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FILTERING AND PARAMETRIZATION ISSUES IN FEEDBACK RELEVANT IDENTIFICATION BASED ON FRACTIONAL MODEL REPRESENTATIONS

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Abstract

In view of consecutive control design, approximate identification of linear models is performed on the basis of a feedback relevant performance criterion. The resulting closed loop identification problem is handled by identifying stable coprime factors of a possibly unstable plant, while the model class is restricted to contain models of a prespecified McMillan degree. It is shown that the formulated performance criterion can be handled by imposing a constraint on the model parametrization. A solution to deal with this restriction based on an update algorithm is presented and illustrated by an example.

1 Introduction

Induced by the fact that dynamical models obtained from system identification are used as a basis for model based control design, there is a growing interest in merging the problems of identification and control. This has been the motivation to develop methods for a so-called feedback relevant identification, which implies that the feedback relevant dynamical behaviour of the plant operating in a closed loop configuration has to be estimated in order to design enhanced controllers [8, 19].

To perform a feedback relevant identification, closed loop experiments from the real plant P_o are a prerequisite to come up with a model $\hat{P}$ suitable for control design [9, 12]. The need for closed loop experiments is induced by a feedback relevant criterion to be minimized for the estimation of a model $\hat{P}$. Since the controller that creates the closed loop configuration can (yet) be unknown, a simultaneous optimization of identification and control design has been proposed in [1] or [11]. In addition, it has been widely motivated to separate the two stages of identification and control design and to use an iterative scheme [17]. Recently developed iterative schemes can be found in [12], [16], [18] or [22].

In this paper the *identification stage* in such an iterative scheme will be discussed. During the identification a feedback relevant criterion will be minimized on the basis of closed loop observations of the plant P_o, to obtain a model $\hat{P}$ of a *prespecified* McMillan degree. Both the feedback relevant criterion and the usage of closed loop data can be handled by the algebraic theory of fractional representations [21]. Similar approaches based on a Youla parametrization can also be found in [10] or [12] but these techniques lack the ability to prespecify the McMillan degree of the model $\hat{P}$ being estimated.

By estimating a stable factorization of the model $\hat{P}$ directly, similar as in [18, 20], the McMillan degree of the model $\hat{P}$ can be controlled. However, minimizing the feedback relevant criterion for a *fixed order* model $\hat{P}$ leads to an additional restriction on the factorization to be estimated. A possible solution to deal with this restriction based on an update algorithm is presented and illustrated by an example.

2 Preliminaries

Let P be used to denote either the plant P_o or the model $\hat{P}$, then the feedback configuration of P and a controller C is denoted with $\mathcal{T}(P,C)$ and defined as the connection structure depicted in Figure 1.

If P equals P_o in Figure 1, then the signals u and y reflect respectively the inputs and outputs of the plant P_o, where v is an additive noise on the output y of the plant. It is presumed that the noise v is uncorrelated with the external reference signals r_1, r_2 and can be modelled by a monic stable and stably invertible noise filter H having a white noise input e [13]. The signals u and y are

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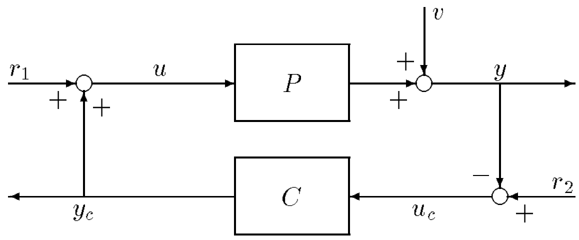

Fig. 1: feedback connection structure $\mathcal{T}(P,C)$

being measured and r_1, r_2 (and consequently u_c, y_c) are possibly at our disposal.

