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FREQID - Frequency domain identification toolbox for use with Matlab[‡§]

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Abstract. This paper presents a new MATLAB toolbox, called FREQID. FREQID is an abbreviation of FREQuency domain IDentification and can be used to estimate linear (multivariable) discrete or continuous time models on the basis of frequency response data. Within FREQID, a model is being estimated by applying a (frequency weighted) curve fit procedure on the available frequency response. To simplify the operations involved with choosing frequency dependent weightings, model order selection and model evaluation, FREQID is equipped with a Graphical User Interface (GUI). The usage of the GUI and the way in which models can be estimated within FREQID is the core of this paper.

Keywords. system identification; frequency domain; multivariable systems; curve fitting

1 Introduction

The availability of measured frequency responses as a commencement to estimate linear models has gained considerable attention in the research on system identification. First of all this is due to the fact that estimating a model on the basis of a frequency response has several advantages compared to a time domain approach, see e.g. Ljung (1993) or Pintelon *et al.* (1994). Additionally, many engineers have a strong inclination towards a frequency domain related identification procedure, as the "shape" or quality of the model can be influenced directly in the frequency domain by the usage of so-called frequency dependent weightings.

This paper describes the usage of FREQID, a toolbox for use with MATLAB (version 4.2c) for performing identification on the basis of frequency response measurements. FREQID is an abbreviation of FREQuency domain IDentification, which is supposed to cover the main purpose of this software: it can handle the estimation of both discrete and continuous time (multivariable) models on the basis of a frequency response in which the frequency vector can be arbitrarily spaced.

Estimating a model in FREQID is done by a curve fitting procedure. In such a procedure, a model is being estimated by fitting the frequency response of the model on a measured frequency response. Within the curve fitting a frequency dependent weighting can be used to emphasize specific parts of the frequency response, so as to influence the quality or "shape" of the model being estimated.

Compared to the frequency domain identifcation toolbox of Kollár (1994), FREQID focuses solely on frequency response curve fitting. To simplify the operations involved with the estimation and validation of a model, FREQID is equipped with a Graphical User Interface (GUI). This GUI is meant to simplify both the manipulation of frequency domain measurements, the shaping of frequency dependent weightings and the model order selection during the estimation of a model. Furthermore, the GUI serves as a bookkeeper of the models being estimated and enables the user to validate and compare various

‡MATLAB is a registered trademark of the MathWorks, Inc.
§The software described in this paper is available through anonymous ftp at: `ftp-mesc.wbmt.tudelft.nl`, directory `/pub/matlab/freqid`.
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models relatively easily. As the GUI is designed to be user friendly, most of the information described in this paper is apparent from the GUI of FREQID. By pushing the various help-buttons present in the GUI of FREQID, additional information is displayed. Therefore, this paper will only focus on the main elements present in the GUI of FREQID.
For notational convenience and reasons of clarity, different fonts are used to indicate different objects in this paper. Text in various windows of the GUI of FREQID like titles and text on buttons are typeset in this font. Names of files or directories, commands to be typed and editable text in the GUI of FREQID are typeset `in this font`. Finally, most abbreviations will be typeset IN THIS FONT. In this way, the difference between freqid in a title, the command `freqid` to be typed and FREQID as an abbreviation will be unambiguous.
First in section 2, the main window of FREQID will be discussed. This section also shows the possibilities and bookkeeping facilities of the main window and how frequency responses (the data) and models can be imported. Section 3 describes the possibility to estimate a model on the basis of a frequency response using the GUI of FREQID. Subsequently, section 4 presents the available procedures to evaluate the models being estimated. Finally, the paper is ended by a short summary.

2 The main window

2.1 Overview

If the FREQID toolbox has been installed properly, typing the command

```
>> freqid
```

in the MATLAB command window, will invoke the GUI of FREQID. First a small message window will be opened, that contains information on FREQID and the authors who wrote the software. By clicking the continue-button, the main window of FREQID will be opened. This main window is depicted in Figure 1 and consists of the following distinguishable parts.

- At the top of the window a menu bar can be found. Via the options on the menu bar, session files can be loaded or saved, different MATLAB windows can be accessed and the layout of the FREQID windows can be modified.

- At the left top part of the window one can find the Data Board. This is used to store and manipulate the frequency domain measurements and/or frequency dependent weights used for estimating a model. It also contains a data-popup menu.

