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Cover page

Title : On-line damage identification using model based orthonormal functions

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ABSTRACT

In this paper, a new on-line damage identification method is proposed for monitoring the behavior of a dynamical system for structural dynamical changes. The method uses (approximate) knowledge of the dynamical system to be monitored in the form of a so-called linear dynamical reference model. The reference model is then used for two purposes. Firstly, the model is used to formulate a linear combination of model based orthonormal functions. This linear combination can be estimated and updated via a simple least squares optimization that can be implemented recursively for an on-line implementation. Secondly, the model with its model based orthonormal functions are used to detect plant parameter variations by monitoring the estimated parameters.

INTRODUCTION

In many of the existing fault detection or damage identification algorithms, in general both a parameter estimation technique and a reference model are used to detect plant parameter variations and to formulate an algorithm for plant damage detection [1, 12]. As can be seen from the methods presented in e.g. [2] or [7], a separation is made between the parameter estimation technique and the reference model or reference parameters.

This separation is indicated in Figure 1, where a schematic picture of the possible configuration of a monitoring or detection algorithm has been given. From Figure 1 it can be seen that a parameter estimation technique is used to analyse on-line measured data of input/excitation signal u and output signal y coming from the (unknown) dynamical system G_0 to be monitored. Subsequently, a reference model $\bar{G} = G(\bar{\theta})$ is used to monitor the system behavior of G_0. This can be done by monitoring parameter residuals θ_ε (differences between reference model parameters $\bar{\theta}$ and estimated parameters $\hat{\theta}$) and/or signal

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residuals ε generated by the reference model $\bar{G}$.

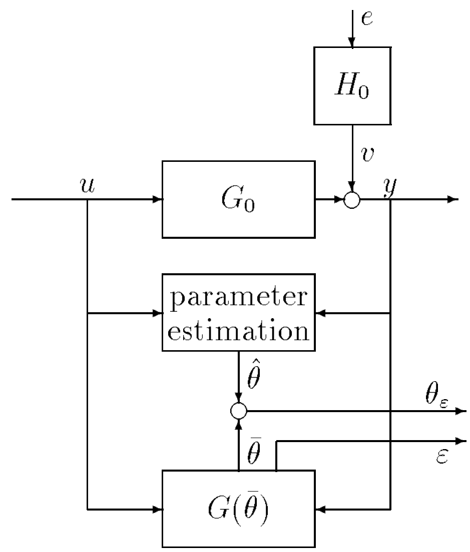

Figure 1: Parameter estimation (identification) and reference model $\bar{G}$ in a monitoring or detection algorithm

The parameter estimation technique may require an optimization to come up with a parameter estimate $\hat{\theta}$. Depending on the parametrization used during the optimization and the nature of the parameters present in the reference model $\bar{G}$, this optimization might require a number crunching non-linear optimization that is hard to implement in an on-line algorithm. Although the parameter estimation can be used to update the reference model $\bar{G}$ and detect plant variations, the knowledge of the dynamics in the reference model $\bar{G}$ is not used to facilitate the parameter estimation.

To accommodate the identification and the damage detection, the damage identification method proposed in this paper uses a reference model $\bar{G}$ that does facilitate and simplify the parameter estimation. To accomplish that objective, the reference model is formulated as a linear combination of so-called model based orthonormal functions [4]. It can be shown that the linear combination of the model based orthonormal functions can be estimated via a simple least squares optimization that has a unique and explicit computable minimum [9]. The estimation of the linear combination of the model based orthonormal functions estimation can be computed recursively for on-line implementations.

MODEL BASED ORTHONORMAL FUNCTIONS

Using Model Based Knowledge

Consider a reference model $\bar{G}$ as depicted in Figure 1. For analysis purposes, we assume that such a reference model is a linear time invariant (LTI) stable dynamical system that is given by the following (discrete time) minimal state space realization

$$\begin{aligned} qx(t) &= \bar{A}x(t) + \bar{B}u(t) \\ \bar{y}(t) &= \bar{C}x(t) + \bar{D}u(t) \end{aligned} \tag{1}$$

where q indicates the time shift operator with $qx(t) = x(t + \Delta T)$ and ΔT indicates the sampling time.

