theta} \right|_{\theta=\theta_*}$$

then subject to the technical conditions:

- model structure is linear and uniformly stable
- data are generated by stable linear filtering of quasistationary signals with finite moments of  $4+\delta$
- $\theta_N \rightarrow \theta_*$  w.p.1 as  $N \rightarrow \infty$  for bounded  $\theta_*$
- $\frac{\partial^2 V_\theta}{\partial \theta^2} > 0$
- $\sqrt{N} \bar{E} \left\{ \frac{1}{N} \sum_{t=1}^N \psi(t, \theta_*) \varepsilon(t, \theta_*) - \bar{E} \psi(t, \theta_*) \varepsilon(t, \theta_*) \right\} \rightarrow 0$  as  $N \rightarrow \infty$

the following asymptotic variance expression of the parameter estimate  $\hat{\theta}_N$  can be given (Ljung 1999):

$$\sqrt{N}(\hat{\theta}_N - \theta_*) \sim N(0, P_\theta)$$

$$P_\theta = [V''(\theta_*)]^{-1} Q [V''(\theta_*)]^{-1} \quad (8)$$

$$Q = \lim_{N \rightarrow \infty} N \bar{E} \{ [V'(\theta_*, N)] [V'(\theta_*, N)]^T \}$$

The background and technical implications behind the variance expression formula are emphasized by mentioning that the result deals with the asymptotic normality and variance of the parameter error. Furthermore it must be stressed that it is possible to extend the parameter variance expression to the frequency domain in a direct way.

$$\text{Cov} \begin{bmatrix} \hat{G}_N(e^{j\omega}) \\ \hat{H}_N(e^{j\omega}) \end{bmatrix} \sim \frac{n}{N} \begin{bmatrix} \Phi_u(\omega) & \Phi_{eu}(\omega) \\ \Phi_{ue}(\omega) & \lambda_0 \end{bmatrix}^{-1} \quad (9)$$

The frequency domain expression of (9) treats the transfer function estimation error distribution. The limitations of the variance expression are pointed out to the students by mentioning that the frequency domain variance expression in (9):

- is asymptotically valid only as  $N \rightarrow \infty$
- relies on weak convergence — no large deviations

- fundamentally assumes  $(G_0, H_0)$  has a parametrization with  $\theta = \theta_0$  and  $\theta_N \rightarrow \theta_0$
- indicates that the covariance increases with the number of parameters  $n$ , and decreases with the number of data points  $N$
- depends in a sensible way on noise to signal ratio
- the exact calculation depends on the criterion used and provides the measure of asymptotic efficiency of the estimator.

Although limited in application, the variance expression results in (8) and (9) and the bias expressions in (4) and (7) give insight in the variance and bias tradeoff in experiment design and system identification. More detailed information on (closed-loop) variance expressions are summarized in the paper by Gevers *et al.* (2001) that is presented to the students during the course. It is illustrated that the spectrum of the reference or input signal can be used to influence bias and variance aspects during experiment design. For closed-loop experiments, the controller  $K$  also provides a valuable tool to influence the signal properties.

Post processing of the data (after the actual experiments) can be done by the choice of a suitable data filter  $L$  and the choice of the model class in the identification of plant model  $G_\theta$  and noise model  $H_\theta$ . Important design variables are the model structure with the number of parameters and the possibility to use an independent parametrization of plant and noise dynamics. But it must be stressed that identification on the basis of closed-loop data requires special attention: standard open-loop identification techniques that directly use the input  $u$  and output  $y$  signals of the plant is bound to give biased result. What remains is the motivation to perform closed-loop experiments, despite the pitfalls of biased estimation.

## 4. IDENTIFICATION AND CONTROL

### 4.1 Why closed-loop experiments?

To argue for the role of experiments measured in closed-loop, several basic results are mentioned to the students that motivate the usefulness of closed-loop experiments. One of the first steps towards the interaction between identification and control has been made in Åström and Wittenmark (1971) and Gevers and Ljung (1986). In Gevers and Ljung (1986) it is mentioned from a variance point of view that optimal models can be found via a Prediction Error estimation method that uses closed-loop experiments and appropriate data filters. As such, the usefulness of closed-loop experiments as opposed to open-loop experiments to model a plant was shown to be fruitful. Unfortunately, the desired closed-loop experiments and the appropriate data filter contains knowledge of the controller yet to be designed.