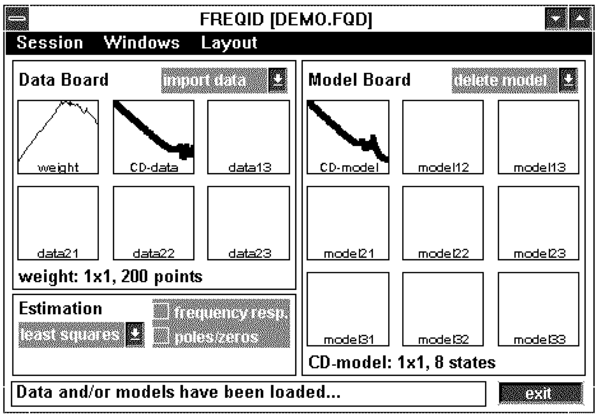

Fig. 1: Main window of FREQID

- The right part of the window contains the Model Board. Similar to the Data Board, this is used to store and manipulate models and contains a model-popup menu.

- The left bottom part of the main window is reserved for Estimation (and evaluation) of models.

- Finally, at the bottom of the window the status line is displayed. This is used to display all kinds of messages to the user. In Figure 1 the user is notified of the fact that Data and/or models have been loaded..., just after a session file called `DEMO.FQD` was loaded.

A more detailed description on importing data, models and mouse actions defined within the main window of FREQID can be found in the following sections.

2.2 Importing data

The starting point for estimating models within FREQID is the availability of a frequency response that needs to be fitted. Subsequently, the Data Board can be used to store and manipulate the frequency response and/or frequency domain weights used for estimating a model. For this purposes, a data-popup menu and specific mouse actions (clicking, drag & drop) are defined within the Data Board depicted in Figure 1.
Frequency responses (or frequency dependent weightings) can be imported from a file or from the MATLAB workspace onto the Data Board in three different formats:

- MVFR matrix (MFD tools) In such a MultiVariable Frequency Response (MVFR) matrix, a frequency domain measurement (single- or multivariable) is stacked columnwise for each frequency point separately. The frequency vector (always in [rad/s]) corresponding to it, must be specified separately. This format is also supported by the Multivariable Frequency Domain (MFD) toolbox, (Maciejowski, 1990).

- FREQFUNC matrix (IDENT tools) This is a format to store frequency domain data supported by the System Identification ToolBox (SITB), (Ljung, 1995). Such a matrix already contains the corresponding frequency vector.

- Varying matrix (MU tools) This is the format supported by the μ-analysis and synthesis toolbox, (Balas *et al.*, 1995). Such a matrix already contains the frequency vector.

To import frequency responses on the Data Board, the import data option of the data-popup menu can be used. Invoking this menu option yields the window depicted in Figure 2. From Figure 2 the three

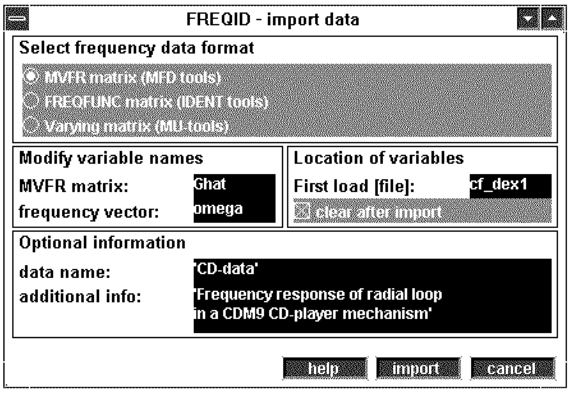

Fig. 2: Import data window.

different formats discussed above can be recognized, whereas for bookkeeping purposes, the name of the data and some additional information can be specified.

2.3 Obtaining models

Quite similar to the Data Board, the Model Board has been defined on the main window of FREQID. Importing a model on the Model Board can be done by estimating a model on the basis of a frequency response available on the Data Board. However, the discussion of this option is postponed until section 3. Additionally, a model can be imported by the import model option of the model-popup menu. Invoking this menu option yields the window depicted in Figure 3. Three different formats are supported to

