

CONNECTING SYSTEM IDENTIFICATION AND ROBUST CONTROL BY A FACTORIZATION APPROACH

Raymond A. de Callafon¹ and Paul M.J. Van den Hof

*Mechanical Engineering Systems and Control Group,
Delft University of Technology, Mekelweg 2, 2628 CD Delft, The Netherlands
E-mail: r.a.decalfon@wbmt.tudelft.nl*

Abstract. In this paper a framework for connecting system identification and robust control design via a factorization approach is being introduced. Within the scope of this framework, models are represented in an estimated set of models that is used subsequently to design a robust performing controller. The set of models is structured by means of a nominal coprime factorization along with an allowable perturbation written in terms of a Youla-Kucera parametrization. This set of models is shown to be particularly suitable for both identification and control design purposes.

Keywords. coprime factorization; robust control; system identification

1. INTRODUCTION

An approach to obtain a well performing and robust feedback controller for a plant with unknown dynamics is the application of a system identification technique and a subsequent model-based control design. In the literature many approaches have been listed where an iterative minimization of an identification criterion and a control design criterion are used to arrive at a feedback controller of restricted complexity (Gevers, 1993; Van den Hof and Schrama, 1995). The iterative optimization is employed in order to converge to an optimal performing controller that can be applied successfully to the unknown plant, see e.g. (Lee *et al.*, 1993; Schrama and Bosgra, 1993; Zang *et al.*, 1995).

Inevitably, a model found by system identification techniques will be an inaccurate or approximate representation of the plant to be controlled. This is due to the fact that the data used for identification purposes only represents a finite time, possibly disturbed, observation of the plant causing the knowledge of the plant to be incomplete. Additionally, approximate modelling of the plant

may be required in order to arrive at low complexity model based controllers. As pointed out in Hjalmarsson *et al.* (1995), convergence and especially optimality of a controller that emerges from an iterative scheme becomes questionable in case of inaccurate modelling. Furthermore, in many of the available iterative approaches robustness of the controller with respect to the incomplete knowledge of the plant is lacking, as most of the attention is focused on nominal performance specifications.

Both the bias and variance aspects play an important role in modelling via system identification. In these paper, both aspects are considered by employing a set of models to represent the incomplete knowledge of the plant. From an identification point of view, such a set of models may consist of all models that are either validated (Ljung, 1987) or cannot be invalidated by the available data (Smith *et al.*, 1997). A robust control paradigm can deal with such a set of models, provided that it has been formulated properly. A proper formulation of such a set of models is by means of a nominal model, along with an allowable model perturbation (Boyd and Barrat, 1991). Currently, system identification methods are available to estimate such a set of mod-

¹ The research of Raymond de Callafon is supported by the Dutch Institute of Systems and Control (DISC).

els (Mäkilä *et al.*, 1995; Ninness and Goodwin, 1995). The availability of a set of models provides the opportunity to monitor the performance robustness of a controller that is applied to the unknown plant. In this way, either the performance can be assessed *a posteriori* (when a controller is implemented) or guaranteed *a priori* (before implementing a controller). These properties will be illuminated in this paper by the characterization of a set of models that is particularly useful for both identification and robust control design purposes. In this way it is possible to design enhanced controllers, for which the performance robustness can be monitored and improved subsequently. This paper is an abbreviated version of de Callafon and Van den Hof (1997).

2. PROBLEM DESCRIPTION

The notation P will be used to denote any linear time invariant system. The actual plant is denoted by P_o and the model denoted by \hat{P} . Furthermore, let \mathcal{P} be used to denote a set of models and C to represent a feedback controller. The subscript i is applied to \hat{P} , \mathcal{P} or C in order to indicate that the variables depend on the i th step in a subsequent step of system identification and control design. As such, C_i is used to denote a controller (currently) implemented on the unknown plant P_o , whereas C_{i+1} is used to indicate a new, to be designed, controller. A control objective function is denoted by $J(P, C)$ and the notion of performance cost will be characterized by the value of a norm $\|J(P, C)\|$: a smaller value of $\|J(P, C)\|$ indicates better performance (Van den Hof and Schrama, 1995).

