

Control relevant identification for H_∞ -norm based performance specifications[‡]

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Abstract. For consecutive model-based control design, approximate identification of linear models should be performed on the basis of a feedback-relevant criterion, compatible with the control design. For an H_∞ -norm based control design, a procedure is presented to estimate a possibly unstable and feedback controlled plant by using an H_∞ -norm based feedback-relevant identification criterion. It is shown that the formulated identification problem can be handled by taking (noisy) closed loop frequency domain measurements of the plant and fitting a model of a prespecified McMillan degree, which is parametrized in a stable factorization, using a certain non-linear constrained minimization. The procedure is illustrated by an example.

Keywords. system identification; coprime factors; curve fitting; parametrization.

1 Introduction

The motivation for developing methods for so-called feedback relevant or control relevant system identification is induced by the fact that linear models found by identification techniques are exploited usually in a model-based control design. Any model found by system identification techniques will be inevitably approximative and the validity of the model will highly depend on its intended use. In this perspective the identification procedure being used, will be subjected to several requirements if the model being estimated is to be used mainly for control design. These requirements boil down to the fact that the feedback relevant dynamical behaviour of a plant operating in a closed loop configuration has to be estimated, in order to design an enhanced and robust control system (Gevers, 1993; Van den Hof and Schrama, 1994).

To tackle the problem of designing an enhanced and robust control system for a plant with unknown

dynamics, a simultaneous (off-line) optimization of identification and model-based control design criteria would be required, as formulated in Bayard *et al.* (1992). On the other hand, it has been widely motivated to separate the two stages of identification and control design and to use an iterative scheme of identification and model-based control design (Schrama, 1992). In such an iterative scheme, closed loop experimental conditions are indispensable to obtain data from the (possibly unstable) plant (Hakvoort *et al.*, 1994; Hjalmarsson *et al.*, 1994a; Lee *et al.*, 1993).

Compatible criteria in both the identification and control design are a prerequisite in order to ensure performance improvement of the plant to be controlled during the subsequent iterations of identification and model-based control design (Bayard *et al.*, 1992). For ‘classical’ (weighted) H_2 -norm based identification criteria, control performance specifications are implicitly restricted to H_2 /LQG-type control design criteria, see e.g. Hakvoort *et al.* (1994), Lee *et al.* (1993), Zang *et al.* (1992) or Gevers (1993) for a nice overview. For H_∞ -norm based control performance specifications this unleashes the need for an H_∞ -norm based identification procedure. Furthermore, compatible (weighted) H_∞ -norm based

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criteria in both the identification and the control design of an iterative scheme, opens the possibility to incorporate (performance) robustness considerations (Doyle *et al.*, 1992) to ensure performance improvement of the plant to be controlled (Bayard *et al.*, 1992).

In this paper a procedure is presented to estimate a linear multivariable discrete or continuous time model with a prespecified McMillan degree that fits (noisy) frequency response data of a (possibly unstable) plant operating under closed-loop conditions on the basis of an H_∞ -criterion. The H_∞ -criterion will be approximated by a pointwise evaluation of frequency response data, which is the main motivation in this paper to consider frequency domain data of the plant. Furthermore, convergence aspects of an iterative scheme of identification and model-based control design employing an H_∞ -norm control performance are being discussed. A unified approach to handle both stable and unstable plants is obtained by estimating a model via a *stable* factorization similar as in Lee *et al.* (1993) or Schrama and Bosgra (1993). Alternative approaches with an H_∞ -criterion can also be found in the area of identification in H_∞ , see e.g. Gu and Khargonekar (1992) or Helmicki *et al.* (1991). In these approaches a stable transfer function having some worst-case optimality properties is being derived on the basis of (noisy) frequency response data. A drawback is the lack of ability to prespecify the McMillan degree of the model being estimated, which may result in relatively high order models (Friedman and Khargonekar, 1994).

As standard curve fit procedures do not guarantee stability of the resulting estimate of the factorization, a canonical parametrization given in (Ober, 1991) will be used in this paper to parametrize all stable, minimal and balanced state space systems with distinct Hankel singular values of a prespecified McMillan degree. An alternative curve fit procedure with conditions on stability of the estimate can also be found in Hakvoort and Van den Hof (1994). The procedure in Hakvoort and Van den Hof (1994) is based on a maximum amplitude criterion, which has a close connection with an H_∞ -criterion only in the case of fitting a stable scalar transfer function. Using the parametrization results of Ober (1991), it is shown that the minimization of the feedback-relevant H_∞ -criterion can be handled by a non-linear constrained minimization, where the parameters lie in a convex set.