It is assumed that the feedback connection structure is well posed, that is $\det[I+CP] \not\equiv 0$. In this way the mapping of $[r_2\ r_1]^T$ to $[y\ u]^T$ is given by the transfer function matrix $T(P,C)$ with

$$T(P,C) := \begin{bmatrix} P \\ I \end{bmatrix} [I+CP]^{-1} \begin{bmatrix} C & I \end{bmatrix} \quad (1)$$

and the data coming from the closed loop system $\mathcal{T}(P_o,C)$ can be described by

$$\begin{bmatrix} y \\ u \end{bmatrix} = T(P_o,C) \begin{bmatrix} r_2 \\ r_1 \end{bmatrix} + \begin{bmatrix} I \\ -C \end{bmatrix} [I+P_oC]^{-1} v \quad (2)$$

In case of an *internally* stable closed loop system $\mathcal{T}(P,C)$, all four transfer function matrices in $T(P,C)$ will be stable [2, 7] which implies $T(P,C) \in \mathrm{I\!R}\mathcal{H}_\infty$ for a real rational P, where $\mathrm{I\!R}\mathcal{H}_\infty$ denotes the set of all rational stable transfer functions.

Using the theory of fractional representations, P will be expressed as a ratio of two stable mappings N and D. Following [21], P has a right coprime factorization (*rcf*) (N,D) over $\mathrm{I\!R}\mathcal{H}_\infty$ if there exists X, Y, N and D such that $P = ND^{-1}$ and $XN+YD=I$. In addition, a *rcf* (N,D) is normalized if it satisfies $N^*N+D^*D=I$, where * denotes the complex conjugate transposed. Dual definitions apply for left coprime factorizations (*lcf*).

3 Merging identification and control

The characterization of a closed loop performance criterion plays a crucial role in the analysis of feedback relevant identification. A characterization by means of an objective function that depends on the plant P_o and the controller C can be formalized as follows [19]. Let $\mathcal{X}$ denote a complete normed space, where $\|\cdot\|_\mathcal{X}$ is the norm defined on $\mathcal{X}$. Let $J(P_o,C)$ be any function with an image in $\mathcal{X}$, then an objective function can defined by $\|J(P_o,C)\|_\mathcal{X}$.

In this respect a norm based control design can be formulated by the minimization of $\|J(P_o,C)\|_\mathcal{X}$. Since the actual plant P_o is unknown, the control design can be based on minimization of an objective function $\|J(\hat{P},C)\|_\mathcal{X}$, by the introduction of a (nominal) model $\hat{P}$. The resulting controller, denoted by $C_{\hat{P}}$, is said to satisfy a (nominal) control objective if $\|J(\hat{P},C_{\hat{P}})\|_\mathcal{X} \leq \gamma$, where γ is some prespecified non-negative real number.

The introduction of a model $\hat{P}$ also opens the possiblity to lower and upper bound $\|J(P_o,C_{\hat{P}})\|_\mathcal{X}$ by employing the following triangular inequalities [18]

$$\begin{gathered} \left| \|J(\hat{P},C_{\hat{P}})\|_\mathcal{X} - \|J(P_o,C_{\hat{P}}) - J(\hat{P},C_{\hat{P}})\|_\mathcal{X} \right| \\ \leq \|J(P_o,C_{\hat{P}})\|_\mathcal{X} \leq \\ \|J(\hat{P},C_{\hat{P}})\|_\mathcal{X} + \|J(P_o,C_{\hat{P}}) - J(\hat{P},C_{\hat{P}})\|_\mathcal{X} \end{gathered}$$

From the second inequality it can be seen that

$$\|J(\hat{P},C_{\hat{P}})\|_\mathcal{X} + \|J(P_o,C_{\hat{P}}) - J(\hat{P},C_{\hat{P}})\|_\mathcal{X} \leq \gamma \quad (3)$$

is a sufficient condition in order to have a controller $C_{\hat{P}}$ which satisfies the control objective on the real plant P_o. Clearly, the first term on the left hand side in (3) can be minimized by the norm based control design for a fixed model $\hat{P}$. The minimization of the mismatch $\|J(P_o,C) - J(\hat{P},C)\|_\mathcal{X}$ for a fixed controller C can be viewed as a feedback relevant identification problem.