As mentioned before, the reference model in (1) represents the (approximate) knowledge of the dynamical behavior of the unknown system G_0 that needs to be monitored. This knowledge can also be represented in an input/output form using the z-transform of the difference equation $\bar{G}(q)$

$$\bar{y}(z) = \bar{G}(z)u(z)$$

where $\bar{G}(z)$ is given by

$$\begin{aligned} \bar{G}(z) &= \bar{D} + \bar{C}(zI - \bar{A})^{-1}\bar{B} \\ &= \bar{D} + \sum_{k=1}^{\infty} \bar{G}_k z^{-k}, \text{ with } \bar{G}_k = \bar{C}\bar{A}^{k-1}\bar{B}. \end{aligned} \tag{2}$$

The so-called Markov parameters $\bar{G}_k$ in the strictly proper part of $\bar{G}(z)$ in (2) form a linearly weighted combination of the orthonormal basis function z^{-k}. As a result, the (approximate) knowledge of the dynamical behavior of the unknown system G_0 that needs to be monitored is represented solely in the parameters $\bar{G}_k$ in (2). The orthonormal basis function z^{-k} only represent a time shift of k data samples.

To anticipate on the results mentioned in the next section, it can be mentioned here that the linear appearance of the model parameters $\bar{G}_k$ in (2) will be exploited for identification purposes. However, an alternative and more sensible linear parametrization of the reference model $\bar{G}(z)$ can be obtained by extending the orthonormal basis z^{-k} to include knowledge of the dynamical behavior of the model $\bar{G}(z)$. Such an extension will lead to a more generalized Markov expansion

$$\bar{G}(z) = \bar{D} + \sum_{k=1}^{\infty} L_k V_k(z) \tag{3}$$

where $V_k(z)$ for $k = \{1, 2, \ldots, \infty\}$ denotes a set of orthonormal basis function that include the dynamics of the (state space) model given in (1).

Construction of Orthonormal Basis Functions

The main results on the construction of an alternative set of orthonormal basis functions $V_k(z)$ for the expansion in (3) are summarized in this section. More details can be found in [4], [6] or [9]

In order to incorporate knowledge of dynamics in the orthonormal basis functions $V_k(z)$ in (3), the state space realization of the model given in (1) is used to construct an inner function $G_b(z) = D + C(zI - A)^{-1}B$ that satisfies $G_b(z)^* G_b(z) = I$. The state space realization of $G_b(z)$ can be constructed in such a way that

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^T \begin{bmatrix} A & B \\ C & D \end{bmatrix} = I \tag{4}$$

holds [8]. The state space realization of $G_b(z)$ can be computed from the model given in (1) by the construction of matrices C, D and a state-space similarity transformation T such that (4) can be satisfied.

With (4) it is straightforward to show that for a state space dimension n, the n functions $\phi_1(z), \ldots, \phi_n(z)$ of $V_1(z)$ given by

$$V_1(z) := \begin{bmatrix} \phi_1(z) \\ \vdots \\ \phi_n(z) \end{bmatrix} = (zI - A)^{-1}B$$

are mutually orthonormal in the standard $\mathcal{H}_2$ sense. Basically, the orthonormal functions are generated as state trajectories of balanced realization of an inner function. Subsequently, the consecutive multiplication of $V_1(z)$ with the inner function $G_b(z)$

$$V_k(z) := V_1(z)G_b^k(z) \tag{5}$$

generates a orthonormal set of basis functions $V_k(z)$. It can be shown that the set $V_k(z)$ for $= 1, 2, \ldots, \infty$ constitutes an orthonormal basis in $\mathcal{H}_2$ for the strictly proper part of $\bar{G}(z)$ in (2) [4]. This set of orthonormal basis functions is a generalization of the classical Laguerre [10] and Kautz [11] functions.