The paper is organized as follows. Firstly some preliminaries and a motivation for the usage of a H_∞ -criterion will be motivated respectively in section 2 and section 3. Next, the framework for the identification of stable factorizations will be pre-

sented in section 4 along with the results on the parametrization of stable, minimal and balanced systems, as introduced in Ober (1991) and further elaborated in Chou (1994), and the resulting minimization problem will be presented. Finally, the procedure is illustrated by an example in section 5 and the paper will end with concluding remarks.

2 Preliminaries

Let P be used to denote either the plant P_o or the model \hat{P} , then the feedback configuration of P and a controller C is denoted with $\mathcal{T}(P, C)$ and defined as the connection structure depicted in Figure 1. If P

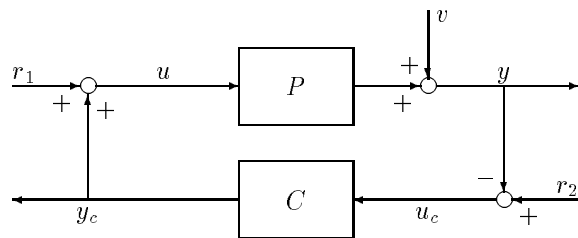


Fig. 1: feedback connection structure $\mathcal{T}(P, C)$

equals P_o in Figure 1, then the signals u and y reflect respectively the inputs and outputs of the plant P_o , where v is an additive noise on the output y of the plant. It is presumed that the noise v is uncorrelated with the external reference signals r_1, r_2 and can be modelled by a monic stable and stably invertible noise filter H having a white noise input ϵ (Ljung, 1987). The signals u and y are being measured and r_1, r_2 (and consequently u_c, y_c) are possibly at our disposal.

It is assumed that the feedback connection structure is well posed, that is $\det[I + CP] \neq 0$. In this way the mapping of $[r_2 \ r_1]^T$ to $[y \ u]^T$ is given by the transfer function matrix $T(P, C)$ with

$$T(P, C) := \begin{bmatrix} P \\ I \end{bmatrix} [I + CP]^{-1} \begin{bmatrix} C & I \end{bmatrix}, \quad (1)$$

and the data coming from the closed loop system $\mathcal{T}(P_o, C)$ can be described by

$$\begin{bmatrix} y \\ u \end{bmatrix} = T(P_o, C) \begin{bmatrix} r_2 \\ r_1 \end{bmatrix} + \begin{bmatrix} I \\ -C \end{bmatrix} [I + P_o C]^{-1} v. \quad (2)$$

In case of an *internally* stable closed loop system $\mathcal{T}(P, C)$, all four transfer function matrices in $T(P, C)$ will be stable which implies $T(P, C) \in \mathbb{RH}_\infty$ for a real rational P , where \mathbb{RH}_∞ denotes the set of all rational stable transfer functions.

Using the theory of fractional representations, P will be expressed as a ratio of two stable mappings N and D . Following Vidyasagar (1985), P has a right coprime factorization (*rcf*) (N, D) over \mathbb{RH}_∞ if there exists X, Y, N and D such that $P = ND^{-1}$ and $XN + YD = I$. In addition, a *rcf* (N, D) is normalized if it satisfies $N^*N + D^*D = I$, where $*$ denotes the complex conjugate transpose. Dual definitions apply for left coprime factorizations (*lcf*).

3 Motivation for H_∞ -criterion

3.1 Norm-based control design and identification

In the analysis of feedback relevant identification, the characterization of a closed loop (nominal) performance criterion plays a crucial role. If again the symbol P is used to denote either the actual plant P_o or the model \hat{P} to be estimated, this criterion can be formalized as follows (Van den Hof and Schrama, 1994). Let \mathcal{X} denote a complete normed space, where $\|\cdot\|_{\mathcal{X}}$ is the norm defined on \mathcal{X} . Let $J(P, C)$ be any function with an image in \mathcal{X} , then an objective function can be defined by $\|J(P, C)\|_{\mathcal{X}}$.

In this paper the normed space \mathcal{X} is chosen to be the space \mathbb{RH}_∞ and consequently the norm function $\|J(P, C)\|_{\mathcal{X}}$ is the H_∞ -norm. The control objective function $J(P, C) \in \mathbb{RH}_\infty$ to be minimized in a norm based control design is taken to be

$$\|J(P, C)\|_\infty := \|W_2 T(P, C) W_1\|_\infty \quad (3)$$

with $W_2, W_1 \in \mathbb{RH}_\infty$, which is a weighted form of the closed loop dynamics described by the transfer function matrix $T(P, C) \in \mathbb{RH}_\infty$ given in (1).