Argumentation of optimal models for control design from a bias point of view is built on the observation

that an approximate identification of a model is allowed, as long as the approximate model  $G_\theta$  takes into account its intended use – the design of a high performing controller  $K$  for the actual plant  $G_0$ . In case a norm function is used to characterize the performance of a feedback system, the performance of a controller  $K$  applied to the actual plant  $G_0$  can be delineated by  $\|J(G_0, K)\|$ . Even if a controller  $K$  is available, the performance  $\|J(G_0, K)\|$  cannot be evaluated precisely, as the plant  $G_0$  is unknown. From a bias point of view, the role of the model  $G_\theta$  can be seen as providing upper and lower bounds for  $\|J(G_0, K)\|$  via triangular inequalities

$$\left| \|J(G_\theta, K)\| - \|J(G_0, K) - J(G_\theta, K)\| \right| \leq \|J(G_0, K)\| \\ \|J(G_0, K)\| \leq \|J(G_\theta, K)\| + \|J(G_0, K) - J(G_\theta, K)\|$$

that were presented in Schrama (1992). For a given controller  $K$ , the minimization of  $\|J(G_0, K) - J(G_\theta, K)\|$  provides a tight upper and lower bound for  $\|J(G_0, K)\|$  and constitutes a so-called control relevant identification problem (Van den Hof and Schrama 1995). In this identification problem, a model  $G_\theta$  is found by minimizing the difference between closed-loop performance criteria. Obviously, closed-loop experiments are required to solve such an identification problem.

#### 4.2 Dealing with closed-loop experiments

Now that the setting and motivation for closed-loop experiments has been established, the methodologies for dealing with closed-loop data are presented. The direct approach consists of applying a standard open-loop prediction error method directly to the input  $u(t)$  and output  $y(t)$  signals, ignoring any possible feedback and the reference signal  $r(t)$ . From the analysis in Section 2.3 it is obvious that this approach leads to estimation results in case of approximate identification for which the bias cannot be tuned explicitly.

Following the analysis of identification on the basis of closed-loop data, possible solutions and methods to the closed-loop identification problem are presented in the course. The closed-loop identification methods are presented by providing a short overview of the method and a copy of the papers that summarize the details of the methodology. It is beyond the scope of this paper to present the methods detail, but it can be mentioned here that in the presentation of these methods, a distinction is made between the following approaches:

- The first class of methods presented in the course is based on a reparametrization of the closed-loop identification problem.
- The second class of methods presented to deal with closed-loop data are two-stage methods, where the estimation of approximate models on the basis of closed-loop data is solved in two estimation steps.

For the first class of methods, reparametrization is done by using the knowledge of the controller and parametrizing the closed-loop transfer function in terms of the controller and the open-loop model to be estimated. Methodologies that are reviewed in the course are the indirect estimation method, tailor-made parametrization (Landau and Karimi 1997) and recursive estimation methods (de Bruyne *et al.* 1999).

In the prediction error framework, the reparametrization of the closed-loop transfer function involves the minimization of a closed-loop prediction error

$$\varepsilon_{cl}(t, \theta) = y(t) - P(q, \theta)r(t)$$

where  $P(q, \theta)$  can be parametrized using a customized parametrization

$$P(q, \theta) = \frac{G(q, \theta)}{1 + K(q)G(q, \theta)} \quad (10)$$

using the explicit knowledge of the controller  $K(q)$ . Alternatively,  $P(q, \theta)$  can be freely parametrized and the knowledge of the controller is used to recompute the model  $G(q, \theta)$  via

$$G(q, \theta) = \frac{P(q, \theta)}{1 - P(q, \theta)K(q)} \quad (11)$$

In the customized parametrization (10) the order of the model  $G(q, \theta)$  can be controlled. The computation of the model  $G(q, \theta)$  via (11) in general increases the model order due to the free parametrization of the closed-loop transfer function  $P(q, \theta)$ .

