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IDENTIFICATION AND CONTROL OF A COMPACT DISC MECHANISM USING FRACTIONAL REPRESENTATIONS

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Abstract. This paper discusses the approximate and feedback relevant parametric identification of the radial servo system present in a Compact Disc player. In this application the problem of approximate identification based on data from closed loop experiments will be analyzed to find a finite dimensional linear time invariant discrete time model, suitable for model-based control design. The feedback relevant identification in this paper is based on the algebraic theory of fractional representations, which has lead to a framework for equivalent open loop identification of (normalized) coprime plant factors and a manageable approximate transfer function estimation of the feedback controlled plant. A mixed worst-case/probabilistic approach to model uncertainty quantification is used to construct an upper bound on the uncertainty of the normalized coprime factors being estimated. Both the nominal model and the uncertainty description are used to design an enhanced robust controller.

Key Words. System identification; robust control; coprime factorizations; mechanical servo systems.

1. INTRODUCTION

In the field of systems and control there is a growing interest in merging the problems of control design and identification, induced by the fact that dynamical models obtained from system identification, will be used as a basis for control design. In this field of research it is widely recognized that models found by system identification and used for control design are necessarily approximative. On the one hand exact modelling can be impossible or too costly, on the other hand control design methods can get unmanageable if they are applied to models of high complexity. Since the validity of any approximate model hinges on its intended use, the identification procedure being applied, will be subjected to several requirements, to estimate models suitable for control design. These requirements boil down to the fact that the best (nominal) model $\hat{P}$, suitable for control design cannot be derived from closed loop experiments alone (Schrama, 1992*a*). Furthermore, a (nominal) model $\hat{P}$ is just an approximation of the plant P, so the controller based on the model $\hat{P}$ has to be robust against dissimilarities between $\hat{P}$ and P. This has been a motivation for the development of identification techniques that estimate an upper bound on a model error as in e.g. Helmicki *et al.* (1991), Goodwin and Ninnes (1991) and de Vries and Van den Hof (1993), which can be used in a robust control design paradigm.

Performance and robustness can be conflicting factors and hence robust performance of the feedback system can only be a high performance if a nominal model $\hat{P}$ has been estimated with care. This can be achieved by a feedback relevant approximative identification, which implies that the relevant dynamical behaviour of the plant P operating in a closed loop configuration has to be estimated (Schrama, 1992*a*). Since the controller is (yet) unknown, it has been motivated to use an iterative scheme of identification and control design, using the controller of step $i-1$, denoted with C to estimate a model $\hat{P}$ for step i and to design an improved controller $C_{\hat{P}}$ based on this model $\hat{P}$ see for example Zang *et al.* (1992), Hakvoort *et al.* (1994), Lee *et al.* (1993) and Schrama (1992*b*) or Gevers (1993), Van den Hof and Schrama (1994) for some overview. In this paper we will discuss one step in such an iterative scheme involving an identification of $\hat{P}$ using an experimental situation where a controller C (from the previous iteration) is used to control the plant P, and the design of an enhanced controller $C_{\hat{P}}$.

The intention of this paper is to focus on the closed loop approximate identification of the radial positioning mechanism of a Compact Disc (CD) player, denoted with the plant P. An increasing amount of CD players will be used in portable applications having severe shock disturbances. The properties of a CD player, operating in these conditions, can be improved by designing a high performance controller. The identification is based on the algebraic theory of fractional representations, which has led to a framework for equivalent open loop identification of (normalized) coprime plant factors, see for example Hansen (1989), Schrama (1992*b*) and Van den Hof *et al.* (1993). This framework offers a manageable approximate transfer function estimation of the feedback controlled plant P, where the identification criterion can be tuned to become specifically feedback relevant. Additionally, the procedure presented in de Vries and Van den Hof (1993) is used for a quantification of the resulting model error. Both the nominal model and the model error will be utilized to design an enhanced robust controller using the procedure presented in Bongers and Bosgra (1990) resulting in a successful implementation of the controller.

2. COMPACT DISC MECHANISM

The CD mechanism considered here consists of a turn table DC–motor for the rotation of the Compact Disc and a radial arm in order to follow the spiral track of the disc. An OPU (Optical Pick–up Unit) is mounted on the end of the balanced radial arm to read the digitally coded signal, recorded on the track of the reflective disc. Schematically the CD mechanism is given in Fig. 1.