The actual plant P_o under consideration is unknown and hence the minimization of (3) cannot be solved straightforwardly for $P = P_o$. Instead of minimizing $\|J(P_o, C)\|_\infty$ directly, generally $\|J(P_o, C)\|_\infty$ is minimized iteratively (Gevens, 1993) using a sequence of controllers C_i , based on observations of the plant P_o , that satisfy at least

$$\|J(P_o, C_i)\|_\infty \leq \gamma_i \text{ and } \gamma_i \leq \gamma_{i-1}. \quad (4)$$

See e.g. Hjalmarsson *et al.* (1994b) for an approach on direct tuning of controllers C_i based on a 2-norm performance specification. If, on the other hand, a model \hat{P}_i (obtained by identification techniques) is introduced and used to design the controller C_i in the i th step of the iterative scheme, then generally an upper bound on $\|J(P_o, C)\|_\infty$ is minimized iteratively. The upper bound is inspired by the following triangular inequality (Schrama, 1992).

$$\|J(P_o, C_i)\|_\infty \leq \|J(\hat{P}_i, C_i)\|_\infty + \|J(P_o, C_i) - J(\hat{P}_i, C_i)\|_\infty \quad (5)$$

The right hand side of (5) can be minimized by minimizing $\|J(\hat{P}_i, C)\|_\infty$ by a norm based control design,

$$C_i := \min_C \|J(\hat{P}_i, C)\|_\infty \quad (6)$$

while the minimization of the second term can be viewed as an identification problem (Schrama, 1992)

$$\hat{P}_i := \min_P \|J(P_o, C_i) - J(P, C_i)\|_\infty \quad (7)$$

Unfortunately, the controller C_i is based on the model \hat{P}_i via (6), while \hat{P}_i will depend on the controller C_i via (7) and both are unknown (yet). Hence, the introduction of the model \hat{P}_i in the right hand side of (5) does not help in minimizing the upper bound directly. Usually one tries to alternate on the minimization of (6) and (7) leading off with some initial controller, hoping that a sequence of controllers will be obtained that satisfy at least (4).

3.2 Enforcing performance enhancement

In order to have a sequence of controllers C_i found by (6) that satisfy (4), conditions on both the models P_i and the controllers C_i should be derived. These conditions can be found by evaluating the triangular inequality on the previous step $i - 1$ of the iteration as follows.

$$\|J(P_o, C_{i-1})\|_\infty \leq \|J(\hat{P}_i, C_{i-1})\|_\infty + \|J(P_o, C_{i-1}) - J(\hat{P}_i, C_{i-1})\|_\infty \quad (8)$$

Now the minimization of the second term of the right hand side of (8), similar as in (7), can indeed be seen as an identification problem, since C_{i-1} from the previous iteration is assumed to be known. The first term on the right hand side of (8) is now simply an evaluation of the control objective function, using the model being estimated. In this perspective the following iterative scheme of identification and control can be considered.

Proposition 3.1 *Given a controller C_{i-1} and a γ_{i-1} such that $\|J(P_o, C_{i-1})\|_\infty \leq \gamma_{i-1}$. Consider the following iterative scheme.*

1. Estimate a model \hat{P}_i by the minimization

$$\hat{P}_i := \min_P \|J(P_o, C_{i-1}) - J(P, C_{i-1})\|_\infty \quad (9)$$

and accept the model \hat{P}_i only if $\beta_i \leq \gamma_{i-1}$, with

$$\beta_i := \underbrace{\|J(\hat{P}_i, C_{i-1})\|_\infty}_a + \underbrace{\|J(P_o, C_{i-1}) - J(\hat{P}_i, C_{i-1})\|_\infty}_b. \quad (10)$$

2. Design a controller C_i on the basis of \hat{P}_i by the minimization

$$C_i := \min_C \|J(\hat{P}_i, C)\|_\infty \quad (11)$$

and accept the controller only if $\gamma_i \leq \beta_i$, with

$$\begin{aligned} \gamma_i := & \underbrace{\|J(\hat{P}_i, C_i)\|_\infty}_{(c)} + \\ & + \underbrace{\|J(P_o, C_i) - J(\hat{P}_i, C_i)\|_\infty}_{(d)} \end{aligned} \quad (12)$$

3. $i := i + 1$, goto 1

Then the sequence of controllers C_i satisfy (4).