Throughout the paper, the control objective function is restricted to a function $J(P, C) \in \mathbb{R}\mathcal{H}_\infty$. In order to optimize the \mathcal{H}_∞ -norm based performance $\|J(P_o, C)\|_\infty$ for the plant P_o , a sub-optimal \mathcal{H}_∞ -controller design can be used (Doyle *et al.*, 1989; Boyd and Barrat, 1991). In such a design, a controller is found by subsequently lowering an upper bound γ on $\|J(P_o, C)\|_\infty$. In terms of the indexed controllers C_i and C_{i+1} this can be formulated by a subsequent design of controllers that satisfy

$$\|J(P_o, C_{i+1})\|_\infty \leq \gamma_{i+1} < \|J(P_o, C_i)\|_\infty \leq \gamma_i \quad (1)$$

The unavoidable incomplete knowledge of the plant P_o can be represented by a set of models \mathcal{P} , as indicated in section 1. In order to guarantee $P_o \in \mathcal{P}$, additional prior information on the plant P_o must be introduced. This is due to the fact that $P_o \in \mathcal{P}$ cannot be validated solely on the basis of finite time, possibly disturbed, observations coming from the plant P_o (Mäkilä *et al.*, 1995; Ninness and Goodwin, 1995)

Using a set \mathcal{P} that satisfies $P_o \in \mathcal{P}$ opens the possibility to formulate an upper bound γ_i for $\|J(P_o, C_i)\|_\infty$ *a posteriori* (when a controller is implemented). Additionally, the set \mathcal{P} can be exploited to design a new controller C_{i+1} that is guaranteed to improve the upper

bound *a priori* (before implementing a controller). The idea of using a set of models to design such a robust performing, sub-optimal controller can be characterized by the following problem formulation.

Problem 2.1 *Let a plant P_o and a controller C_i form a stable feedback connection. To evaluate $\|J(P_o, C_i)\|_\infty \leq \gamma_i$, consider the following step.*

- (a) *Use experimental data and prior information on both the data and the plant P_o to estimate a set of models \mathcal{P}_i such that $P_o \in \mathcal{P}_i$ and determine*

$$\gamma_i = \sup_{P \in \mathcal{P}_i} \|J(P, C_i)\|_\infty \quad (2)$$

Subsequently, consider the following steps.

- (b) *Design a controller C_{i+1} such that*

$$\|J(P, C_{i+1})\|_\infty \leq \gamma_{i+1} < \gamma_i \quad \forall P \in \mathcal{P}_i \quad (3)$$

- (c) *Use (new) experimental data and prior information on both the data and the plant P_o to estimate a set of models \mathcal{P}_{i+1} such that $P_o \in \mathcal{P}_{i+1}$ and*

$$\|J(P, C_{i+1})\|_\infty \leq \gamma_{i+1} \quad \forall P \in \mathcal{P}_{i+1} \quad (4)$$

In problem 2.1, step (b) reflects the design of a robust controller according to (1). Both step (a) and (c) contain the estimation of a set of models \mathcal{P} . The quality of the models P within the sets is evaluated by the performance specification $\|J(P, C)\|_\infty$, where step (a) and (c) differ only in the feedback controller C being used. Obviously, to provide a feasible procedure for handling problem 2.1, the following items have to be addressed.

- The control objective function $J(P, C)$ that is used throughout problem 2.1.
- The way in which a set of models \mathcal{P} is being structured so as to be able to design a robust controller and to evaluate the performance.
- Identification procedure to estimate and validate a set of models \mathcal{P} in step (a) and (c) of problem 2.1.
- The design of a robust controller in step (b) of problem 2.1.

The remaining part of the paper is devoted to the discussion of the items mentioned above.