The Compact Disc mechanism is a feedback controlled system. Following the track on the Compact Disc involves basically two control loops. Firstly a control loop using a radial actuator in order to position the laser spot orthogonal to the track. Secondly a control loop using a focus actuator in

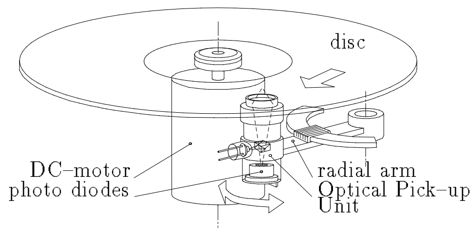

Fig. 1. Schematic view of CD mechanism

order to focus the laser spot on the track. While the CD mechanism is actually a 2-input 2-output system, see also de Callafon *et al.* (1993), for this study we concentrate on the identification and control of the radial servo system only, with the motivation that the radial and focus servo loop are nearly decoupled (Draijer *et al.*, 1992).
The closed loop bandwidth of the Compact Disc radial servo loop present is approximately 450 Hz, see also Steinbuch *et al.* (1992). Increasing the bandwidth to 800 Hz to improve low frequent disturbance rejection by designing a controller on an 'old model' leads to excessive peaking of the sensitivity function. In order to design an improved controller for the radial servo loop, a more accurate (nominal) model together with a characterization of the model error for stability robustness assessment will be estimated, which is the main topic of this paper.

3. PRELIMINARIES AND NOTATIONS

The closed loop system of the radial servo loop in a Compact Disc player can be rewritten into the general feedback system $T(P,C)$[1] given in Fig. 2, which will be used throughout this paper. In here $v(t)$ reflects the additive noise on the output

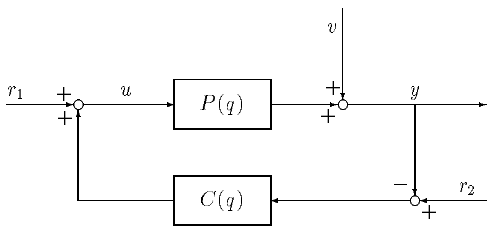

Fig. 2. Feedback system $T(P,C)$

$y(t)$ of the plant $P(q)$, which is supposed to be uncorrelated with the external references signals $r_1(t)$ and $r_2(t)$ entering the closed loop system. From an identification point of view, the signals $u(t)$ and $y(t)$ are measured, $v(t)$ is unknown and $r_1(t)$ and $r_2(t)$ are possibly at our disposal.
Using Fig. 2, the data from the closed loop system $T(P,C)$ will be described with the following equations.

$$\begin{bmatrix} y \\ u \end{bmatrix} = T(P,C)\begin{bmatrix} r_2 \\ r_1 \end{bmatrix} + + \begin{bmatrix} I \\ -C \end{bmatrix} [I+PC]^{-1}He \quad (1)$$

where the additive noise $v(t)$ has been modelled by a monic, stable and stably invertible noise filter $H(q)$ with a white noise input $e(t)$ and $T(P,C)$ reflects the general feedback matrix

$$T(P,C) = \begin{bmatrix} P \\ I \end{bmatrix} [I+CP]^{-1} \begin{bmatrix} C & I \end{bmatrix} \quad (2)$$

Using the theory of fractional representations, a plant P is expressed as a ratio of two stable proper mappings N and D. Similarly as in Vidyasagar (1985) we will use the following definitions, where $\mathcal{RH}_\infty$ denotes the set of all rational stable transfer functions.

Definition 3.1
Let $N,D \in \mathcal{RH}_\infty$, then the pair (N,D) is called right coprime over $\mathcal{RH}_\infty$ if there exist right Bezout factors $X,Y \in \mathcal{RH}_\infty$ such that $XN+YD=I$. The pair (N,D) is a right coprime factorization (rcf) of P if $\det\{D\} \neq 0$, $P=ND^{-1}$ and (N,D) is right coprime.

Definition 3.2
A right coprime factorization (N_n,D_n) is called a normalized right coprime factorization (nrcf) if it satisfies

$$N_n^* N_n + D_n^* D_n = I$$

where * denotes the complex conjugate transposed.

For (normalized) left coprime factorizations we have dual definitions. Using definition 3.1 the algebraic theory provides a representation of all stabilizing controllers C for the plant P in terms of coprime factorizations, known as the Youla parametrization. Similar to Hansen (1989) and Schrama (1992*b*) we use the dual result of this parametrization, given in the following lemma.