Proof: The inequality given in (10) is an upper bound for $\|J(P_o, C_{i-1})\|_\infty$, while (12) is an upper bound for $\|J(P_o, C_i)\|_\infty$. $\|J(P_o, C_i)\|_\infty \leq \gamma_i$ and $\|J(P_o, C_{i-1})\|_\infty \leq \beta_i$ with $\gamma_i \leq \beta_i \leq \gamma_{i-1}$. \square

Note that proposition 3.1 is a rather general set-up of an iterative scheme to generate a sequence of controllers that satisfies (4). In this set-up (10) and (12) reflect respectively a model and a controller validation test in order to enforce (4). The numerical values of (b) and (c) are simply the minimizing values of respectively the minimization (9) and (11), while (a) is just an evaluation of the control objective function. The contribution of (d) can be overestimated by incorporating an estimation of the model mismatch between \hat{P}_i and P_o , see also (Bayard *et al.*, 1992). Moreover, this model mismatch can be used in the minimization (11) of step 2 by formulating a *robust* control design problem (Doyle *et al.*, 1992).

For notational convenience the *known* controller C_{i-1} (coming from the previous iteration) and implemented on the actual feedback controlled plant P_o , will be simply denoted by C in the sequel of this paper. With the choice of the objective function given in (3), the minimization (9) to estimate a model \hat{P}_i can now be formulated as $\hat{P}_i := P(\hat{\theta}_i)$ with

$$\begin{aligned} \hat{\theta}_i = \arg \min_{\theta \in \Theta} & \|\Delta T(P_o, P_i(\theta), C)\|_\infty \text{ and} \\ & \Delta T(P_o, P_i(\theta), C) = \\ & W_2[T(P_o, C) - T(P_i(\theta), C)]W_1. \end{aligned} \quad (13)$$

For the minimization of (11) one is referred to standard H_∞ -control design problems present in the literature (Bongers and Bosgra, 1990; McFarlane and Glover, 1990). In the remaining part of this paper, the minimization (13) will be considered. The minimization will be based on closed loop data coming from the plant and employing a stable factorization of the model to be estimated.

4 Estimation and parametrization of stable factorizations

4.1 Identification of coprime factors

A coprime factorization of a plant P_o operating under closed loop conditions can be accessed by performing a filtering of the signals present in the closed loop system (Van den Hof *et al.*, 1995) and defined by

$$x := F[r_1 + Cr_2] = F[u + Cy]. \quad (14)$$

By considering the map from x in (14) onto $col(y, u)$, a stable right factorization $(P_o S_{in} F^{-1}, S_{in} F^{-1})$ of P_o is readily available from the data of the closed loop controlled plant. For exact details, one is referred to Van den Hof *et al.* (1995) or de Callafon and Van den Hof (1995), but in order to evaluate the usefulness of the filtering (14), the following result can be given.

Lemma 4.1 *Let the plant P_o and a controller C form an internally stable feedback system $T(P_o, C)$ and denote $S_{in} = [I + CP_o]^{-1}$. Then the following statements are equivalent.*

- (i) $(P_o S_{in} F^{-1}, S_{in} F^{-1})$ is a rcf.
- (ii) there exists a rcf (N_x, D_x) of an auxiliary model P_x with $T(P_x, C) \in \mathcal{RH}_\infty$ such that

$$F = [D_x + CN_x]^{-1} \quad (15)$$

Both conditions imply $F \begin{bmatrix} C & I \end{bmatrix} \in \mathcal{RH}_\infty$.

Proof: See Van den Hof *et al.* (1995) or de Callafon and Van den Hof (1995). \square

Using the notation

$$N_{o,F} := P_o S_{in} F^{-1}, \quad D_{o,F} := S_{in} F^{-1}, \quad (16)$$

Lemma 4.1 characterizes the freedom in choosing the right coprime factorization $(N_{o,F}, D_{o,F})$ of the plant P_o by the choice of any stable right factorization of any auxiliary model P_x that is internally stabilized by the controller C . Using the rcf $(N_{o,F}, D_{o,F})$ of the plant P_o , the error $\Delta T(P_o, P_i(\theta), C)$ in (13) can be expressed in the following way.

Lemma 4.2 *Let P_o and C create an internally stable feedback system $T(P_o, C)$ and let $(N_{o,F}, D_{o,F})$ be the rcf of P_o given in (16), where F is given in (15). Consider any $P_i(\theta)$, then*

- (i) for all $\theta \in \Theta$ there exists a rcf $(N_i(\theta), D_i(\theta))$ of $P_i(\theta)$ such that $D_i(\theta) + CN_i(\theta) = F^{-1}$.