3. PERFORMANCE AND SET OF MODELS

3.1 Control objective

A feedback connection of a system P and a feedback controller C is denoted by $T(P, C)$ and defined as the connection structure depicted in Figure 1. Accordingly, the feedback connection of the plant P_o and the controller C_i (currently being implemented) is denoted by $T(P_o, C_i)$. It is assumed that a connection $T(P, C)$ is well posed, that is $\det(I + CP) \neq 0$ (Boyd and Barrat, 1991) and the mapping from the signals $col(r_2, r_1)$

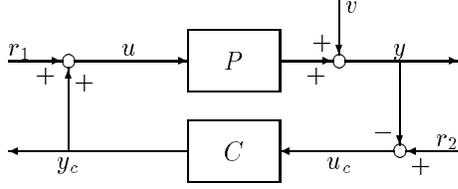


Fig. 1. Feedback connection structure $T(P, C)$.

onto $\text{col}(y, u)$ is given by the transfer function matrix $T(P, C)$ with

$$T(P, C) := \begin{bmatrix} P \\ I \end{bmatrix} (I + CP)^{-1} \begin{bmatrix} C & I \end{bmatrix}, \quad (5)$$

As a result, the signals in the connection $T(P_o, C_i)$ can be described by

$$\begin{bmatrix} y \\ u \end{bmatrix} = T(P_o, C_i) \begin{bmatrix} r_2 \\ r_1 \end{bmatrix} + \begin{bmatrix} I \\ -C_i \end{bmatrix} (I + P_o C_i)^{-1} v \quad (6)$$

where the signals u and y reflect respectively the inputs and outputs of the feedback controlled plant P_o . For identification purposes, it is presumed that the noise v is uncorrelated with the external reference signals r_1, r_2 and that it can be modelled as the output of a monic stable and stably invertible noise filter H_0 having a white noise input e (Ljung, 1987).

An internally stable closed loop system $T(P, C)$ is equivalent to $T(P, C) \in \mathbb{RH}_\infty$ (Schrama and Bosgra, 1993). The control objective function $J(P, C) \in \mathbb{RH}_\infty$ is taken to be a weighted form of $T(P, C)$ and defined as follows

$$\|J(P, C)\|_\infty := \|U_2 T(P, C) U_1\|_\infty \quad (7)$$

where U_2 and U_1 are (square) weighting functions. Although it is impossible to transform any desirable control design objective into a norm function $\|J(P, C)\|_\infty$, the performance characterization (7) is fairly general and has wide applicability. At first instance, the weighting functions U_1 and U_2 are assumed to be given and fixed in order to be able to compare the control objectives $J(P, C_i)$ and $J(P, C_{i+1})$ in the subsequent steps of problem 2.1.

3.2 Structure of the set of models

A set of models to represent the incomplete knowledge of the plant P_o is usually structured by means of a nominal model \hat{P} , along with an allowable model perturbation. However, knowledge of the controller C_i (currently implemented on the plant P_o) can also be used in the construction of the set of models in this paper. For that purpose, the allowable model perturbation is formulated in terms of a (dual) Youla-Kucera parametrization, see e.g. (Lee *et al.*, 1993). Such a set of models has some favourable properties that are illuminated below.

To characterize the set of models, the (possibly unstable) transfer function of \hat{P} and C_i are expressed as a

ratio of two stable transfer functions that constitute a right coprime factorization (*rcf*) (Vidyasagar, 1985). In this way, a unified approach to handle both stable and unstable models and controllers can be obtained. Using a *rcf* $(N_{c,i}, D_{c,i})$ of C_i and a nominal model \hat{P}_i with a *rcf* (\hat{N}_i, \hat{D}_i) that satisfies $T(\hat{P}_i, C_i) \in \mathbb{RH}_\infty$, the set \mathcal{P}_i is defined by

$$\mathcal{P}_i = \{P \mid P = (\hat{N}_i + D_{c,i}\Delta_i)(\hat{D}_i - N_{c,i}\Delta_i)^{-1} \quad (8)$$

$$\text{with } \Delta_i \in \mathbb{RH}_\infty \text{ and } \|\hat{V}_i\Delta_i\hat{W}_i\|_\infty < \gamma_i^{-1}\}$$

where γ_i denotes the upper bound given in (2). Δ_i reflects an allowable model perturbation such that $P_o \in \mathcal{P}_i$ whereas \hat{V}_i, \hat{W}_i denote stable and stably invertible weighting functions used to normalize the upper bound on $\hat{V}_i\Delta_i\hat{W}_i$ to γ_i^{-1} . A similar definition can also be given for the set \mathcal{P}_{i+1} in step (c) of problem 2.1, using a nominal model \hat{P}_{i+1} and the (newly designed) controller C_{i+1} .