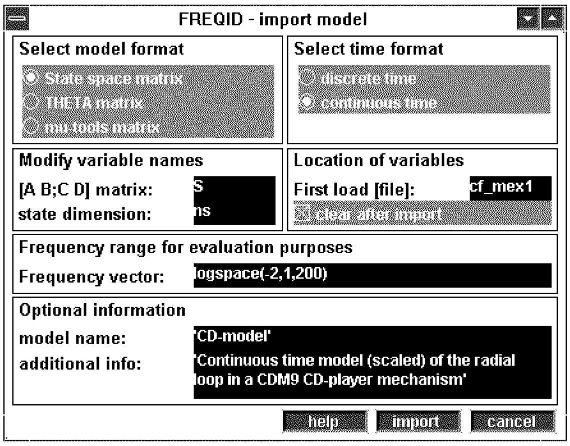

Fig. 3: Import model window.

import a model (discrete or continuous time) from a file or the MATLAB workspace onto the Model Board of FREQID:

- State space matrix In this format the state space matrices `A`, `B`, `C` and `D` of a model (single- or multivariable) are stacked in a system matrix `S=[A B;C D]`. In order to be able to re-extract the state space, the size of the matrix `A` (state space dimension) must be given too.

- THETA matrix. This is a format to store a model as supported by the System Identification toolbox, (Ljung, 1995).

- mu-tools matrix. This is the format supported by the μ-analysis and synthesis toolbox, (Balas *et al.*, 1995).

To import a model, a frequency vector must be added in order to evaluate the frequency response of the model. Finally, the name of the model and some additional information can be specified for bookkeeping purposes, see also Figure 3.

2.4 Mouse actions

Once some frequency response (data) or a model has been loaded successfully, it will appear as an icon in one of the boxes present in the main window depicted in Figure 1. This icon is formed by plotting the Bode amplitude diagram of the first element of the (multivariable) frequency response of either the data or the model. The icon can now be selected simply by clicking on the corresponding box. If the icon has been selected, a fat line will be drawn in the corresponding box, see e.g. the second box in Figure 1. Information on the icon can also be accessed by a simple click on the name. By a simple drag & drop action, an icon can also be copied.

3 Estimating a model

The estimation of a model is done by performing a curve fit on a frequency response available on the main window of FREQID. Depending on the parametrization of the model, the curve fitting generally involves a non-linear optimization that needs to be solved. Currently, two different curve fitting routines are implemented within FREQID. These routines are available by invoking the estimation-popup menu present in the main window of FREQID. A short summary of the two methods is listed below.

least squares estimation

The least-squares estimation routine implemented aims to minimize the 2-norm of a (weighted) difference between the frequency response of the model and the data. A frequency dependent weighting is a so-called Schur-weighting in which the weighting is specified for each transfer function separately. The (multivariable) model is parametrized by either a left or right Matrix Fraction Description (MFD), which reduces to a simple numerator/denominator representation for estimating scalar models. For a more detailed discussion on the procedure, one is referred to de Callafon *et al.* (1996).

maximum amplitude

The maximum amplitude routine implemented aims at minimizing the (weighted) maximum difference between the frequency response of the model and the data, element wise. Again the weighting can be specified for each transfer function separately. The (multivariable) model is parametrized by a combined diagonal left and right Matrix Fraction Description (MFD), which reduces to a simple numerator/denominator representation for estimating scalar models. For a more detailed discussion on the procedure, one is referred to Hakvoort and Van den Hof (1994).

For both the methods discussed above, an iteration based on the Sanathanan-Koerner procedure (Sanathanan and Koerner, 1963) is used to tackle the non-linear minimization involved. Although there is no direct guarantee of convergence, the method generaly leeds to usefull models. Furthermore it is reasonably fast and due to the subsequent convex optimization steps it supports the estimation of relatively high order models. The procedure to estimate a model using the GUI of FREQID is nearly the same for both methods. Furthermore, the least squares estimation routine is included in FREQID, whereas for the maximum amplitude criterion the installation of the MATLAB optimization toolbox, version 1.0c, is a prerequisite (Hakvoort, 1994). Therefore, only the least squares estimation routine will be illustrated here.

3.1 Least squares estimation

Once data has been loaded and selected, invoking the least squares option from the estimation-popup menu in the main window of FREQID will present the least squares estimation window on the screen. An overview of this window is depicted in Figure 4. In the least squares estimation algorithm, a multi-