(ii) $\|\Delta T(P_o, P_i(\theta), C)\|_\infty$ in (13) equals

$$\left\| W_2 \left(\begin{bmatrix} N_{o,F} \\ D_{o,F} \end{bmatrix} - \begin{bmatrix} N_i(\theta) \\ D_i(\theta) \end{bmatrix} \right) F [C \ I] W_1 \right\|_\infty \quad (17)$$

where $(N_i(\theta), D_i(\theta))$ is any rcf of $P_i(\theta)$ that satisfies (i).

Proof: See de Callafon and Van den Hof (1995) \square

With the result mentioned in lemma 4.2 it can be seen that $\Delta T(P_o, P_i(\theta), C)$ in (13) is simply a *weighted* difference between the rcf $(N_{o,F}, D_{o,F})$ and $(N_i(\theta), D_i(\theta))$ respectively of the plant P_o and the model $P_i(\theta)$. However, the weighting contains the filter F , which depends on the factorization $(N_i(\theta), D_i(\theta))$ of the model. In de Callafon and Van den Hof (1995) an iterative procedure has been presented to update the filter F on the basis of the factorization $(N_i(\hat{\theta}), D_i(\hat{\theta}))$ being estimated, to satisfy (i) in Lemma 4.2.

Since the rcf $(N_{o,F}, D_{o,F})$ can be accessed via the map from x in (14) onto $col(u, y)$, complex (noisy) frequency domain data can be obtained. The data of the factorization $(N_{o,F}, D_{o,F})$ will be denoted by $(N(\xi_j), D(\xi_j))$, where ξ_j for $j = 1, 2, \dots, l$ denotes a (prespecified) frequency grid $\xi_j = e^{i\omega_j}$ for discrete time systems and $\xi_j = i\omega_j$ for continuous time systems. In this way the minimization of the H_∞ -criterion (13) will be approximated by performing a pointwise evaluation of (17). The actual minimization problem can now be formalized as follows.

$$\min_{\theta} \max_{j=1, \dots, l} \bar{\sigma} \{ W_2(\xi_j) \left(\begin{bmatrix} N(\xi_j) \\ D(\xi_j) \end{bmatrix} - \begin{bmatrix} N_i(\theta, \xi_j) \\ D_i(\theta, \xi_j) \end{bmatrix} \right) \cdot F(\xi_j) [C(\xi_j) \ I] W_1(\xi_j) \} \quad (18)$$

where $\bar{\sigma}\{\cdot\}$ denotes the maximum singular value. To ensure stability of the estimate $(N_i(\hat{\theta}), D_i(\hat{\theta}))$, a special parametrization presented in Ober (1991) of minimal, stable and balanced state space representations for the factorization $(N_i(\theta), D_i(\theta))$ will be used. Combining the requirements for minimality and stability along with a balancing property of the state space representation, will lead to parameter constraints that can be dealt with relatively easily.

4.2 Parametrization of stable factorizations

The parametrization results on stable, minimal and balanced state space realization in Ober (1991) and further elaborated in Chou (1994), are based on continues time systems having multiple common Hankel singular values. For discrete time systems an indirect state space parametrization can be based

on a Möbius transformation, since this transformation preserves both stability, minimality and the balanced property of the continues time state space realization. Furthermore, in this paper only the case of distinct Hankel singular values will be discussed, which can be considered to be the generic case (Chou, 1994).

Lemma 4.3 Let $G(s)$ be defined by

$$G(s) := \begin{bmatrix} N(s) \\ D(s) \end{bmatrix}$$

where $(N(s), D(s))$ is a rcf of the $p \times m$ rational transfer function $P(s)$, then the following statements are equivalent

1. $G(s)$ is a $(p+m) \times m$ stable rational transfer function matrix of McMillan degree n , with n distinct Hankel singular values.
2. $G(s)$ has a state space representation represented by the matrices \bar{A} , \bar{B} , \bar{C} and \bar{E} with

$$\bar{B} := \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} \text{ with } b_j = [b_{ij}] \in \mathbb{R}^{1 \times m} \text{ and } b_{1j} > 0 \text{ for } 1 \leq j \leq n \quad (19)$$

$$\bar{C} := [c_1 \ \dots \ c_n] \text{ with } c_j = u_j [b_j b_j^T]^{1/2}, \quad u_j \in \mathbb{R}^{(p+m) \times 1} \text{ and } u_j^T u_j = 1 \text{ for } 1 \leq j \leq n \quad (20)$$

$$\bar{A} := [a_{ij}] \in \mathbb{R}^{n \times n}, \quad a_{ij} = -\frac{b_j b_j^T}{2\sigma_j} \text{ for } i = j, \quad \text{and } a_{ij} = \frac{\sigma_j b_i b_j^T - \sigma_i c_i^T c_j}{\sigma_i^2 - \sigma_j^2} \text{ for } i \neq j, \quad \sigma_{j+1} > \sigma_j > 0 \text{ for } 1 \leq j \leq n-1 \quad (21)$$

$$\bar{E} := [e_{ij}] \in \mathbb{R}^{(p+m) \times m}$$

which is a balanced state space representation having distinct Hankel singular values σ_j .