Lemma 3.3
Let P_x be an auxiliary model and C a controller such that $T(P_x,C) \in \mathcal{RH}_\infty$ and let (N_x,D_x) and (N_c,D_c) be a rcf of respectively P_x and C. Then $P=ND^{-1}$ satisfies $T(P,C) \in \mathcal{RH}_\infty$ if and only if $\exists\ R \in \mathcal{RH}_\infty$ with

$$\begin{aligned} N &= N_x + D_c R \\ D &= D_x - N_c R \end{aligned}$$

Proof: see Schrama (1992*b*). ■

Although a rcf of an auxiliary plant P_x is not unique, the factors N and D given in lemma 3.3 are uniquely associated to a rcf of the auxiliary model P_x. This is stated in the following lemma.

Lemma 3.4
Let the plant P, the auxiliary model P_x and a controller C be such that $T(P,C) \in \mathcal{RH}_\infty$ and $T(P_x,C) \in \mathcal{RH}_\infty$, and let (N_x,D_x) be a rcf of P_x, then the coprime factors D and N given in lemma 3.3 are uniquely determined by

$$\begin{aligned} N &= P[I+CP]^{-1}[I+CP_x]D_x \\ D &= [I+CP]^{-1}[I+CP_x]D_x \end{aligned}$$

Proof: Since both $P=ND^{-1}$ and $P_x=N_xD_x^{-1}$ have been stabilized by the controller C we can apply lemma 3.3 to P and P_x, yielding

$$N = N_x + D_c R \quad (3)$$
$$D = D_x - N_c R \quad (4)$$

Substituting $N_c = CD_c$ for any rcf (N_c,D_c) of the controller C into (4) and adding C times (3) with (4) gives

$$D + CN = D_x + CN_x \quad (5)$$

Next with $P=ND^{-1}$ we find $D=[I+CP]^{-1}[D_x+CN_x]$ and with $P_x=N_xD_x^{-1}$ this yields $D=[I+CP]^{-1}[I+CP_x]D_x$. N follows from $N=PD$. ■

4. IDENTIFICATION AND CONTROL

The general feedback matrix $T(P,C)$ has been recognized as an important feedback property of the closed loop system (Bongers and Bosgra, 1990; Maciejowski, 1989). It induces a feedback relevant topology, see also Schrama (1992*b*), meaning that if two such operators are alike, the corresponding feedback controlled systems will have similar performances. Moreover, the control design being used in this paper is

[1] for notational convenience the time shift operator q will be omitted without mentioning

based on the minimization of the ∞-norm of the $T(P,C)$ matrix, see Bongers and Bosgra (1990) for details. Therefore the difference between $T(P,C)$ and $T(\hat{P},C)$ forms a so-called feedback relevant mismatch, caused by the difference between nominal model $\hat{P}$ and plant P.

Considering any norm or distance function $\|\cdot\|$ and applying the triangle inequality to $\|T(P,C)-T(\hat{P},C)\|$ yields:

$$\|T(P,C)\| \leq \|T(\hat{P},C)\| + \|T(P,C)-T(\hat{P},C)\| \tag{6}$$

$$\|T(P,C)\| \geq \left| \|T(\hat{P},C)\| - \|T(P,C)-T(\hat{P},C)\| \right| \tag{7}$$

From (6) and (7) we see that by posing the following requirement

$$\|T(P,C)-T(\hat{P},C)\| \ll \|T(\hat{P},C)\| \tag{8}$$

similar performances for the controlled plant P and the controlled model $\hat{P}$ can be derived, see also Schrama (1992*b*). Therefore, minimizing the difference $|T(P,C)-T(\hat{P},C)|$ can be seen as a feedback relevant identification of the plant P.

5. IDENTIFICATION OF COPRIME FACTORS

5.1. *Motivation*

The major problem arising from an approximate identification using closed loop experiments, is the correlation of the additive noise v with the input u of the system, see also Fig. 2. Additionally, an explicit expression of the approximation of $\hat{P}$ is needed, to tune the bias distribution of the model $\hat{P}$ being estimated in a feedback relevant way. The framework for identification used in this paper, is based on the algebraic theory of fractional representations by estimating coprime factors of the plant. Several authors have worked on this subject, see for example Hansen (1989), Schrama (1992*b*) and Lee *et al.* (1993). The motivation for using fractional representations from an *identification point of view* can be summarized as follows.