Due to the close connection with the dual Youla-Kucera parametrization, the uncertainty set \mathcal{P}_i in (8) contains only models that are stabilized by the controller C_i (being implemented on the plant P_o) regardless of the value γ_i . This advantage, observed also by Sefton *et al.* (1990), is not shared by alternative uncertainty characterizations such as an open loop additive uncertainty description. An additional advantage of the set of models given in (8) is the fact that an affine expression in the allowable model perturbation is obtained in order to evaluate the performance specification $U_2 T(P, C) U_1$ for all models $P \in \mathcal{P}_i$. This property will be illuminated in the following section.

4. EVALUATION OF PERFORMANCE

4.1 Computations via LFT's

The fairly general framework of Linear Fractional Transformations (LFT's) opens the possibility to rewrite the set of models of (8) into a form to which standard results for performance evaluation can be applied. For that purpose, an upper LFT

$$\mathcal{F}_u(Q, \Delta) := Q_{22} + Q_{21}\Delta(I - Q_{11}\Delta)^{-1}Q_{12}$$

can be adopted to rewrite \mathcal{P}_i of (8) as below. Obviously, the LFT representation of the set \mathcal{P}_{i+1} mentioned in problem 2.1 can be obtained in a similar way.

Corollary 4.1 *The set of models \mathcal{P}_i given in (8) can be written as*

$$\mathcal{P}_i = \{P \mid P = \mathcal{F}_u(Q_i, \Delta)$$

$$\text{with } \Delta \in \mathbb{RH}_\infty \text{ and } \|\Delta\|_\infty < \gamma_i^{-1}\}$$

$$\text{where } Q_i = \left[\begin{array}{c|c} \hat{W}_i^{-1} \hat{D}_i^{-1} N_{c,i} \hat{V}_i^{-1} & \hat{W}_i^{-1} \hat{D}_i^{-1} \\ \hline (D_{c,i} + \hat{P}_i N_{c,i}) \hat{V}_i^{-1} & \hat{P}_i \end{array} \right] \quad (9)$$

With the LFT representation of the set of models \mathcal{P}_i in corollary 4.1, the performance of a controller C^2 applied to any model $P \in \mathcal{P}_i$ again can be written in terms of an LFT.

Lemma 4.2 Consider the set \mathcal{P}_i defined in (8) and a controller C such that the map $J(P, C) = U_2 T(P, C) U_1$ is well-posed for all $P \in \mathcal{P}_i$. Then

$$\mathcal{P}_i = \{P \mid J(P, C) = \mathcal{F}_u(M_i, \Delta)\}$$

$$\text{with } \Delta \in \mathbb{R}\mathcal{H}_\infty, \|\Delta\|_\infty < \gamma_i^{-1}\}$$

where the entries of M_i are given by

$$\begin{aligned} M_{11} &= -\hat{W}_i^{-1}(\hat{D}_i + C\hat{N}_i)^{-1}(C - C_i)D_{c,i}\hat{V}_i^{-1} \\ M_{12} &= \hat{W}_i^{-1}(\hat{D}_i + C\hat{N}_i)^{-1}[C \ I]U_1 \\ M_{21} &= -U_2 \begin{bmatrix} -I \\ C \end{bmatrix} (I + \hat{P}_i C)^{-1} (I + \hat{P}_i C_i) D_{c,i} \hat{V}_i^{-1} \\ M_{22} &= U_2 \begin{bmatrix} \hat{N}_i \\ \hat{D}_i \end{bmatrix} (\hat{D}_i + C\hat{N}_i)^{-1} [C \ I] U_1 \end{aligned} \quad (10)$$

Proof: By algebraic manipulation, see de Callafon and Van den Hof (1997). \square