Proof: Direct application of theorem 2.1 in Ober (1991) for a system with distinct Hankel singular values. \square

Splitting up \bar{C} and \bar{E} respectively into $[\bar{C}_N^T \ \bar{C}_D^T]^T$ and $[\bar{E}_N^T \ \bar{E}_D^T]^T$, the operation $P = ND^{-1}$ leads to a state space realization (A, B, C, E) of P given by

$$\begin{aligned} A &= \bar{A} - \bar{B} \bar{E}_D^{-1} \bar{C}_D & B &= \bar{B} \bar{E}_D^{-1} \\ C &= \bar{C}_N - \bar{E}_N \bar{E}_D^{-1} \bar{C}_D & E &= \bar{E}_N \bar{E}_D^{-1} \end{aligned} \quad (22)$$

It should be noted that (19) and (21) reflect parameter constraints that can be dealt with relatively

easy. In standard pseudo canonical (overlapping) parametrizations (Ljung, 1987) more complicated parameter constraints should have been specified to enforce stability. Unfortunately, (20) reflect a non-linear equality constraint to parametrize the elements of a unitary vector.

To circumvent (20), in Chou (1994) a parametrization based on rotating actions has been proposed. This parametrization parametrizes *almost* all unitary vectors in $\mathbb{R}^{k \times 1}$ for $k > 1$, without a non-linear constraint. Since $p + m > 1$, this can be applied without objections and it will be shown here that *all* unitary vectors $\in \mathbb{R}^{(p+m) \times 1}$ can be parametrized.

Lemma 4.4 *Let $\mathcal{U} := \{\bar{u} \in \mathbb{R}^{(p+m) \times 1} \mid \bar{u}^T \bar{u} = 1\}$, $\phi := [\varphi_1 \cdots \varphi_{p+m-1}] \in \mathbb{R}^{(p+m-1) \times 1}$ and the set $\Phi := \{\phi \in \mathbb{R}^{(p+m-1) \times 1} \mid \varphi_i \in (-\pi/2, \pi/2] \text{ for } 1 \leq i < p + m - 1, \varphi_{p+m-1} \in (-\pi, \pi]\}$. Consider the map $f : \mathbb{R}^{(p+m-1) \times 1} \rightarrow \mathbb{R}^{(p+m) \times 1}$ given by*

$$u = \prod_{i=1}^{p+m-1} x_i, \text{ with} \quad (23)$$

$$x_i := \begin{bmatrix} \cos(\varphi_i) x_{i-1} \\ \sin(\varphi_i) \end{bmatrix}, \quad x_0 = 1$$

then

- (a) $f(\phi) \in \mathcal{U}$ for all $\phi \in \mathbb{R}^{(p+m-1) \times 1}$
- (b) the map $f : \mathbb{R}^{(p+m-1) \times 1} \rightarrow \mathcal{U}$ is surjective
- (c) the map $f : \Phi \rightarrow \mathcal{U}$ is bijective.

Proof: (a) The fact $f(\phi) \in \mathcal{U}$, $\forall \phi \in \mathbb{R}^{(p+m-1) \times 1}$ can be found by induction, see also Chou (1994).

(b) Take any $u := [u_1 \cdots u_{p+m}]^T \in \mathcal{U}$ and define k to denote the index of the *first* non-zero entry u_k in u . Using the map (23) for verification, the elements φ_i of ϕ can be given by $\varphi_i = \pi/2$ for $1 \leq i \leq k - 1$,

$$\varphi_i = \tan^{-1} \left(\frac{u_{i+1}}{u_k} \prod_{j=k}^{i-1} \cos(\varphi_j) \right)$$

for $k \leq i < p + m - 1$ and finally $\varphi_{p+m-1} = \text{sign}(u_{p+m})[1 - \text{sign}(u_k)]\pi/2 + \text{sign}(u_k) \sin^{-1}(u_{p+m})$, where $\text{sign}(\cdot)$ denotes the sign function defined by $\text{sign}(x) := 1$ if $x \geq 0$ and $\text{sign}(x) := -1$ if $x < 0$. Clearly, $\exists \phi \in \mathbb{R}^{p+m-1}$, which proves (b).