- We can handle unstable plants P and controllers C.
- The closed loop identification problem can be recasted into an equivalent open loop identification of coprime plant factors. Consequently, the results in approximate open loop identification, based on Prediction Error methods like in Ljung (1987), can be exploited.
- The fractional representation offers an approximate transfer function estimate $\hat{P}$ of the feedback controlled plant P which is feedback relevant in terms of the general feedback configuration $T(P,C)$.

Clearly, the first item is evident since coprime factorizations are defined to be stable, see also definition 3.1. The last two items will be illuminated in the following sections; a thorough treatment can also be found in Schrama (1992*b*).

5.2. *Equivalent Open Loop Identification*

By using the equation of the data generating system given in (1) and the definition $r := r_1 + Cr_2$ we have

$$r = r_1 + Cr_2 = u + Cy \tag{9}$$

Using this signal r, (1) can be simplified to

$$\left[\begin{array}{c} y \\ u \end{array}\right] = \left[\begin{array}{c} P[I+CP]^{-1} \\ [I+CP]^{-1} \end{array}\right] r + \left[\begin{array}{c} [I+PC]^{-1} \\ -C[I+PC]^{-1} \end{array}\right] v$$

If the controller C (internally) stabilizes the plant P, all elements of the matrix $T(P,C)$ will be stable. Hence both $P[I+CP]^{-1}$ and $[I+CP]^{-1}$ will be stable and can considered to be a right coprime fractional representation (N,D) of the plant P. Moreover, the signal r defined in (9) is uncorrelated with the noise v of the closed loop system given in Fig. 2. This gives rise to an equivalent open loop identification problem by estimating a stable right coprime factorization of the plant using r as input and $[y\ u]^T$ as output.

However, a right coprime factorization (rcf) is not unique and one can incorporate this freedom similarly as in Van den Hof *et al.* (1993), by introducing an additional filtering of the signal r, with $x := Fr$. Again using (1) this yields a rcf (N,D) of the plant P, with

$$\left\{ \begin{array}{lcl} N & = & P[I+CP]^{-1}F^{-1} \\ D & = & [I+CP]^{-1}F^{-1} \end{array} \right. \tag{10}$$

where the filter F denotes the additional freedom in the rcf of the plant P. Restricting the two factors in (10) to be stable and coprime and using the fact that $T(P,C) \in \mathcal{RH}_\infty$, the freedom in F can be characterized by employing the dual Youla parametrization given in lemma 3.3, where P_x is any auxiliary model satisfying $T(P_x,C) \in \mathcal{RH}_\infty$. The result is already given in lemma 3.4 as a uniqueness property of the rcf of the plant P in terms of the rcf of the auxiliary model P_x. Combining the results of lemma 3.4 and (10) we find the filter F to be

$$F = D_x^{-1}[I+CP_x]^{-1} = [D_x + CN_x]^{-1} \tag{11}$$

which brings us to the equivalent open loop identification problem of feedback controlled plants, stated in the following proposition.

Proposition 5.1
Let a plant P be stabilized by a controller C with a rcf (N_c, D_c) and let r_1 and r_2 be statistically independent of v, then (1) can be rewritten into

$$\left[\begin{array}{c} y \\ u \end{array}\right] = \left[\begin{array}{c} N \\ D \end{array}\right] x + \left[\begin{array}{c} I \\ -C \end{array}\right] [I+PC]^{-1}He \tag{12}$$

where (N,D) is the rcf of the plant P as in lemma 3.4 and

$$x = F[C\ I]\left[\begin{array}{c} y \\ u \end{array}\right] \tag{13}$$

where F is given in (11).

Proof: see Van den Hof *et al.* (1993) ■

Since x is statistically independent of the noise v, the identification of the plant P from closed loop measurements u and y is equivalent to an open loop identification of N and D in proposition 5.1. Based on the results of proposition 5.1 we propose the following equivalent open loop identification problem of the plant P. The coprime factors (N,D) of the plant P are being estimated through the signals $u(t)$, $y(t)$ and the reconstructed signal $x(t)$ given in (13) by applying an output error (OE) model structure, having a fixed noise filter:

$$\begin{array}{c} \mathcal{M}: \ \varepsilon(t,\theta) = \left[\begin{array}{c} y(t) \\ u(t) \end{array}\right] - \left[\begin{array}{c} N(q,\theta) \\ D(q,\theta) \end{array}\right] x(t) \\ \varepsilon_f(t,\theta) = L(q) I_{2\times 2}\varepsilon(t,\theta) \end{array} \tag{14}$$

where $\varepsilon(t,\theta)$ is the one-step ahead prediction error (Ljung, 1987) and $L(q)$ is an additional filtering of the prediction error.