4.2 Worst case performance

With lemma 4.2, the performance $\|J(P, C)\|_\infty$ of a controller C applied to all models $P \in \mathcal{P}_i$ can be evaluated by the upper LFT

$$\|M_{22} + M_{21}\Delta(I - M_{11}\Delta)^{-1}M_{12}\|_\infty \quad (11)$$

for all $\Delta \in \mathbb{R}\mathcal{H}_\infty$ with $\|\Delta\|_\infty \leq \gamma_i^{-1}$. Note that the entries of the transfer function M in (10) are determined solely by the controller C , the variables used to represent the set \mathcal{P}_i in (8) and the weightings U_2, U_1 of the performance specification (7). As a special entry of M , one can recognise M_{11} as the lower LFT $\mathcal{F}_l(Q, -C)$, whereas M_{22} equals the (nominal) performance specification $U_2 T(\hat{P}_i, C) U_1$. Evaluation of (11) can be done by employing the structured singular value μ (Packard and Doyle, 1993).

The definition of $\mu(\cdot)$ depends on an underlying (diagonal) structure Δ (Packard and Doyle, 1993; Zhou *et al.*, 1996). Typically, such a structure complies with the size and structure of the transfer function M as used in $\mathcal{F}_u(M, \Delta)$. For reasons of clarity, the structure Δ is restricted to have a diagonal form that consists of two unstructured uncertainty blocks only. The structured singular value $\mu(\cdot)$ with respect to such a structure Δ is denoted by $\mu_\Delta(\cdot)$. A formal definition and a discussion of the properties of the structured singular value is beyond the scope of this paper and one can be referred to the book by Zhou *et al.* (1996). The following result can be used to evaluate (11) for all models within a set of models.

Theorem 4.3 Consider the set \mathcal{P}_i defined in (8) and a controller C such that $T(\hat{P}_i, C)$ is well-posed, internally stable and satisfies $U_2 T(\hat{P}_i, C) U_1 \in \mathbb{R}\mathcal{H}_\infty$. Then, for all $P \in \mathcal{P}_i$, the feedback system $T(P, C)$ is well-posed, internally stable and satisfies $\|U_2 T(P, C) U_1\|_\infty \leq \gamma_i$ if and only if

$$\mu_\Delta(M_i) \leq \gamma_i \quad (12)$$

where the entries of M_i are given in (10)

Proof: Lemma 4.2 connects the upper LFT $\mathcal{F}_u(M_i, \Delta)$ with $U_2 T(P, C) U_1$ for all $P \in \mathcal{P}_i$. The main loop theorem as presented in Zhou *et al.* (1996) yields the necessary and sufficient condition for $\|\mathcal{F}_u(M_i, \Delta)\|_\infty \leq \gamma_i$ to hold for all $P \in \mathcal{P}_i$. \square

Referring to problem 2.1, substituting $C = C_i$ in (10) can be used for the performance assessment in step (a). On the other hand, substitution of $C = C_{i+1}$ in (10) can be used to check and guarantee performance robustness of C_{i+1} in step (b). Similar results can be derived also for the set of models \mathcal{P}_{i+1} as used in step (c) of problem 2.1. Finally, it can be observed from (10) that substitution of $C = C_i$ yields $M_{11} = 0$. This implies that the controller C_i applied to the (identified) set of models \mathcal{P}_i satisfies stability robustness (Zhou *et al.*, 1996), regardless of the value of γ_i . This was one of the motivations already mentioned in section 3.2 to use the uncertainty set (8). Moreover, for $C = C_i$ the upper LFT $\mathcal{F}_u(M, \Delta)$ modifies into

$$M_{22} + M_{21}\Delta M_{12} \quad (13)$$

which is an affine expression in Δ . As a result, finding the smallest possible allowable model perturbation Δ such that $P_o \in \mathcal{P}_i$ (via system identification techniques) will effectively minimize the worst case performance too.

5. IDENTIFICATION PROBLEM

5.1 Estimation of a set of models

Estimating the set \mathcal{P}_i such that γ_i in (2) is as small as possible in step (a), could be achieved by minimizing

$$\sup_{P \in \mathcal{P}_i} \|J(P, C_i)\|_\infty \quad (14)$$

subjected to the condition $P_o \in \mathcal{P}_i$. As such, the identification problems of step (a) and (c) of problem 2.1 are similar and differ only in the controller being implemented on the plant P_o .

