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Fixed Order H_∞ Control Design for Dual–Stage Hard Disk Drives

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1 Introduction

Crucial to the increase in aerial densities of future Hard Disk Drives (HDD) is the increase of the number of Tracks Per Inch (TPI). In order to achieve very high track densities new high bandwidth and very accurate servo systems will be necessary. One possible solution is the use of dual–stage actuators, in which a high-bandwidth and highly accurate micro-actuator (MA) is used in combination with a traditional Voice Coil Motor (VCM). In this case the nature of the servo controller becomes multivariable since both the VCM and the MA have to be controlled simultaneously.

The design of servo controllers for track following with dual–stage actuators has been an active area of research in the last few years. One may divide the current research into two categories: classical control approaches [1] and modern control approaches [2], that use optimal control theory and optimization tools to compute feedback controllers.

Modern control methods were specifically developed for multivariable systems and offer significant advantages over classical control methods. On the down side, they usually lead to controllers which are of order equal to or greater than the plant they were designed for. The aim of this paper is to present a systematic approach for the design of a fixed order servo controller and to illustrate this design on a piezoelectric MA designed by Hutchinson Technologies Inc (HTI). The complexity of the servo controller is limited to address costs and implementation issues in a commercial HDD. The systematic approach for such a low order servo controller design is developed in this paper by utilizing H_∞-optimization with a choice of specific closed loop weighting functions and additional constraints on the order of the controller.

2 Control Design

The block diagram in Figure 1 shows the controller architecture and weighting filters to be used for design purposes. P_{vcm} and P_{ma} are the transfer functions from VCM and MA input to head position output respectively, C is the controller to be designed, w is the track position and e is the Position Error Signal (PES). W_S is used to shape the sensitivity transfer function so as to obtain the desired error reduction, W_R determines the frequency separation of the actuators, and W_T and W_V are used to bound the overall and vcm energy respectively.

The controller C is to be designed so that all the signals

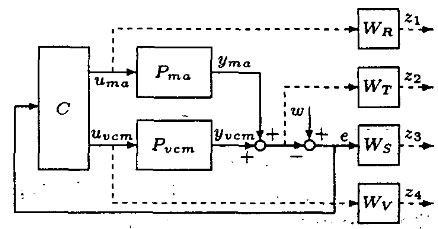

Figure 1: Plant with Weighting Filters

of interest will be bounded above by their weighting functions. This can be accomplished by requiring that $\|T_{zw}\|_\infty < 1$ where T_{zw} is the transfer function from w to vector z of outputs of the weighting filters. The use of an H_∞-norm based control design also allows a reasonably easily incorporation of modeling uncertainties in the servo control design. In this way, robust performing low order controllers can be designed that take into account inevitable product variability or modeling uncertainties that are present in commercial hard disk drives.

3 Fixed Order H_∞ Controllers

Let a minimal realization of a stabilizable and detectable (generalized) discrete time plant P be given by the following state space realization

$$\begin{aligned} x(k+1) &= Ax(k) + B_1 w(k) + B_2 u(k) \\ z(k) &= C_1 x(k) + D_{11} w(k) + D_{12} u(k) \\ y(k) &= C_2 x(k) + D_{21} w(k) \end{aligned} \quad (1)$$

where $A \in \mathbf{R}^{n \times n}$. For the dual-stage application the generalized plant P represents the connection of all the transfer functions in Figure 1 without the controller C. The existence of H_∞ optimal or suboptimal controllers of order n_c is fully characterized by the following result [3] which follows from the Bounded Real Lemma.

Theorem 1 *Let $\mathcal{N}_x$ and $\mathcal{N}_y$ denote orthonormal bases of the null spaces of (B_2^T, D_{12}^T) and (C_2, D_{21}) respectively. There exists a controller of order n_c which stabilizes the system and yields $\|T_{wz}\|_\infty < \gamma$ if and only if there exist symmetric matrices X and Y such that*

$$\mathcal{N}_x^T \begin{pmatrix} AXA^T - X + \frac{1}{\gamma}B_1B_1^T, & A^TXC_1^T + \frac{1}{\gamma}B_1D_{11}^T \\ C_1XA + \frac{1}{\gamma}D_{11}B_1^T, & C_1XC_1^T + \frac{1}{\gamma}D_{11}D_{11}^T - \gamma I \end{pmatrix} \mathcal{N}_x < 0 \quad (2)$$

$$\mathcal{N}_y^T \begin{pmatrix} A^TYA - Y + \frac{1}{\gamma}C_1^TC_1, & A^TYB_1 + \frac{1}{\gamma}C_1^TD_{11} \\ B_1^TYA + \frac{1}{\gamma}D_{11}^TC_1, & B_1^TYB_1 + \frac{1}{\gamma}D_{11}^TD_{11} - \gamma I \end{pmatrix} \mathcal{N}_y < 0 \quad (3)$$

$$\begin{pmatrix} X & I \\ I & Y \end{pmatrix} \geq 0 \quad (4)$$

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together with the rank constraint

$$\mathbf{Rank}\begin{pmatrix} X & I \\ I & Y \end{pmatrix} \leq n + n_c \tag{5}$$