Alternative Representation of Reference Model

Both the model structure given in (2) and (3) exhibit a linearly weighted combination of a set of orthonormal basis functions that will be favorable for identification purposes. Obviously, only a finite number m of expansion coefficients $\bar{G}_k$ in (2) or L_k in (3) can be used to represent the system G_0 via the reference model $\bar{G}$. Therefore, the reference model used in the (damage) detection algorithm discussed in this paper is represented by the following series expansion

$$\bar{G} = G(z, \bar{\theta}) = \bar{D} + \sum_{k=1}^{m} L_k V_k(z), \text{with } \bar{\theta}^T := [\bar{D} \ L_1 \ L_2 \ \cdots \ L_m] \tag{6}$$

where $V_k(z)$ are the orthonormal basis functions mentioned in (5) and $\bar{\theta}$ are (a finite number of) generalized Markov parameters that are used to model the dynamics of the system G_0 to be monitored.

As mentioned before, the orthonormal basis functions $V_k(z) = z^{-k}$ in (2) only represent a time shift q^{-k}. Especially in the case of a moderately damped system G_0, the reference model $\bar{G}$ requires a high value for m to accomplish an accurate description of the dynamics of the system G_0 by the reference model $\bar{G}$ [5].

However, by incorporating the (approximate) knowledge of the dynamics in the set of orthonormal basis function, a more accurate dynamical representation is obtained for the system G_0 by the reference model $\bar{G}$ for a fixed value of m in (6) [4]. In fact, it can be shown that if the set of orthonormal basis functions has precisely captured the dynamics of G_0, then $m = 1$ in (6) is sufficient to accomplish $\bar{G} = G_0$.

Obviously, the orthonormal basis function only represent approximate knowledge about the dynamics of the system G_0 to be monitored and therefore $m > 1$ is needed to represent the dynamics of the system G_0 in the reference model $\bar{G}$. In order to find the parameter $\bar{\theta}$ of the reference model, a least squares estimation technique can be used. More details on the identification can be found in the next section.

DAMAGE IDENTIFICATION

Least-Squares Estimation

In order to formulate a damage identification problem, the parameter estimation routine depicted in Figure 1 will be discussed in this section. The main idea is that the identification method should deliver parameters estimates $\hat{\theta}$ on the basis of (noisy) input/output data $\{u, y\}$ that can be used for damage detection by comparing $\hat{\theta}$ with the parameters $\bar{\theta}$ of the reference model. Considering the structure and the benefits of the reference model $\bar{G}$ mentioned in (6), the model G for parameter estimation can be parametrized in a similar way. Hence

$$G(z, \theta) = \theta_0 + \sum_{k=1}^{m} \theta_k V_k(z), \text{ with } \theta^T := [\theta_0 \; \theta_1 \; \ldots \; \theta_m] \tag{7}$$

where θ denotes the parameter to be estimated/optimized and $V_k(z)$ are the orthonormal basis functions mentioned in (5).

For the estimation of a parameter $\hat{\theta}$, the prediction error framework of [5] can be adopted. Denoting the prediction error by $\varepsilon(t, \theta)$, then the model structure $G(z, \theta)$ in (7) yields the following prediction error

$$\varepsilon(t, \theta) := y(t) - [\theta_0 + \sum_{k=1}^{m} \theta_k V_k(q)]u(t) \tag{8}$$

where $V_k(q)$ denote the inverse z-transform of the orthonormal basis functions $V_k(z)$. It should be noted that due to the linear parametrization of the model $G(q, \theta)$, (8) can be rewritten in a linear regression from

$$\varepsilon(t, \theta) = y(t) - \theta^T \varphi(t), \text{ with } \varphi^T(t) = u^T(t)[I \; V_1^T(q) \; V_2^T(q) \; \cdots \; V_m^T(q)]. \tag{9}$$