According to (8), the set \mathcal{P}_i is determined by a factorization (\hat{N}_i, \hat{D}_i) of a nominal model \hat{P}_i and the weighting functions (\hat{V}_i, \hat{W}_i) . Minimizing (14) using the variables $(\hat{N}_i, \hat{D}_i, \hat{V}_i, \hat{W}_i)$ simultaneously is impracticable. Furthermore, the variables should have limited complexity as the complexity of Q_i in (9) will directly influence the complexity of the model-based controller being computed (Boyd and Barrat, 1991). Therefore, minimization

² C is used to denote either C_i or C_{i+1}

of (14) is tackled by estimating the *rcf* (\hat{N}_i, \hat{D}_i) and the pair (\hat{V}_i, \hat{W}_i) separately; estimation of a nominal factorization and estimation of an allowable model perturbation. Due to this separation only an upper bound on (14) can be minimized. However, (standard) tools to estimate both a nominal model and an uncertainty can be used.

5.2 Estimation of a nominal model

Estimation of a nominal model involves the estimation of $\hat{P}_i = \hat{N}_i \hat{D}_i^{-1}$, subjected to internal stability of the feedback connection $\mathcal{T}(\hat{P}_i, C_i)$, such that (14) is being minimized. At this stage, the variables \hat{V}_i and \hat{W}_i are unknown and assumed to vary freely in order to satisfy $P_o \in \mathcal{P}_i$. Consequently, the set \mathcal{P}_i is still unknown and (14) cannot be computed. However, for any $P \in \mathcal{P}_i$ $\|J(P, C_i)\|_\infty$ can be evaluated by considering the following upper bound for $\|J(P, C_i)\|_\infty$:

$$\|J(P_o, C_i)\|_\infty + \|J(P, C_i) - J(P_o, C_i)\|_\infty$$

As $\|J(P_o, C_i)\|_\infty$ in (5.2) does not depend on the nominal model \hat{P}_i , a *rcf* (\hat{N}_i, \hat{D}_i) of a nominal model can be found by minimizing

$$\|J(P, C_i) - J(P_o, C_i)\|_\infty \quad (15)$$

Estimation of a *rcf* of a nominal model of limited complexity by minimizing (15) on the basis of closed loop experiments obtained from the connection $\mathcal{T}(P_o, C_i)$ has been studied extensively in (de Callafon and Van den Hof, 1995b) and (Van den Hof *et al.*, 1995). An approach to minimize (15) on the basis of frequency domain data can be found in de Callafon and Van den Hof (1995a).

5.3 Estimation of allowable model perturbation

The *rcf* (\hat{N}_i, \hat{D}_i) of the nominal model is now fixed to the estimate obtained above. Estimation of an allowable model perturbation involves the characterization of an upper bound on Δ_i in (8) via (\hat{V}_i, \hat{W}_i) such that (14) is being minimized and $P_o \in \mathcal{P}_i$. For that purpose, first the allowable model perturbation Δ_i in (9) is determined. Subsequently, stable and stably invertible weightings \hat{V}_i and \hat{W}_i can be determined that normalize the upper bound on $\Delta := \hat{V}_i \Delta_i \hat{W}_i$ to γ_i^{-1} as indicated in (8). Lemma 4.2 connects the upper LFT $\mathcal{F}_u(M_i, \Delta)$ with $U_2 T(P, C) U_1$ for all $P \in \mathcal{P}_i$. As mentioned in (13), for $C = C_i$ (the controller currently being implemented on the plant P_o) the upper LFT $\mathcal{F}(M, \Delta_i)$ reduces to

$$M_{22} + M_{21} \hat{V}_i \Delta_i \hat{W}_i M_{12} \quad (16)$$

With (10) it can be observed that M_{22} , $M_{21} \hat{V}_i$ and $\hat{W}_i M_{12}$ solely depend on the performance weight U_2 and U_1 , the nominal *rcf* (\hat{N}, \hat{D}) and the controller C_i and its *rcf*. All these variables are fixed and an affine relation in

Δ_i is obtained; minimizing Δ_i frequency wise will effectively minimize (16). The system identification used for this purpose will be discussed by first considering the following proposition.