For a fixed value of γ the constraints (2)–(4) are Linear Matrix Inequalities (LMI) and define a convex set in the variables (X, Y). It should be noted that the rank constraint is satisfied trivially when $n_c = n$ (full order case), which corresponds to the design of a controller that has the same order as the plant P. In the design of a reduced order controller or reduced order case, the rank constraint makes the problem non convex, so that efficient semi-definite programming (SDP) techniques are not applicable and other methods are needed. Given any solution (γ, X, Y) of (2)–(5), it is possible to construct a n_c-th order H_∞ controller by solving the Bounded Real Lemma inequality [3] for the controller data. This problem can be solved using SDP.

3.1 Solution via Numerical Optimization

Based on the above theorem, the synthesis of H_∞ controllers of order $n_c < n$ involves finding (γ, X, Y) so that the LMI's (2)–(4) and the rank constraint are satisfied. The rank constraint (5) is equivalent to requiring that the $k = n - n_c$ smallest eigenvalues of be zero. One method of enforcing the rank constraint is to enforce that $\lambda_k(4)$ be sufficiently small, and to then minimize with respect to γ. The problem formulated in these terms has the advantage that it can potentially yield a $n_c th$ order controller with the least performance degradation. This approach can be stated as:

$$\min_{\gamma, X, Y} \gamma \qquad \text{subject to:} \tag{6}$$

$$\lambda_{max}(2) < 0\,,\ \lambda_{max}(3) < 0\,,\ \lambda_{min}(4) > 0\,,\ \lambda_k(4) \leq \alpha$$

The value of α should be chosen small enough so that the rank constraint is approximately satisfied so that a controller of the desired order is obtainable. The minimization problem (6) is non-differentiable but is solved using standard constrained optimization routines.

4 Results

Figure 2 shows the magnitude plots for the transfer functions P_{vcm} and P_{ma} sampled at 20kHz. With the technique mentioned in Section 2 a full ($16th$) order controller was computed from the augmented plant model of 16 states. A low fixed order controller was also computed using the technique mentioned in Section 3, this yielded a $5th$ order controller that shows satisfactory performance. A comparison between the full and fixed order controllers is given in Figure 3where the controller transfer functions are shown. Both the full and fixed order controllers notch out the 9kHz suspension sway mode. The low order controller has sacrificed performance in terms of lower gain at low frequencies due to the limited design freedom in the controller to control and stabilize resonance modes in the dual-stage actuator. Table 1 summarizes some frequency domain and step response characteristics. It can be noted that

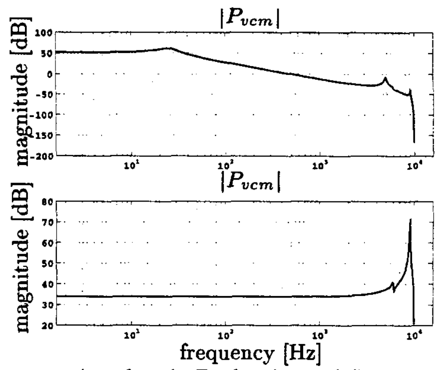

Figure 2: Amplitude Bode plots of P_{vcm} and P_{ma}

the 5th order controller performs very well and is completely adequate for application as a dual-stage servo controller. Compared to the full order controller 5th order controller has a settling time which is slightly worse and has more off-track ringing, but it does achieve a better over-shoot.

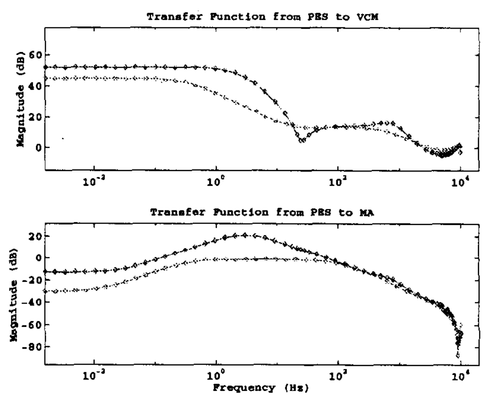

Figure 3: Controller Bode Plot: ◇ Full Order(16), ○ Fixed Order(5)

	BW kHz	GM dB	PM deg	rise ms	set ms	over %
$C(16)$	1.2	9.7	47	0.1	0.45	27
$C(5)$	1.0	6.6	54	0.1	0.7	23

Table 1: Performance Characteristics

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[3] P. Gahinet and P. Apkarian. A linear matrix inequality approach to H_∞ control. *Int. Journal of Robust and Nonlinear Control*, 4(4):421–448, July-August 1994.