By applying a least squares identification criterion

$$\begin{array}{rcl} \hat{\theta} & = & \arg\min_{\theta \in D_\mathcal{M}} V_N(\theta, Z^N), \\ V_N(\theta, Z^N) & = & \dfrac{1}{2N} \displaystyle\sum_{t=0}^{N-1} tr\left\{\varepsilon_f^T(t,\theta)\varepsilon_f(t,\theta)\right\} \end{array} \tag{15}$$

where Z^N reflects the observed data of length N and tr is the trace operator, we can write down an equivalent frequency domain representation of the least squares criterion given in (15) having an OE-model structure, see also Ljung (1987). By employing Parseval's relationship and using (12) and (14) we find

$$\lim_{N\to\infty} V_n(\theta, Z^n) = \bar{V}(\theta), \text{ with}$$
$$\bar{V}(\theta) = \frac{1}{4\pi}\int_{-\pi}^{\pi} tr\left\{\left[\begin{array}{c} N(e^{i\omega}) - N(e^{i\omega},\theta) \\ D(e^{i\omega}) - D(e^{i\omega},\theta)\end{array}\right]^T L(e^{i\omega})^T \right. \quad (16)$$
$$\left. L(e^{i\omega})\left[\begin{array}{c} N(e^{i\omega}) - N(e^{i\omega},\theta) \\ D(e^{i\omega}) - D(e^{i\omega},\theta)\end{array}\right]\Phi_x(\omega)\right\} \,\mathrm{d}\omega$$

where $\Phi_x(\omega)$ is the auto spectral density of the signal $x(t)$. The usage of the frequency domain representation will be scrutinized in the following subsection.

5.3. *Feedback relevant identification*

The difference $\Delta T(P,\hat{P},C) = T(P,C) - T(\hat{P},C)$ introduced in section 4 can be seen as feedback relevant mismatch between the plant P and the nominal model $\hat{P}$, when using the control design introduced in Bongers and Bosgra (1990). This mismatch $\Delta T(P,\hat{P},C) = T(P,C) - T(\hat{P},C)$ can be expressed in terms of coprime factors, which is stated in the following lemma.

Lemma 5.2
Let a plant P with a rcf (N,D), an auxiliary model P_x with a rcf (N_x,D_x) and a nominal model $\hat{P}$ with a rcf $(\hat{N},\hat{D})$[2] form stable feedback systems $T(P,C)$, $T(P_x,C)$ and $T(\hat{P},C)$, then the mismatch $\Delta T(P,\hat{P},C) = T(P,C) - T(\hat{P},C)$ can be expressed as

$$\begin{aligned}\Delta T(P,\hat{P},C) &= \left[\begin{array}{c} N-\hat{N} \\ D-\hat{D}\end{array}\right][D_x + CN_x]^{-1}[C\ I] \\ &= \left[\begin{array}{c} N-\hat{N} \\ D-\hat{D}\end{array}\right]\tilde{F}\end{aligned} \quad (17)$$

with $\tilde{F} = F[C\ I]$ and F as in (11).

Proof: Since both $T(P,C)$ and $T(\hat{P},C)$ form stable feedback systems, lemma 3.3 can be applied, using P_x as an auxiliary model. Consequently (5) yields

$$D + CN = D_x + CN_x \quad (18)$$
$$\hat{D} + C\hat{N} = D_x + CN_x \quad (19)$$

Rewriting $T(P,C)$ and $T(\hat{P},C)$ in the form

$$T(P,C) = \left[\begin{array}{c} N \\ D\end{array}\right][D+CN]^{-1}[C\ I]$$

and using (18) and (19), gives the feedback relevant mismatch in (17) combined with the filter $\tilde{F} = [C\ I]F$ with F given in (11). ∎