Under the assumption that N time samples are available, a parameter estimate $\hat{\theta}_N$ can be determined by the following least-squares optimization

$$\hat{\theta}_N := \min_{\theta \in \Theta} \frac{1}{N} \sum_{t=1}^{N} \varepsilon^T(t, \theta) \varepsilon(t, \theta) \tag{10}$$

where $\varepsilon(t,\theta)$ is the prediction error given in (8). Due to the linear parametrization in (9) the least-squares criterion in (10) can be solved analytically, yielding

$$\hat{\theta}_N = \left[\frac{1}{N}\sum_{t=1}^{N}\varphi(t)\varphi^T(t)\right]^{-1}\frac{1}{N}\sum_{t=1}^{N}\varphi(t)y(t) \tag{11}$$

for the parameter estimate $\hat{\theta}_N$, provided that

$$\frac{1}{N}\sum_{t=1}^{N}\varphi(t)\varphi^T(t)$$

is invertible. This condition is satisfied on the assumption of persistent excitation of the (filtered) input signal $\varphi(t)$ [5].

It can be noted here that the analytical solution given in (11) can also be implemented recursively. A recursive formula for the parameter estimates can be found in [5] and is given by

$$\begin{array}{rcl}\hat{\theta}_t & = & \hat{\theta}_{t-1} + R^{-1}(t)\varphi(t)[y(t) - \varphi^T(t)\hat{\theta}_{t-1}] \\ R(t) & = & \lambda(t)R(t-1) + \varphi(t)\varphi^T(t)\end{array} \tag{12}$$

where $\lambda(t)$ indicates an (exponential) forgetting factor on the prediction error $\varepsilon(t,\theta)$ [5]. A recursive formula with an efficient matrix inversion can also be formulated to avoid inverting $R(t)$ at each time step.

Following [5], under weak conditions the parameter estimate $\hat{\theta}_N$ converges for $\lim_{N\rightarrow\infty}$ with probability 1 to the asymptotic estimate

$$\hat{\theta} := \bar{E}\{\varphi(t)\varphi^T(t)\}^{-1}\bar{E}\{\varphi(t)y(t)\} \tag{13}$$

where $\bar{E}\{\cdot\}$ denotes

$$\bar{E}\{x(t)\} := \lim_{N\rightarrow\infty}\sum_{t=1}^{N}E\{x(t)\}$$

and $E\{\cdot\}$ is the usual expectation operator [5]. The weak conditions include persistence of excitation of the input u. Furthermore, the assumption on the input u and the noise v (with noise shaping filter H_0 and unit variance white noise input e, depicted in Figure 1) being uncorrelated, is included in these conditions.

The asymptotic estimate in (13) plays an important role in the characterization of the parameter $\bar{\theta}$ of the reference model $\bar{G}$. Obviously, in case the (fixed) parameter $\bar{\theta}$ of the reference model $\bar{G}$ is obtained by an identification experiment on the basis of a large number of data points N, the parameter $\bar{\theta}$ of the reference model $\bar{G}$ in (6) will equals the asymptotic estimate $\hat{\theta}$ given in (13). For analysis purpose, it is assumed that such a parameter estimate $\bar{\theta} = \hat{\theta}$ for the reference model $\bar{G}$ is available.

Moreover, it can be shown that under fairly weak conditions

$$\sqrt{N}(\hat{\theta}_N - \bar{\theta}) \rightarrow \mathcal{N}(0,Q) \text{ as } N \rightarrow \infty$$

where $\mathcal{N}(0, Q)$ denotes a Gaussian distribution with zero mean and a covariance matrix Q that is given by

$$Q = \bar{E}\{\varphi(t)\varphi^T(t)\}^{-1}\bar{E}\{\phi(t)\phi^T(t)\}\bar{E}\{\varphi(t)\varphi^T(t)\}^{-1} \tag{14}$$

where $\phi(t)$ denotes

$$\phi_(t) = \sum_{k=0}^{\infty} H_k\,\varphi(t+k)$$

which represents the convolution of $\varphi(t)$ with the Markov parameter H_k of the noise shaping filter H_0 depicted in Figure 1 see e.g. [9].