For the second class or the two-stage methods, the first step is used to create a (noise free) instrument that will be used in the second step to recast the closed-loop identification problem into an open-loop one. The instrument in the first step of the two-stage methods is typically a filtered closed-loop signal. With the closed-loop data given in (5), an estimate of the (input) sensitivity function  $S_{in}$  can be obtained by minimizing the closed-loop prediction error

$$\varepsilon_1(t, \theta) = u(t) - S(q, \theta)r(t)$$

in the first step of the method, where  $S(q, \theta)$  is used to model the (input) sensitivity function  $S_{in}(q)$  as accurately as possible. The estimated model  $\hat{S}(q) = S(q, \hat{\theta})$  can be used to create a filtered input signal

$$\hat{u}(t) := \hat{S}(q)r(t)$$

that will be uncorrelated with the noise  $e(t)$  present on the (unfiltered) input signal  $u(t)$ . The (noise free) instrument can be used to perform an equivalent open-loop identification problem by minimizing the prediction error

$$\varepsilon_2(t, \theta) = y(t) - G(q, \theta)\hat{u}(t)$$

in the second step of the method. Closed-loop identification methods that fall under this category are presented in the course and include the instrumental variable method (Ljung 1999), the two-stage method (Van den Hof and Schrama 1993) and the coprime factor based methods (Van den Hof *et al.* 1995, Anderson 1998).

### 4.3 Link between identification and control

Estimating approximate models suitable for control requires closed-loop experiments to approximate the closed-loop and control-relevant aspects of the system. This is illustrated by evaluating the bias expressions associated to the different the closed-loop identification methods. It is shown that for most of the methods presented in the course, the closed-loop prediction error exhibits a spectrum  $\Phi(\omega, \theta)$  that is given by

$$|G_0 - G_\theta|^2 \Phi_r \frac{|S_{in}|^2 |L|^2}{|H_\star|^2} \quad (12)$$

for a prediction error model estimation with a fixed noise filter  $H_\star$ . It can be observed that (12) is equal to the first term in (7) and the closed-loop identification methods have eliminated the bias effects due to the closed-loop noise. This allows for an explicit tuning of the bias expression of the model  $G_\theta$ .

The merits of approximate identification and the use of closed-loop data in estimating approximate models can be found in the triangular inequalities that represent the interaction between model based control and identification of models for control. The idea of alternately minimizing  $\|J(G_\theta, K) - J(G_0, K)\|$  in via system identification and  $\|J(G_\theta, K)\|$  via a control design problem forms a basis for many of the iterative schemes or control relevant identification approaches listed in the literature (Van den Hof and Schrama 1995). In such an iterative scheme, the control relevant identification of a (nominal) model  $G_\theta$  and the design of a model-based controlled  $K$  are applied iteratively with the aim to minimize the overall performance  $\|J(G_0, K)\|$  of the feedback controlled plant  $G_0$ .

### 4.4 Case studies

To illustrate the work that has been done in the field of control relevant identification and model based control design, a short overview of iterative methods is presented at the end of the course. The iterative identification and control methods are presented by means of applications and case studies that illustrate the effectiveness of closed-loop identification methods and model-based control design to obtain high performance feedback control systems.

The case studies that are presented at the end of the course are the “control relevant identification and servo design for a compact disc player” and the “iterative identification and control: a sugar cane crushing mill” that both can be found in Albertos and Sala (2002). Both case studies illustrate the use of closed-loop experiments and model-based control design to enhance the performance of a feedback controlled system. The case studies are used to demonstrate in concrete form the principles of closed-loop identification presented during the course.

## 5. SUMMARY

This paper shows the organization of a course that focuses on estimation techniques for closed-loop or feedback controlled systems. The course gives both a theoretical and practical introduction to closed-loop identification methods and control relevant experimentally based modeling. Instead of focusing on models that are optimized for standard prediction or simulation objectives, models are optimized for closed-loop control objectives. As part of the course, two case studies are reviewed: a sugar cane crushing mill and the identification of a marginally stable electromechanical system.

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