Proposition 5.1 Consider C_i with *rcf* $(N_{c,i}, D_{c,i})$ and \hat{P}_i with *rcf* (\hat{N}_i, \hat{D}_i) . Let $\mathcal{T}(P_o, C_i)$ and $\mathcal{T}(\hat{P}_i, C_i)$ be internally stable and define

$$\begin{aligned} x &:= (\hat{D}_i + C_i \hat{N}_i)^{-1} [C_i \ I] \begin{bmatrix} y \\ u \end{bmatrix} \\ z &:= (D_{c,i} + \hat{P}_i N_{c,i})^{-1} [I \ -\hat{P}_i] \begin{bmatrix} y \\ u \end{bmatrix} \end{aligned} \quad (17)$$

then

$$z = \Delta_i x + D_{c,i} (I + P_o C_i)^{-1} v \quad (18)$$

where $\Delta_i \in \mathbb{RH}_\infty$ and x is uncorrelated with v .

Proof: Equation (18) and the property $x \perp v$ can be verified by algebraic manipulation, see Van den Hof and Schrama (1995). The property $\Delta_R \in \mathbb{RH}_\infty$ follows from the (dual) Youla-Kucera parametrization. \square

Oroposition 5.1 gives rise to an equivalent open loop identification problem of the stable dual Youla-Kucera parameter Δ_i (Lee *et al.*, 1993). However, the dual Youla parameter is being used here *only* to construct the set \mathcal{P}_i of (8) such that $P_o \in \mathcal{P}_i$. An uncertainty estimation routine such as the procedure described by Hakvoort (1994) can be used to obtain a frequency dependent upper bound for Δ_i . Application of this procedure results in a frequency dependent upper bound $\delta(\omega)$

$$\|\Delta_i(\omega)\| \leq \delta(\omega) \text{ with probability } \geq \alpha \quad (19)$$

where α is a prechosen probability. In the multivariable case, the upper bound (19) can be obtained for each transfer function. Subsequently, stable and stably invertible weighting filters \hat{V}_i and/or \hat{W}_i can be constructed to normalize the upper bound on $\hat{V}_i \Delta_i \hat{W}_i$ to γ_i^{-1} (Hakvoort, 1994).

6. CONTROLLER DESIGN

To complete the analysis of problem 2.1, the estimated set \mathcal{P}_i should be used for control design. In order to satisfy (3) a controller C_{i+1} can be designed minimizing

$$\sup_{P \in \mathcal{P}_i} \|J(P, C)\|_\infty \quad (20)$$

Basically, (20) constitutes a (standard) \mathcal{H}_∞ -norm based control design, wherein the worst case performance is being optimized. For that purpose, a μ -synthesis via a so-called D-K iteration (Zhou *et al.*, 1996) can be used. In order to use the available techniques on μ -synthesis, the transfer function M_i in (10) should be represented as a lower fractional transformation $\mathcal{F}_l(G_i, C)$, where the controller $C = C_{i+1}$ to be computed has been extracted

from the expression of M_i given in (10). As a result, G_i will depend on the performance weightings (U_2, U_1), the uncertainty weightings (\hat{V}_i, \hat{W}_i), the *ref* (\hat{N}_i, \hat{D}_i) of \hat{P}_i and the *ref* ($N_{c,i}, D_{c,i}$) of the (previous) controller C_i . An expression of G_i can be found in de Callafon and Van den Hof (1997).

7. CONCLUSIONS

Unavoidable incomplete knowledge due to finite time and possibly disturbed observations coming from an unknown plant, requires the plant to be modelled by a set of models. An estimated set of models can be used subsequently to design a robust performing controller. To obtain an improved and robust performing controller a framework is proposed in which the system identification of a set of models and the subsequent design of a robust controller have been merged. Within this framework, the performance characterization, the structure and identification of the set of models and the subsequent robust controller design is addressed shortly. The set of models is structured by means of a nominal coprime factorization along with an allowable perturbation written in terms of a Youla-Kucera parametrization. This fractional approach enables a unified approach to the identification of stable and unstable plants on the basis of closed loop experiments. Furthermore, the structure and estimation of the set of models is tuned towards the performance specification being used.