Lemma 5.2 makes the feedback relevant mismatch between P and $\hat{P}$ a linear function of the difference between the rcf of the model $\hat{P}$ and the corresponding rcf of the plant P. However (19) introduces an additional parametrization constraint on the rcf $(\hat{N},\hat{D})$ of the model $\hat{P}$ that has to be taken into account while performing an identification of the nominal model $\hat{P}$.

$$D(q,\theta) + CN(q,\theta) = D_x + CN_x \quad (20)$$

By replacing the norm operator $\|\cdot\|$ by the H_2–norm (Maciejowski, 1989) the following quadratic feedback relevant performance criterion $J_f(\theta)$, based on (17), can be defined, if the parametrization constraint given in (20) is satisfied

$$J_f(\theta) \stackrel{\text{def}}{=} \frac{1}{4\pi}\int_{-\pi}^{\pi} tr\left\{\left[\begin{array}{c} N(e^{i\omega}) - N(e^{i\omega},\theta) \\ D(e^{i\omega}) - D(e^{i\omega},\theta)\end{array}\right]^T\right.$$
$$\left.\left[\begin{array}{c} N(e^{i\omega}) - N(e^{i\omega},\theta) \\ D(e^{i\omega}) - D(e^{i\omega},\theta)\end{array}\right]\tilde{F}^T(e^{i\omega})\tilde{F}(e^{i\omega})\right\}\,\mathrm{d}\omega \quad (21)$$

Comparing the feedback relevant performance criterion $J_f(\theta)$ in (21) with the frequency domain representation of the least squares OE-minimization given in (16), we present the following proposition for feedback relevant identification.

Proposition 5.3
Criterion $J_f(\theta)$ in (21) and $\bar{V}(\theta)$ in (16) can be made compatible by satisfying the parametrization condition given in (20) and taking the filter L in (14) such that

$$|L(e^{i\omega})|^2 = c_1\frac{1+|C(e^{i\omega})|^2}{\Phi_r(\omega)}$$

where $c_1 \neq 0$ is an arbitrary constant and $\Phi_r(\omega)$ is the auto spectral density of the signal $r(t)$ given in (9).

Proof: If the parametrization condition in (20) is satisfied, then we have by (18) and (20) that $D(q) + C(q)N(q) = D(q,\theta) + C(q) + N(q,\theta)$ and consequently $\Delta T(P,\hat{P},C)$ can be written in the form given in (17) for a parametrized model $(N(q,\theta), D(q,\theta))$, yielding (21). The expression for the filter L can be found by direct verification of (16) and (21), using the cyclical property of the trace operator. ∎

5.4. *Parametrization Constraints and Identification of Normalized Coprime Factors*

One of the conditions in order to perform a feedback relevant identification as stated in proposition 5.3, is to satisfy the parametrization constraint given in (20). To avoid this parametrization problem we propose the following iterative scheme

- In step $i-1$ identify the coprime factors denoted by $(N(q,\theta_{i-1}), D(q,\theta_{i-1}))$ without the parametrization constraint given in (20).
- Update the coprime factorization (N_x,D_x), using the estimate of $(N(q,\theta_{i-1}), D(q,\theta_{i-1}))$ in order to update the filter F in (11)
- In step i re-identify the coprime factors denoted by $(N(q,\theta_i), D(q,\theta_i))$ using the new filter F.

The condition (19) can be updated trivially according to $N_x(q) = N(q,\theta_{i-1})$ and $D_x(q) = D(q,\theta_{i-1})$, but in fact any combination of N_x and D_x satisfying (20) can be used for updating. In order to find (N_x,D_x) for updating we add an *additional equation* for (N_x,D_x) by making (N_x,D_x) a normalized right coprime factorization (nrcf) as in definition 3.2, yielding

$$D_x + CN_x = D(q,\theta_{i-1}) + CN(q,\theta_{i-1}) \quad (22)$$
$$D_x^* D_x + N_x^* N_x = I \quad (23)$$

The usage of a nrcf has some favourable properties, since a nrcf has minimal order and is unique up to postmultiplication with an unimodulair matrix (Vidyasagar, 1985). In order to approximate the solution to (22) and (23) we use a similar approach as in Van den Hof *et al.* (1993) of estimating coprime factors $(N(q,\theta), D(q,\theta))$ and updating the nrcf of the auxiliary model P_x by computing a nrcf of the model $P(q,\theta) := N(q,\theta)D^{-1}(q,\theta)$. If the iterative scheme converges then we satisfy the parametrization constraint (20) and by (23) we have obtained an estimate of the nrcf of the plant P. In order to estimate the coprime factors $(N(q,\theta), D(q,\theta))$ we use a linear regression scheme based on generalized orthonormal basis functions (Heuberger, 1991). This yields an analytical solution of the least squares prob-

[2] for notional convenience, the parameter $\hat{\theta}$ being estimated will be omitted without mentioning

lem given in (15) using an FIR-model structure (Ljung, 1987).