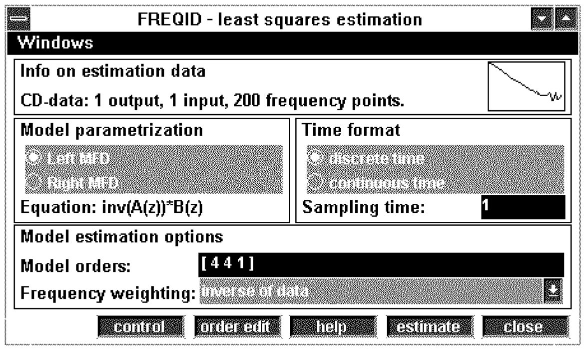

Fig. 4: Least squares estimation window

variable model is parametrized by a Matrix Fraction Description (MFD), using the inverse of a square and monic A-polynomial and a B-polynomial. Before starting up the estimation of a model, the user can specify the time format (discrete or continuous time) and the parametrization of the model (left or right MFD). For the left MFD, the inverse of the monic A-polynomial appears at the output of the model, whereas for the right MFD, the inverse appears at the input. For a scalar system, both parametrizations are the same and reflect an ordinary numerator/denominator parametrization. Subsequently, the model orders or number of parameters to be estimated can be specified, for which a separate order editor is available.
Finally, the weighting to be used during the estimation (curve fitting) of the model can be specified in the frequency weighting-popup menu. Default, the weighting is chosen to be the inverse of the data, so as to minimize a relative error instead of an absolute error. Additional choices include none (unit weighting to minimize an unweighted, absolute error) or advanced. The advanced weighting option enables the user to load and/or modify frequency domain weightings relatively easily. One is referred to section 3.2 for a more detailed discussion on the usage of advanced weightings.

A simple click on the estimate button will start the minimization. Progress on the iteration to fit the frequency response is displayed in the MATLAB command window. Some options associated with the Sanathanan-Koerner iteration are available under a control-button, see also Figure 4. If the minimization has been completed successfully, the model can be imported on the Model Board. Before importing the model, options associated to the frequency range for evaluation purposes, the name of the model and the additional information on the model being estimated can be modified.

3.2 Advanced weightings

The weighting used in the least squares estimation can be any frequency dependent weighting, having the same size as the frequency response used for curve fitting. It is applied *element wise* in case of multivariable frequency response. The GUI of FREQID allows the import and/or modification of a weighting relatively easily by opening a weighting window, in which each element of a (multivariable) weighting can be edited.
The weighting window can be opened by selecting the advanced weighting option in the frequency weighting-popup menu depicted in Figure 4. The weighting window will start up with the *default* weighting: inverse of data, so as to minimize relative errors during curve fitting. However, any weighting can be imported and edited elementwise in the weighting window. A snap shot of the weighting window for editing an element is depicted in Figure 5. The vertical dashed-dotted lines in

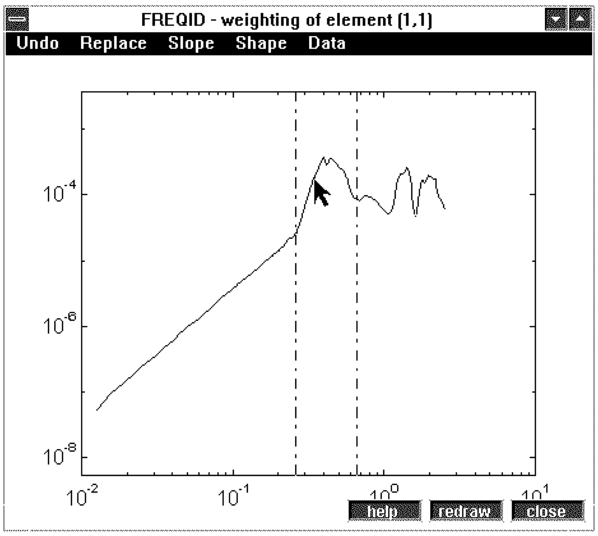

Fig. 5: Editable weighting per element

Figure 5 are used to select the frequency range to be edited for the specific element. These lines can be moved by a simple drag & drop mouse action. The different options on the menu bar of the window depicted in Figure 5 can be used to modify the weight. In this way, the frequency weighting can for example be smoothed, integrated and differentiated. Additionally, the weighting between the two vertical dash-dotted lines can also be modified by a drag & drop mouse action, as indicated in Figure 5. The shape of the weighting caused by this drag & drop action can be influenced by the different options available under the Shape option on the menu bar. In this way, FREQID offers the possibility to tune the frequency dependent weighting in a very flexible way.