In this paper the normed space $\mathcal{X}$ is chosen to be the space $\mathrm{I\!R}\mathcal{H}_\infty$ and the control objective function $\|J(\hat{P},C)\|_\infty \in \mathrm{I\!R}\mathcal{H}_\infty$ is taken to be

$$\begin{aligned} \|J(\hat{P},C)\|_\infty &:= \|W_oT(\hat{P},C)W_i\|_\infty \\ \text{and } C_{\hat{P}} &:= \arg\min_C \|W_oT(\hat{P},C)W_i\|_\infty \end{aligned} \quad (4)$$

with W_o, $W_i \in \mathrm{I\!R}\mathcal{H}_\infty$. The objective function given in (4) represents a large class of ∞-norm based control design schemes and the usage of the weightings W_o, W_i is inspired by the ability to create a trade off between conflicting requirements and constraints always present [3]. With the choice of the objective function given in (4), the minimization of the performance degradation $\|J(P_o,C) - J(\hat{P},C)\|_\mathcal{X}$

$$\hat{\theta} = \arg\min_\theta \|W_o[T(P_o,C) - T(P(\theta),C)]W_i\|_\infty \quad (5)$$

will be regarded as the feedback relevant identification problem.

The minimization (5) on the basis of data coming from the plant P_o operating under closed loop conditions can be handled by the framework of fractional representations, which additionally gives a unified appraoch to handle stable and unstable plants. How to access a stable factorization of the plant P_o is discussed in section 4, while in section 5 the minimization (5) for a *fixed order* model $P(\theta)$ will be expressed in terms of a fractional representation.

4 Access to stable factorizations

Denoting $r := r_1 + Cr_2$ and $S_i := [I+CP_o]^{-1}$, it follows from (2) that the transfer functions (P_oS_i, S_i), which are accessible from data as r, u and y are measured, can be considered to be a stable (right) factorization of the plant P_o, i.e. $P_0 = [P_oS_i][S_i]^{-1}$. To avoid the presence and estimation of common unstable zeros in the stable right factorization of P_o, the factorization needs to be a *rcf*. Furthermore, a *rcf* is not unique and access to different factorizations would be preferable.

Similar as in [20] or [5], an additional filtering $x := Fr$ can be introduced to fulfill these requirements. With (2) this yields

$$x = F \begin{bmatrix} C & I \end{bmatrix} \begin{bmatrix} r_2 \\ r_1 \end{bmatrix} = F \begin{bmatrix} C & I \end{bmatrix} \begin{bmatrix} y \\ u \end{bmatrix} \qquad (6)$$

and (2) reduces to

$$\begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} P_oS_iF^{-1} \\ S_iF^{-1} \end{bmatrix} x + \begin{bmatrix} S_o \\ -CS_o \end{bmatrix} He \qquad (7)$$

where $(P_oS_iF^{-1}, S_iF^{-1})$ can be considered to be a (right) factorization of the plant P_o.

The freedom in choosing the filter F can be found by restricting $(P_oS_iF^{-1}, S_iF^{-1})$ to be a *rcf* and is summarized below.

Lemma 4.1 *Let P and C form an internally stable feedback system $\mathcal{T}(P,C)$. Then the following statements are equivalent.*

(i) (PS_iF^{-1}, S_iF^{-1}) *is a* rcf.

(ii) *there exists a* rcf (N_x, D_x) *of an auxiliary model* P_x *with* $T(P_x, C) \in \mathbb{R}\mathcal{H}_\infty$ *such that*

$$F = [D_x + CN_x]^{-1} \qquad (8)$$

Both conditions on F imply $F \begin{bmatrix} C & I \end{bmatrix} \in \mathbb{R}\mathcal{H}_\infty$.