5.5. *Stability Robustness*

Additional to the identification of a nominal model employing the equivalent open loop identification of the plant's normalized coprime factors as proposed in section 5.2, we will estimate upper bounds on the additive error of the normalized coprime factors $(\hat{N}, \hat{D})$ being estimated. Upper bounds on these additive errors, denoted by (Δ_N, Δ_D) are obtained by employing the procedure described in de Vries and Van den Hof (1993), which is a mixed deterministic and probabilistic approach leading to frequency depended upper bounds with some prespecified confidence interval. This yields

$$\left.\begin{array}{lcl} |\Delta_N(e^{i\omega})| & \leq & \gamma_1(e^{i\omega}) \\ |\Delta_D(e^{i\omega})| & \leq & \gamma_2(e^{i\omega}) \end{array}\right\} \text{ w.p. } \geq \alpha \tag{24}$$

where α is a prespecified probability. Using this knowledge of an upper bound on (Δ_N, Δ_D) we can check stability robustness properties of the closed loop system by employing the following lemma

Lemma 5.4
Given $\hat{P}$ with a rcf $(\hat{N}, \hat{D})$ and C with a lcf $(\tilde{D}_c, \tilde{N}_c)$ such that $T(\hat{P}, C) \in \mathcal{RH}_\infty$. Define $\hat{\Lambda} := \tilde{D}_c\hat{D} + \tilde{N}_c\hat{N}$, then for all plants P with a rcf (N, D) given by

$$\left[\begin{array}{c} N \\ D \end{array}\right] = \left[\begin{array}{c} \hat{N} \\ \hat{D} \end{array}\right] + \left[\begin{array}{c} \Delta_N \\ \Delta_D \end{array}\right], \ \left[\begin{array}{c} \Delta_N \\ \Delta_D \end{array}\right] \in \mathcal{RH}_\infty$$

and

$$\|[\tilde{N}_c \ \tilde{D}_c] \left[\begin{array}{c} \Delta_N \\ \Delta_D \end{array}\right] \hat{\Lambda}^{-1}\|_\infty < 1 \tag{25}$$

will satisfy $T(P, C) \in \mathcal{RH}_\infty$.

Proof: see Bongers and Bosgra (1990) ∎

6. APPLICATION TO THE CD PLAYER

6.1. *Data Acquisition*

Measurements of the Compact Disc radial servo loop have been obtained from an experimental set up of a Compact Disc player at Philips' Research Laboratories. This experimental set up is used to gather several time sequences of 8192 data points of $u(t)$ and $y(t)$ while injecting a reference signal $r(t) = r_1(t)$ in the radial control loop, see Fig. 2.

6.2. *Estimation of Normalized Coprime Factors*

First we present the result of the identification of a nominal model employing the equivalent open loop identification of the plant's coprime factors as proposed in section 5.2 based on a linear regression scheme using system based orthonormal functions. The results are depicted as Bode plots in Fig. 3.
Compared with the spectral estimate of the plant P, it can be observed from Fig. 3 that the essential dynamics which leads to excessive peaking of the sensitivity function have been captured reasonably well in the 16th order coprime factors.

6.3. *Estimation of model uncertainty*

In order to check stability robustness properties of a newly designed model-based controller, applied to the real plant P, an estimate of upper bounds on the additive error of the coprime factors $(\hat{N}, \hat{D})$ is being estimated according to the procedure described in section 5.5. This leads to $|\Delta_N(e^{i\omega})|$ and $|\Delta_D(e^{i\omega})|$ given within some probability α as in (24).