Knowledge with respect to the noise shaping filter H_0 can be incorporate to get a more accurate estimate of the covariance matrix Q. In case H_0 is assumed to be a (constant) shaping filter, $\phi(t) = \sqrt{\lambda}\varphi(t)$ where λ is the variance of the noise v.

Hence, the estimate $\hat{\theta}_N$ converges to the parameter $\hat{\theta} = \bar{\theta}$, Furthermore, the convergence of $\hat{\theta}_N$ is independent of the noise characteristics present on the output $y(t)$. Only the (size of) the covariance matrix given in (14) depends on the noise characterized by the noise shaping filter H_0. These consistency results are due to the Output Error structure of the model in (8) [5].

As can be seen from (7) and (8), the knowledge of the dynamics in the reference model $\bar{G}$ is used to facilitate the parametrization of the model G and the parameter estimation, yielding an analytical solution for $\hat{\theta}_N$ by exploiting the linear regression structure. The expression for the parameter estimate $\hat{\theta}_N$ and its probability distribution with (the estimate of) the covariance matrix Q

$$\hat{Q}_N = \left[\frac{1}{N}\sum_{t=1}^{N}\varphi(t)\varphi^T(t)\right]^{-1}\left[\frac{1}{N}\sum_{t=1}^{N}\phi(t)\phi^T(t)\right]\left[\frac{1}{N}\sum_{t=1}^{N}\varphi(t)\varphi^T(t)\right]^{-1} \tag{15}$$

play an important role in the detection of parameter changes, discussed in the following section.

Statistical Test of Parameter Changes

The identification method discussed in the previous section delivers (recursive) parameters estimates $\hat{\theta}_N$ on the basis of (noisy) input/output data $\{u, y\}$ that can be used for damage detection. This can be done by comparing $\hat{\theta}_N$ and $\bar{\theta}$, as they both constitute a linear expansion based on the orthonormal function $V_k(z)$ generated by the reference model $\bar{G}$.

As the parameter residual

$$\theta_{\varepsilon,N} = \sqrt{N}(\hat{\theta}_N - \bar{\theta})$$

satisfies a zero mean Gaussian distribution with a covariance matrix Q, a statistical T-test can be used to analyse when a parameter fault or change has occurred. The T-test determines, for a given fault probability α, the acceptance or rejection of the hypothesis that $\theta_\varepsilon = 0$ for M independent observations of θ_ε, see e.g. [3].

This statistical test can be implemented by testing the following inequality

$$-T(\alpha/2,\nu) < \frac{1}{\sqrt{M}}\hat{Q}_N^{-1}\theta_{\varepsilon,N} < T(\alpha/2,\nu)$$

for each new value of the parameter estimate $\hat{\theta}_N$, where $\nu = M-1$ is the degree of freedom, the estimated parameter $\hat{\theta}_N$ given in (10) and $\hat{Q}_N$ the estimated covariance function given in (15).

As mentioned before, the estimation can be performed recursively or on the basis of a batch of N time samples. Similar for the recursive formulae given in (12) for the parameter estimate, a recursive formula for the estimated covariance function can be formulated to implement an on-line recursive T-test for detection of parameter changes. The recursive formulae and the properties for the least-squares estimation for the on-line damage identification are exploited by the linear model structure based on the alternative orthonormal function expansion.

CONCLUSIONS

A new on-line damage identification method is proposed for monitoring the behavior of a dynamical system for structural dynamical changes. The method discussed in this paper uses (approximate) knowledge of the dynamical system to be monitored in the form of a so-called linear dynamical reference model.

The reference model is formulated by parametrizing a linear combination of so-called model based orthonormal functions. This linear combination of orthonormal functions can be estimated and updated via a simple least squares optimization that can be implemented recursively for an on-line implementation. The parametrization and estimation is robust in the presence of colored noise on the data and yields unbiased parameter estimates that can be used for detecting plant parameter variations.

Finally, the parameters of the reference model are compared with the estimated parameters in the linear combination orthonormal functions for damage detection. A standard statistical T-test is used to detect changes in parameter estimates.

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