Proof: See [20] or [4]. □

With the result of lemma 4.1 the following proposition to get access to a *rcf* of the plant P_o on the basis of closed loop signals can be given.

Proposition 4.2 *Let the plant P_o and a controller C create an internally stable feedback system $\mathcal{T}(P_o,C)$, then* (2) *can be rewritten as*

$$\begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} N_{o,F} \\ D_{o,F} \end{bmatrix} x + \begin{bmatrix} I \\ -C \end{bmatrix} [I + P_oC]^{-1} v$$

where x is given in (6)*, F is given in* (8) *and $(N_{o,F}, D_{o,F})$ is the* rcf *of the plant P_o given by*

$$\begin{bmatrix} N_{o,F} \\ D_{o,F} \end{bmatrix} = \begin{bmatrix} P_o \\ I \end{bmatrix} [I + CP_o]^{-1}[I + CP_x]D_x \qquad (9)$$

Proof: By use of (7) with $N_{o,F} := P_oS_iF^{-1}$ and $D_{o,F} := S_iF^{-1}$ and direct application of (8). □

Since x in (6) is uncorrelated with v, proposition 4.2 gives rise to an equivalent open loop identification problem of the *rcf* $(N_{o,F}, D_{o,F})$ of the plant P_o, as also been indicated in [18]. It should be noted that the specific *rcf* $(N_{o,F}, D_{o,F})$ in (9) of the plant P_o that can be accessed, can be influenced by the *rcf* (N_x, D_x) of the auxiliary model P_x used to create the filter F in (8) and their interrelation is mentioned in the following corollary.

Corollary 4.3 *The* rcf $(N_{o,F}, D_{o,F})$ *of the plant P_o given in proposition 4.2 satisfies*

$$[D_{o,F} + CN_{o,F}] = F^{-1} = [D_x + CN_x]. \qquad (10)$$

Proof: With $N_{o,F} = P_oS_iF^{-1}$ and $D_{o,F} = S_iF^{-1}$, $[D_{o,F} + CN_{o,F}] = [I + CP_o]S_iF^{-1} = F^{-1}$ proving equation (10), where F is given in (8). □

Note that the transfer functions $D_{o,F}$ and $N_{o,F}$ seperately are unknown, but (10) indicates that $[D_{o,F} + CN_{o,F}]$ can be replaced by F^{-1}, which is completely known. Furthermore, (9) indicates that $(N_{o,F}, D_{o,F})$ can be of high order, containing redundant dynamics. A sensible choice for the auxiliary model P_x and the corresponding factorization (N_x, D_x) may lead to cancelling of redundant dynamics, which is used in [20] to access and approximate a normalized factorization of the plant P_o.

For sake of analysis and to maintain generality, it is presumed here that an identification procedure based on the data given in proposition 4.2 is able to come up with an estimate $\hat{\theta}$ given by

$$\hat{\theta} = \arg\min_{\theta} \left\| W_1 \left(\begin{bmatrix} N_{o,F} \\ D_{o,F} \end{bmatrix} - \begin{bmatrix} N(\theta) \\ D(\theta) \end{bmatrix} \right) W_2 \right\|_{\mathcal{X}} \qquad (11)$$

where W_1, W_2 are weighting functions and $\|\cdot\|_{\mathcal{X}}$ is a norm function to be specified. The role of the weighting functions W_1, W_2, the norm function $\|\cdot\|_{\mathcal{X}}$ to be used and the parametrization of the factorization $(N(\theta), D(\theta))$ will be scrutinized in the following sections.

5 Estimation of coprime factors

5.1 Feedback relevant identification

For the feedback relevant identification problem introduced in section 3, the mismatch $W_o[T(P_o,C) - T(\hat{P},C)]W_i$ given in (5) needs to be minimized for a model $\hat{P}$ with a prespecified McMillan degree. Using the fractional representations introduced in section 4, this mismatch can be expressed as follows.