(c) It can be verified that the elements φ_i of ϕ_i defined above, restricts ϕ to be an element of Φ and therefore uniquely determines $\phi \in \Phi$. \square

With the result mentioned in lemma 4.4, the parametrization of the unitary vectors $u_j \in \mathbb{R}^{p+m}$ in (20) can be replaced by the alternative parametrization in terms of $\phi \in \mathbb{R}^{p+m-1}$. Putting linear constraints on the elements φ_i such that $\phi \in \Phi$ according to lemma 4.4, will yield a *unique* parametrization. However, due to the periodicity of the elements

φ_i of ϕ , the parametrization will be locally identifiable. Therefore, the constraints on the elements φ_i of ϕ can be omitted during the optimization.

Resuming, the minimization problem given in (18) can be formulated as follows. Let $G_i(\theta)$ be defined as

$$G_i(\theta) := \begin{bmatrix} N_i(\theta) \\ D_i(\theta) \end{bmatrix}$$

having m inputs, $p + m$ outputs and parametrized according to the results mentioned in lemma 4.3 and lemma 4.4. Then $[\bar{A}, \bar{B}, \bar{C}]$ are given by the parameter

$$\theta = [\sigma_1 \cdots \sigma_n \ b_1 \cdots b_n \ \phi_1^T \cdots \phi_n^T], \text{ with}$$

$$\theta \in \Theta \subset \mathbb{R}^{1 \times n(2m+p)}$$

where Θ is determined by the additional constraints given in (19) and (21). These constraints can be rewritten into

$$\begin{aligned} \sigma_n - \sigma_{n-1} > 0, \cdots, \sigma_2 - \sigma_1 > 0, \sigma_1 > 0 \\ b_{11} > 0, b_{21} > 0, \cdots, b_{n1} > 0 \end{aligned} \quad (24)$$

to ensure a minimal, stable and balanced continuous time state space realization of the stable factorization $(N_i(\theta), D_i(\theta))$. In this way the minimization given in (18) is in fact a non-linear constrained minimization, where the parameters lie in a convex set Θ . This can be solved by standard constrained minimization routines, for example available in the optimization toolbox of the MatLab package (MatLab, 1994).

Compared to the curve fit procedure presented in Hakvoort and Van den Hof (1994), an alternative parametrization along with linear parameter constraints is presented here, to enforce stability of the multivariable estimate having multiple outputs. In addition, the frequency grid is used here to evaluate a maximum singular value, instead of a maximum amplitude criterion over all possible transfer functions.

5 Example

In this section, only the results on the minimization of (18) will be illustrated for a fixed filter F and unity weightings W_1 and W_2 . Using lemma 4.3 and lemma 4.4 to parametrize the state space realization of $(N(\theta), D(\theta))$ and the convex parameter constraints given in (24), (18) will be solved by a non-linear constrained optimization.

Consider a 5th order SISO discrete time plant P_o having a DC-gain of 5, zeros located at $0.52 \pm 0.44i$, $0.97 \pm 0.06i$, poles located at $0.76 \pm 0.40i$, $0.99 \pm 0.06i$ and 0.94. The plant P_o is controlled by a discrete time controller C given by

$$C(q) = \frac{2.04q^3 - 1.66q^2 - 1.14q + 1.24}{q^3 - 1.68q^2 + 1.03q - 0.35} \quad (25)$$

As mentioned before, the minimization of (18) will be illustrated and W_1 and W_2 are chosen to be identity. Furthermore, the filter F in (15) is taken to be fixed and based on an auxiliary model P_x that equals P_o , where the factorization (N_x, D_x) is taken to be a discrete time normalized right coprime factorization. Noisy frequency response measurements were generated at 100 frequency points distributed between 10^{-3} and 1Hz and based on time domain observations of x in (14) and $col(y, u)$.

The aim is to fit a 3rd order stable factorization on the frequency response data $(N(e^{i\omega_j}), D(e^{i\omega_j}))$ of the 5th order plant P_o , using the non-linear constrained minimization. In order to start up the non-linear minimization, an ordinary least-squares equation error fit is applied to have an initial estimate of the factorization. The initial estimate happens to be stable and therefore it can be balanced and converted to the parametrization of lemma 4.3 and lemma 4.4. After this, the non-linear constraint optimization is started up. The final result (after 432 iterations) represented in a transfer function of $col(\hat{N}(q), \hat{D}(q))$ is given by

$$\frac{\begin{bmatrix} 0.0005q^3 + 0.1162q^2 - 0.1165q + 0.0594 \\ 0.8802q^3 - 2.1267q^2 + 1.8395q - 0.5871 \end{bmatrix}}{[q^3 - 2.1861q^2 + 1.7704q - 0.5244]}$$

which is of course stable. The amplitude Bode plot of the result is given in Figure 2.