4 Evaluation of models

As a final step in estimating models, the possibility to evaluate a model on the basis of its frequency response or pole/zero plot is available within FREQID. This can be done by using either the frequency resp. or the poles/zeros check-boxes available in the main window of FREQID, see Figure 1. Turning the frequency resp. check-box on, will open the frequency response window, as depicted in Figure 6. The fre-

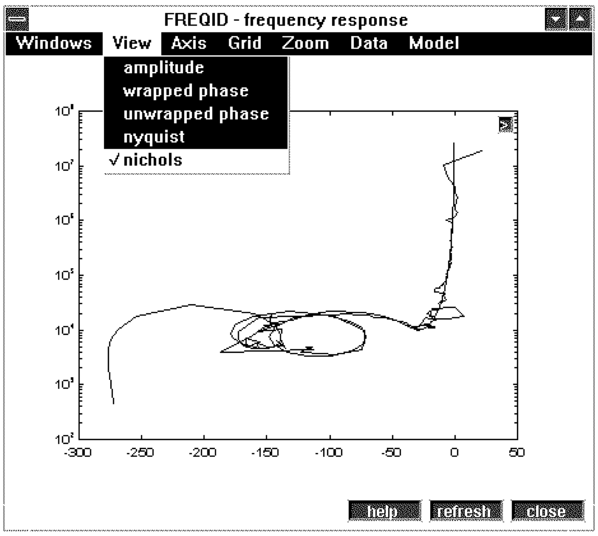

Fig. 6: Frequency response window

quency response window enables the user to view and compare both the measured frequency response and the frequency response of different models in various ways, as indicated by Figure 6. The menu balk offers the possibility to change the axis, add a grid, enable zooming and to plot various measured frequency responses (data) and/or model frequency responses in the same plot.

Next to the frequency domain evaluation of the model, the poles and zeros of the models being estimated can be computed. Turning the poles/zero check-box on in the main window of FREQID, will open the poles/zero window, as depicted in Figure 7. In Figure 7, poles of a model are indicated

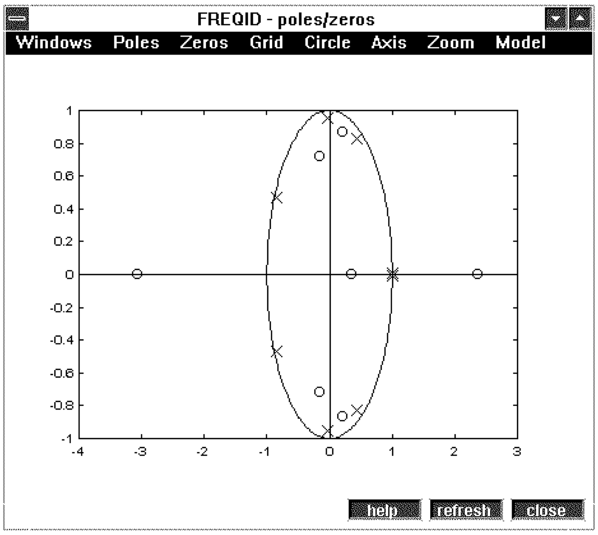

Fig. 7: poles and zeros window

by crosses, whereas zeros are plotted by circles. The pole/zero window enables the user to compare the poles and zeros of different models being estimated. For discrete time models a unit circle is displayed to evaluate stability conditions. The menu balk offers the possibility to toggle pole and/or zero location plot, add a grid, toggle unit circle plot, toggle real/imaginary axis plot, enable zooming and to plot poles/zero locations of different models. As a final remark it can be said here that the stability of a model can also be evaluated by by simple click on the name of the model in the main window of FREQID, see also section 2.4. This will display some additional information on the model, including the stability of the model.

5 Summary

A MATLAB toolbox called FREQID has been presented for estimating discrete or continuous time linear (multivariable) models on the basis of (measured) frequency responses. Within FREQID, a model is being estimated by performing a curve fit routine on the available frequency domain measurement. In the current version of FREQID, this curve fit routine can be either a least-squares or a maximum amplitude criterion.
To simplify the operations involved with choosing frequency dependent weightings, model order selection and model evaluation, FREQID is equipped with a user friendly Graphical User Interface (GUI). Additionally, the GUI serves as bookkeeper of the available frequency domain measurements and the different models being estimated.
The software of FREQID is written for MATLAB version 4.2c and the standard signal and control MATLAB toolboxes are required only. In addition, the optimization toolbox is needed only if the maximum amplitude routine is being used too.

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Any feedback by remarks, suggestions or bug reports are a prerequisite to develop reliable and user friendly software. Therefore, the authors would like to thank David Molenaar for his contribution.

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