Lemma 5.1 *Let P_o and C create an internally stable feedback system $\mathcal{T}(P_o,C)$ and let $(N_{o,F}, D_{o,F})$ be the* rcf *of P_o given by* (10) *where F is any filter satisfying* (8)*. Consider any model $\hat{P}$, then*

(i) *there exists a* rcf $(\hat{N}, \hat{D})$ *of the model $\hat{P}$ such that* $\hat{D} + C\hat{N} = F^{-1}$.

(ii) $W_o[T(P_o,C)-T(\hat{P},C)]W_i$ *equals*

$$W_o\left(\begin{bmatrix} N_{o,F} \\ D_{o,F}\end{bmatrix} - \begin{bmatrix}\hat{N} \\ \hat{D}\end{bmatrix}\right) F \begin{bmatrix} C & I \end{bmatrix} W_i \qquad (12)$$

where $(\hat{N},\hat{D})$ *is a* rcf *of* $\hat{P}$ *that satisfies* (i).

Proof: Take $(N_{o,F}, D_{o,F})$ in (10) as the *rcf* of P_o and $(\hat{N},\hat{D})$ as a *rcf* of the model $\hat{P}$. Using (10) and restricting $[\hat{D}+C\hat{N}] = F^{-1}$, the mismatch $W_o[T(P_o,C)-T(\hat{P},C)]W_i$ equals (12). □

Clearly, lemma 5.1 reflects also a restriction on the factorization $(N(\theta), D(\theta))$ to be estimated. If the filter F is characterized by (8), the restriction

$$D(\theta)+CN(\theta) = D_x + CN_x = F^{-1} \qquad (13)$$

has to be incorporated in the feedback relevant identification of the model $P(\theta) = N(\theta)D(\theta)^{-1}$.

Corollary 5.2 *The identification problem of* (5) *and the estimate* (11) *can be made compatible, by taking* $W_1 = W_o$, $W_2 = F[C\ I]W_i$, $\|\cdot\|_{\mathcal{X}} = \|\cdot\|_\infty$ *and the restriction of* (13), *yielding*

$$\min_{\substack{\theta \\ D(\theta)+CN(\theta)=F^{-1}}} \left\| W_o \begin{bmatrix} N_{o,F}-N(\theta) \\ D_{o,F}-D(\theta)\end{bmatrix} F \begin{bmatrix} C & I \end{bmatrix} W_i \right\|_{\mathcal{X}}$$

Proof: With $W_1 = W_o$, $W_2 = F[C\ I]W_i$ the argument of $\|\cdot\|_{\mathcal{X}}$ in (11) equals the argument of $\|\cdot\|_\infty$ in (5), by substituting the results of lemma 5.1. □

5.2 Minimization with restriction

In order to deal with the restriction (13) in the minimization, basically two approaches can be followed. The first approach is to parametrize $N(\theta)$ and $D(\theta)$ in such a way that (13) is always satisfied.

Proposition 5.3 *Let* P_x *with a* rcf (N_x, D_x) *and* C *with a* rcf (N_c, D_c) *satisfy* $T(P_x, C) \in \mathbb{R}\mathcal{H}_\infty$, *then the parametrization*

$$\begin{array}{rcl} N(\theta) & = & N_x + D_c R(\theta) \\ D(\theta) & = & D_x - N_c R(\theta) \end{array} \qquad (14)$$

with $R(\theta) \in \mathbb{R}\mathcal{H}_\infty$ *complies with the restriction* (13).

Proof: Substitution of (14) in (13). $R(\theta) \in \mathbb{R}\mathcal{H}_\infty$ is needed to ensure $(N(\theta), D(\theta)) \in \mathbb{R}\mathcal{H}_\infty$. □

The parametrization in (14) (or a dual form based on *lcf*'s) is known as the (dual) Youla parametrization [18, 19]. It is used extensively in the literature by estimating the stable factor $R(\theta)$ directly [10, 12] but lacks the ability to prespecify the McMillan degree of the model $\hat{P}$ being estimated. Hence the minimization (5) cannot be achieved for a *fixed order* model. An alternative approach to deal with the restriction (13) is to reverse the problem by parametrizing the filter F such that it satisfies (13).