REFERENCES

- Boyd, S.P. and C.H. Barrat (1991). *Linear Controller design – Limits of Performance*. Prentice-Hall, Englewood Cliffs, New Jersey, USA.
- de Callafon, R.A. and P.M.J. Van den Hof (1995a). Control relevant identification for H_∞ -norm based performance specifications. In *Proc. 34th IEEE Conference on Decision and Control*. New Orleans, USA. pp. 3498–3503.
- de Callafon, R.A. and P.M.J. Van den Hof (1995b). Filtering and parametrization issues in feedback relevant identification based on fractional model representations. In *Proc. 3rd European Control Conference*, Vol. 1. Rome, Italy. pp. 441–446.
- de Callafon, R.A. and P.M.J. Van den Hof (1997). Sub-optimal feedback control by a scheme of iterative identification and control design. *Mathematical Modelling of Systems*, Vol. 3, pp. 77–101.
- Doyle, J.C., K. Glover, P.P. Khargonegar and B.A. Francis (1989). State space solutions to standard H_2 and H_∞ control problems. *IEEE Trans. on Automatic Control*, AC-34, pp. 831–847.
- Gevers, M.R. (1993). Towards a joint design of identification and control? In *Essays on Control: Perspectives in the Theory and its Applications*. H.L. Trentelman and J.C. Willems (Eds.). Birkhäuser, Boston. pp. 111–151.
- Hakvoort, R.G. (1994). System Identification for Robust Process Control: Nominal Models and Error Bounds. PhD thesis. Delft University of Technology, Delft, the Netherlands.
- Hjalmarsson, H., S. Gunnarson and M.R. Gevers (1995). Optimality and sub-optimality of iterative identification and control design schemes. In *Proc. American Control Conference*. Seattle, USA. pp. 2559–2563.
- Lee, W.S., B.D.O. Anderson, R.L. Kosut and I.M.Y. Mareels (1993). A new approach to adaptive robust control. *Int. Journal of Adaptive Control and Signal Processing*, Vol. 7, No. 3, pp. 183–211.
- Ljung, L. (1987). *System Identification: Theory for the User*. Prentice-Hall, Englewood Cliffs, New Jersey, USA.
- Mäkilä, P.M., J.R. Partington and T.K. Gustafson (1995). Worst-case control-relevant identification. *Automatica*, Vol. 31, pp. 1799–1819.
- Ninness, B. and G.C. Goodwin (1995). Estimation of model quality. *Automatica*, Vol. 31, pp. 1771–1797.
- Packard, A. and J.C. Doyle (1993). The complex structured singular value. *Automatica*, Vol. 29, No. 1, pp. 71–109.
- Schrama, R.J.P. and O.H. Bosgra (1993). Adaptive performance enhancement by iterative identification and control design. *Int. Journal of Adaptive Control and Signal Processing*, Vol. 7, No. 5, pp. 475–487.
- Sefton, J.A., R.J. Ober and K. Glover (1990). Robust stabilization in the presence of coprime factor perturbations. In *Proc. 29th IEEE Conference on Decision and Control*. Honolulu, Hawaii. pp. 1197–1198.
- Smith, R., G. Dullerud, S. Rangan and K. Poola (1997). Model validation for dynamically uncertain systems. *Mathematical Modelling of Systems*, Vol. 3, pp. 43–58.
- Van den Hof, P.M.J. and R.J.P. Schrama (1995). Identification and control – closed loop issues. *Automatica*, Vol. 31, No. 12, pp. 1751–1770.
- Van den Hof, P.M.J., R.J.P. Schrama, R.A. de Callafon and O.H. Bosgra (1995). Identification of normalised coprime plant factors from closed-loop experimental data. *European Journal of Control*, Vol. 1, No. 1, pp. 62–74.
- Vidyasagar, M. (1985). *Control System Synthesis: A Factorization Approach*. MIT Press, Cambridge, Massachusetts, USA.
- Zang, Z., R.R. Bitmead and M.R. Gevers (1995). Iterative weighted least-squares identification and weighted LQG control design. *Automatica*, Vol. 31, pp. 1577–1594.
- Zhou, K., J.C. Doyle and K. Glover (1996). *Robust and Optimal Control*. Prentice-Hall, Upper Saddle River, New Jersey, USA.