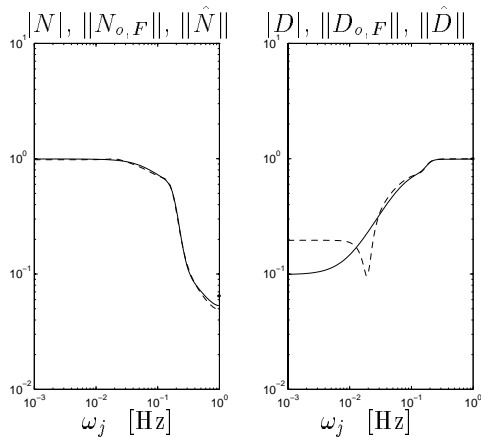


Fig. 2: Amplitude Bode plot of the frequency response data $(N(e^{i\omega_j}), D(e^{i\omega_j}))$ (dotted), of the factorization $(N_{o,F}(e^{i\omega_j}), D_{o,F}(e^{i\omega_j}))$ of the plant P_o (dashed) and 3rd order estimate $(\hat{N}(e^{i\omega_j}), \hat{D}(e^{i\omega_j}))$ (solid)

To illustrate the character of the minimization (18) being performed, a plot of the weighted error

$$\bar{\sigma} \left\{ \left(\begin{bmatrix} N(e^{i\omega_j}) \\ D(e^{i\omega_j}) \end{bmatrix} - \begin{bmatrix} \hat{N}(e^{i\omega_j}) \\ \hat{D}(e^{i\omega_j}) \end{bmatrix} \right) \cdot F(e^{i\omega_j}) \begin{bmatrix} C(e^{i\omega_j}) & I \end{bmatrix} \right\} \quad (26)$$

is depicted in Figure 3. Clearly, this plot indicates the objective to minimize pointwise the maximum value of the error (26) along the available frequency grid.

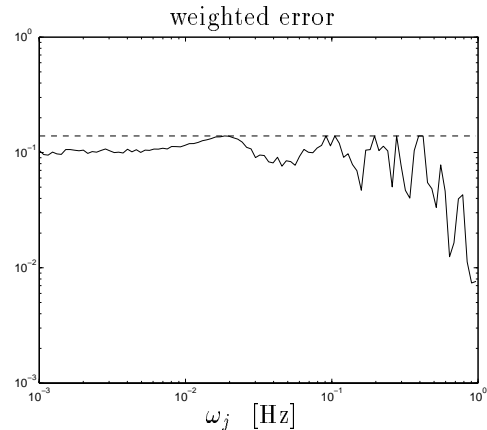


Fig. 3: Amplitude plot of the weighted error given in (26)

For completeness, a Bode plot of the resulting estimate $\hat{P} := \hat{N}\hat{D}^{-1}$ is depicted in Figure 4. From this picture it can be seen that a good estimate is obtained around the cross-over frequency of 0.2Hz. Moreover, it can be verified that the model \hat{P} is also stabilized by the controller given in (25).

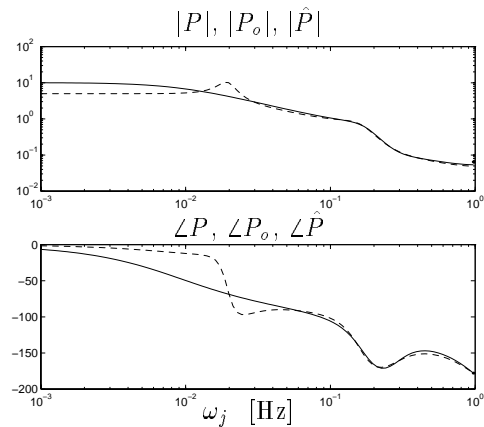


Fig. 4: Bode plot of data $P(e^{i\omega_j})$ (dotted), plant $P_o(e^{i\omega_j})$ (dashed) and 3rd order estimate $\hat{P}(e^{i\omega_j})$ (solid)

6 Conclusions

In this paper a procedure is presented to estimate a possibly unstable and feedback controlled multivariable plant by using an H_∞ -norm based feedback-relevant identification criterion. To handle the H_∞ -norm based criterion, (noisy) closed loop frequency

domain measurements of the plant are used to fit a model, parametrized in a stable factorization of a prespecified McMillan degree, by a pointwise evaluation of the error along a prespecified frequency grid. To ensure stability of the factorization being estimated, a parametrization that enforces a stable, minimal and balanced state space representation is being used. With this parametrization, the H_∞ -norm based identification criterion can be solved by a non-linear constrained optimization.

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