As a result, the feedback relevant identification problem can be formulated as

$$\min_\theta \left\| W_o \begin{bmatrix} N_{o,F(\theta)}-N(\theta) \\ D_{o,F(\theta)}-D(\theta)\end{bmatrix} F(\theta) \begin{bmatrix} C & I \end{bmatrix} W_i \right\|_{\mathcal{X}} \qquad (15)$$

with $F(\theta) = [D(\theta)+CN(\theta)]^{-1}$.

With the parametrization of the filter F, the optimization problem clearly becomes essentially different from the criterion (11) that can be handled by a "standard" identification procedure. This is caused by the fact that the signal x can not be constructed prior to identification now. However, the above criterion gives rise to an iterative scheme where in iteration step i, parameter estimate $\hat{\theta}_i$ is obtained by applying (15) with $F(\theta)$ replaced by $F_i = F(\hat{\theta}_{i-1})$.

Furthermore, it follows directly from (5) that the objective function to be minimized does not distinguish between different *rcf*'s of the plant model $P(\theta)$. In other words, a *rcf* of a model $P(\theta)$ is not unique which leads to a non-unique parametrization of the filter $F(\theta)$ and different *rcf*'s of $P(\theta)$ give the same value of the objective function. This gives us an additional freedom in the construction of the "fixed" filter F, characterized by: $F_i = [D(\hat{\theta}_{i-1})Q + CN(\hat{\theta}_{i-1})Q]^{-1}$, where Q is just any stable transfer function.

In order to reduce this additional freedom, which is required for obtaining convergence of the iterative scheme, the filter F will be updated by restricting the *rcf* of the model to be normalized. In this way

$$F_i = (\bar{D}_{i-1} + C\bar{N}_{i-1})^{-1}$$

where $(\bar{N}_{i-1}, \bar{D}_{i-1})$ is a normalized *rcf* of the model $P(\hat{\theta}_{i-1})$ with $P(\hat{\theta}_{i-1}) = N(\hat{\theta}_{i-1})D(\hat{\theta}_{i-1})^{-1}$, which is unique upto a post multiplication with a unitairy matrix.

Starting off from an initial model estimate with normalized rcf $(\bar{N}, \bar{D})$ by setting $\bar{N}_{i-1} = \bar{N}$ and $\bar{D}_{i-1} = \bar{D}$, the iterative scheme reads as follows.

1. In step i, create $F_i = (\bar{D}_{i-1}+C\bar{N}_{i-1})^{-1}$ and simulate the input x.
2. Estimate $(N(\hat{\theta}_i), D(\hat{\theta}_i))$ by the minimization given in corollary 5.2 *discarding* the parameter restriction (13).
3. Compute a normalized *rcf* $(\bar{N}_i, \bar{D}_i)$ for the model $P(\hat{\theta}_i) := N(\hat{\theta}_i)D^{-1}(\hat{\theta}_i)$, and go back to step 1.

If the iteration converges then $(\bar{D}_i+C\bar{N}_i)^{-1} = F_i$ is independent of i. According to corollary 5.2, the restriction (13) has been satisfied, thus a feedback relevant estimate $\hat{P}$ of the plant P_o has been obtained. A rigorous proof of the convergence of the iteration is not available (yet) but extensive simulations, using a 2-norm in the minimization reveal promising results and will be illustrated in the example of section 6.

5.3 Parametrization

To control the McMillan degree of the model being estimated, the factorization $(N(\theta), D(\theta))$ can be parametrized in state space form as follows.