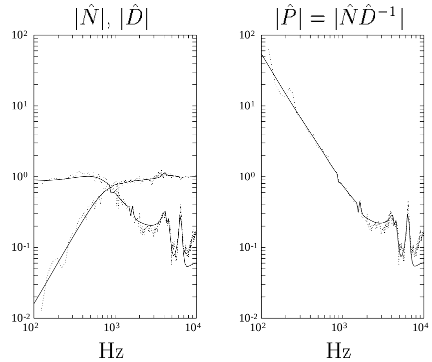

Fig. 3. Amplitude plots of spectral estimate $(\cdots)$ and parametric estimate (—) of 16th order normalized coprime factors $\hat{N}$, $\hat{D}$ (left) and 16th order model $\hat{P} = \hat{N}\hat{D}^{-1}$ (right)

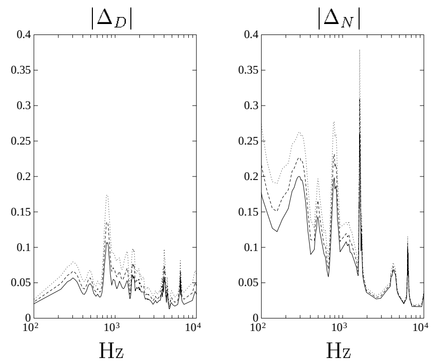

Fig. 4. Upper bounds on additive model error of normalized coprime factors for $\alpha = 90\%$ (—), 99% (- -) and 99.9% $(\cdots)$

The upper bounds are given in Fig. 4 for three different confidence intervals and the results will be used to check stability robustness.

6.4. *Control design and stability robustness*

Both the nominal model and the upper bound on its normalized coprime factors can be used to design an enhanced controller $C_{\hat{P}}$. The control design procedure presented in Bongers and Bosgra (1990) optimizes robustness against additive perturbations on a plants coprime factorization, see Bongers and Bosgra (1990) for details, and this control design can be used to incorporate the upper bounds $|\Delta_N|$ and $|\Delta_D|$ that have been estimated.

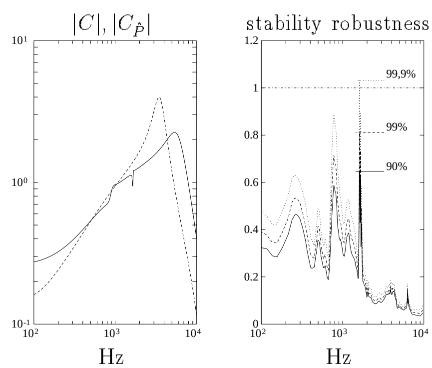

Fig. 5. Improved (—) and old (- -) controller (left) and stability robustness test (right) by evaluation of (25)

Using this control design we have designed an 8th order (improved) controller, which is depicted in Fig. 5 on the left. Moreover, with the knowledge of the upper bound on (Δ_N, Δ_D) with some probability α, we are able to check stability robustness properties of the closed loop plant when applying the newly designed controller. With lemma 5.4 and (25) we obtain the result given in Fig. 5 on the right. From this figure we see that $C_{\hat{P}}$ robustly stabilizes the plant P with a probability $\geq 99\%$. Successful implementation of the controller $C_{\hat{P}}$ using a DSP environment yields a high performance closed loop system with a bandwidth of approximately 800 Hz without excessive peaking of the sensitivity function, leading to improved disturbance rejection. This has been illustrated in Fig. 6

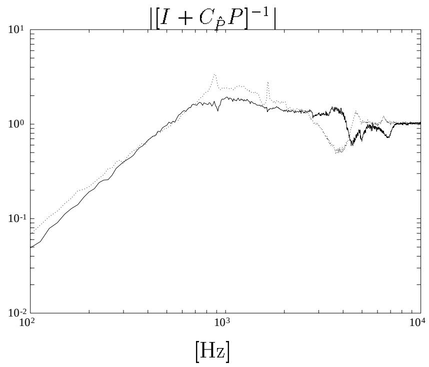

Fig. 6. Improved (—) and old (- -) sensitivity function

7. CONCLUSIONS

In this paper a control relevant parametric identification scheme is applied to a Compact Disc radial servo system, using the well known Prediction Error methods, wherein the difficulty of approximate identification and closed loop experiments has been merged. The problems arising from the closed loop and approximate identification have been handled using a identification based on fractional representations. The identification problem is now initiated in terms of coprime factorizations of the plant and additionally, it yields a manageable approximate identification of the feedback controlled plant. An estimate of the additive error on the coprime factors is used to check stability robustness of a newly designed enhanced controller which has been stressed by successful implementation of the controller.

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