Theorem 5.4 *Let* $(\hat{N}, \hat{D}) \in \mathbb{R}\mathcal{H}_\infty$ *be given by a stable and minimal state space representation*

$$\left(\bar{A}, \bar{B}, \left[\begin{array}{c} \bar{C}_N \\ \bar{C}_D \end{array}\right], \left[\begin{array}{c} \bar{E}_N \\ \bar{E}_D \end{array}\right]\right)$$

with $\det\{\bar{E}_D\} \neq 0$*, then*

(i) $\det\{\hat{D}\} \not\equiv 0$

(ii) $\hat{P} = \hat{N}\hat{D}^{-1}$ *is given by the state space representation* $[A, B, C, E]$ *with* $A = \bar{A} - \bar{B}\bar{E}_D^{-1}\bar{C}_D$*,* $B = \bar{B}\bar{E}_D^{-1}$*,* $C = \bar{C}_N - \bar{E}_N\bar{E}_D^{-1}\bar{C}_D$*,* $E = \bar{E}_N\bar{E}_D^{-1}$

(iii) $(\hat{N}, \hat{D})$ *is a* rcf *of* $\hat{P}$*.*

Proof: Due to the non-singular matrix $\bar{E}_D$, $\hat{D}$ is invertible having a state space representation $(\bar{A} - \bar{B}\bar{E}_D^{-1}\bar{C}_D, \bar{B}\bar{E}_D^{-1}, -\bar{E}_D^{-1}\bar{C}_D, \bar{E}_D^{-1})$ which proves (i). From the state space representation of the operation $\hat{P} = \hat{N}\hat{D}^{-1}$ $n = \dim(\bar{A})$ uncontrollable states can be omitted, leading to $[A, B, C, E]$, which proves (ii). The matrices $\bar{A}$, $\bar{B}$, $\bar{C}_N$, $\bar{C}_D$ and $\bar{E}_N$ can be rewriten as $\bar{A} = A - BK$, $\bar{B} = B\bar{E}_D$, $\bar{C}_N = C - EK$, $\bar{C}_D = -K$ $\bar{E}_N = E\bar{E}_D$, In this way $\hat{N}(z) = ([C - EK][zI - A + BK]^{-1}B + E)\bar{E}_D$ and $\hat{D}(z) = (-K[zI - A + BK]^{-1}B + I)\bar{E}_D$, which is proven to be a *rcf* in [14]. □

The result of theorem 5.4 gives rise to a wide class of parametrizations in the estimation of a *rcf* $(N(\hat{\theta}), D(\hat{\theta}))$. Restricting the estimate to be stable, minimal (and balanced) can be enforced by using the parametrizations given in [15] and further elaborated in [6]. However, a stable and minimal state space estimate with non-singular feedthrough matrix $\bar{E}_D$ will be found in the generic case. This due to the fact that the map form x onto $[y\ u]^T$ is stable according to proposition 4.2. Furthermore, the map from x onto u is given by $[I + CP_o]^{-1}[I + CP_x]D_x$ according to (9), which is non-singular by definition.

6 Example

For the simulation example, consider a fifth order plant $P_o(q^{-1}) = b(q^{-1})/a(q^{-1})$. The denominator $a(q^{-1})$ is a monic polynominal having roots at $0.75 \pm 0.4i$, $0.99 \pm 0.06i$ and 1. The first element in the numerator $b(q^{-1})$ equals 0.1 and the roots are given by $0.98 \pm 0.04i$ and $0.5 \pm 0.4i$. The controller is taken to be $C(q^{-1}) = 1$ and white noise reference signals r_1, r_2 with variance 1, white noise input v with variance 0.1 are used to generate the signals u, y from (2). The initial model $\bar{N}\bar{D}^{-1}$ is a simple integrator $\frac{0.1}{1-q^{-1}}$.

The aim is to estimate a *third order* model $\hat{P}$ that approximates the plant P_o by minimizing (5) on the basis of a 2-norm criterion. Starting with a normalized *rcf* of the initial model, the update algorithm mentioned in section 5.2 is invoked. The minimization given in corollary 5.2 *without* the restriction (13) is performed by a least squares estimate using an output error model structure [13]. The state space model of the factorization $(N(\theta), D(\theta))$ mentioned in theorem 5.4 is parametrized by pseudo canonical overlapping form with observability indices 1, 2 [13].

The update algorithm was invoked 6 times. To inspect the restriction (13) during the update algorithm, a plot of $\|D(\hat{\theta}_i) + CN(\hat{\theta}_i) - F_i^{-1}\|_\infty$ is plotted in Figure 2. From

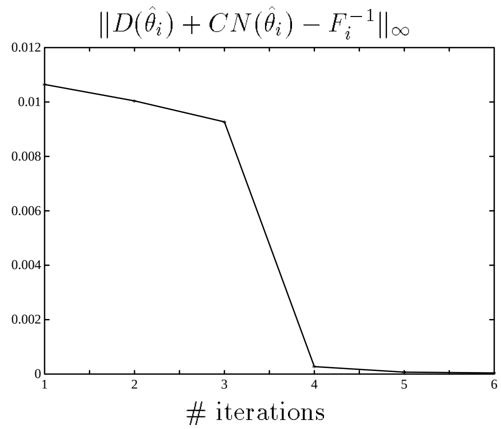

Fig. 2: Evaluation of restriction

Figure 2 can be seen that indeed the restiction (13) is (almost) fulfilled after 4 iterations and that the update algorithm seems to converge.

The factorization $(\hat{N}, \hat{D})$ of the third order factorization $(\hat{N}, \hat{D})$ obtained after the 6 iterations along with the third order model $\hat{P} = \hat{N}\hat{D}^{-1}$ is plotted in Figure 3. The third order (unstable) model $\hat{P}(q^{-1})$ obtained is given by

$$10^{-2}\frac{-0.1256 + 7.9494q^{-1} - 5.3788q^{-2} + 0.1403q^{-3}}{1 - 2.5412q^{-1} + 2.2564q^{-2} - 0.7182q^{-3}}$$

and is stabilized by the controller C. Note that the minimization of (5) induces a weighting on the open loop mismatch between P_o and $\hat{P}$, which is high around the bandwith of 0.2 Hz of the closed loop system.

7 Conclusions

It has been shown that any stable right coprime factorization of the plant can be accessed by a filtering of signals present in the closed loop system and the freedom in the filtering has been characterized by employing the

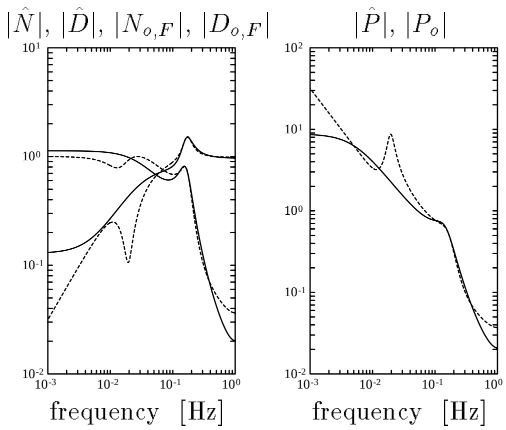

Fig. 3: Bode plot of identified 3rd order coprime factors ($\hat{N}$, $\hat{D}$) and model $\hat{P}$ (solid) and factorization ($N_{o,F}, D_{o,F}$) of the plant P_o (dashed)

knowledge of the controller present during the closed loop experiments.

The feedback relevant estimation of a *fixed order* model based on the fractional approach leads to a restriction on the stable factors to be estimated. This restriction is intrinsic in many schemes on feedback relevant identification but can be written down explicitly in case of the coprime factor identification and boils down to a relation between the filter used to gain access to the coprime factors of the plant and model being estimated. A possible solution to deal with the restriction by updating the filtering is presented and illustrated by an example.

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