Filtering and parametrization issues in feedback relevant identification based on fractional model representations

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Abstract. This paper discusses filtering and parametrization issues involved in the usage of fractional representations in multivariable, approximate and feedback relevant identification of a possibly unstable plant operating under closed loop conditions. The knowledge of the controller is used to access any stable right coprime factorization of the plant by measuring and filtering the signals present in the closed loop system. By exploiting a specific class of parametrizations in the estimation of the stable coprime factorization with a prespecified McMillan degree, a linear time invariant model having the same McMillan degree will be obtained. In addition the approximate and feedback relevant estimation of a *fixed order* linear time invariant model based on coprime factor identification leads to an additional constraint, which can be written down explicitly as a relation between the filtering of the signals present in the closed loop system and the coprime factors of the model being estimated. A possible solution to deal with this constraint based on an update algorithm is presented.

Keywords. system identification; robust control; coprime factors; filtering; parametrization.

1 Introduction

Induced by the fact that dynamical models obtained from system identification are used as a basis for model based control design, there is a growing interest in merging the problems of identification and control. Models found by system identification techniques are necessarily approximative since exact modelling can be impossible or too costly to perform. The validity of any approximative model hinges on the intended use of the model and therefore the identification procedure being used, will be subjected to several requirements to estimate a model suitable for control design thoughtfully. This has been the motivation to develop methods for a feedback relevant identification, which implies that the feedback relevant dynamical behaviour of a plant operating in a closed loop configuration has to be estimated in order to design enhanced controllers (Gevers, 1993; Van den Hof and Schrama, 1994).

To perform a feedback relevant identification, experiments from the real plant, denoted with P_o, operating in a closed configuration are needed to come up with a model, denoted with $\hat{P}$, suitable for control design (Lee *et al.*, 1992; Hakvoort *et al.*, 1994; Hjalmarsson *et al.*, 1994*a*). Since the controller to create the closed loop configuration can (yet) be unknown, a simultaneous optimization of identification and control design has been proposed in Bayard *et al.* (1992) or Hjalmarsson *et al.* (1994*b*). Furthermore, it has been widely motivated to separate the two stages of identification and control design and to use an iterative scheme of identification and model based control design (Schrama, 1992*a*). One of the first papers using this separation can be found in Farison *et al.* (1967) or Schwartz and Steiglitz (1971) and more recent examples of iterative schemes can be found in Zang *et al.* (1992), Rivera

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and Bhatnagar (1993), Bitmead (1993) or Lee *et al.* (1993). In such an iterative scheme the controller of step $i-1$, is used to perform closed loop experiments with the plant P_o and to estimate a feedback relevant model $\hat{P}$. The model $\hat{P}$ is used to design an improved model based controller, denoted by $C_{\hat{P}}$, again to perform closed loop experiments with in step i.

In this paper the *identification stage* in such an iterative scheme will be discussed. The identification is based on the algebraic theory of fractional representations (Vidyasagar, 1985) and involves the feedback relevant identification of a coprime factor realization of a model $\hat{P}$ based on closed-loop observations of the plant P_o using a controller C from the previous iteration (Hansen, 1989; Schrama, 1992*b*; Van den Hof *et al.*, 1993). In order to control the McMillan degree of the linear time invariant model $\hat{P}$, a specific class of parametrizations is used to parametrize the coprime factorization being estimated. Furthermore, the approximate and feedback relevant estimation of a *fixed order* linear time invariant model gives rise to an additional constraint, which can be written down explicitly in case of the coprime factor identification.

The outline of this paper is as follows. In section 2 some preliminary notations and definitions used in the sequel will be given. Section 3 discusses the relation between identification and control design. To deal with the closed loop identification problem, in section 4 the framework of equivalent open-loop identification of a coprime factor representation of the plant P_o will be summarized. Section 5 contains the parametrization aspects on the identification of a coprime factorization itself and the results of performing the identification in a feedback relevant way, leading to an additional parametrization constraint. Possible solutions to cope with this parametrization constraint are summarized. Finally, section 6 contains some concluding remarks.

2 Preliminaries

2.1 Feedback configuration

Throughout this paper the feedback configuration of a plant P and a controller C is denoted with $\mathcal{T}(P, C)$ and defined as the connection structure depicted in Figure 1.

In Figure 1 the signals u and y reflect respectively the inputs and outputs of the plant P, where v is an additive noise on the output y of the plant. The signals u_c and y_c are respectively the inputs and outputs of the controller C, and r_1 and r_2 are external reference signal that are uncorrelated with the additive noise v. From an identification point of view the signals u and y are being measured, v is unknown and r_1, r_2 (and consequently u_c, y_c) are possibly at our disposal.

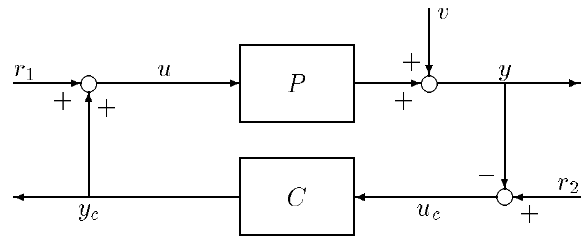

Fig. 1: feedback connection structure $\mathcal{T}(P, C)$

It is assumed that the feedback connection structure is well posed, that is $\det[I + CP] \not\equiv 0$. In this way the closed loop dynamics of the closed loop system $\mathcal{T}(P, C)$ can be described by the mapping of $[r_2 \ r_1]^T$ to $[y \ u]^T$ which is given by the transfer function matrix $T(P, C)$:

$$T(P,C) := \begin{bmatrix} P \\ I \end{bmatrix} [I + CP]^{-1} \begin{bmatrix} C & I \end{bmatrix}. \quad (1)$$

and describing the data coming from the closed loop system $\mathcal{T}(P, C)$ in the following way

$$\begin{bmatrix} y \\ u \end{bmatrix} = T(P,C) \begin{bmatrix} r_2 \\ r_1 \end{bmatrix} + \begin{bmatrix} I \\ -C \end{bmatrix} [I + PC]^{-1} v \quad (2)$$

where the additive noise $v := He$ can be modelled by a monic, stable and stably invertible noise filter H having a white noise input e (Ljung, 1987). In case of an *internally* stable closed loop system $\mathcal{T}(P, C)$, all four transfer function matrices in $T(P, C)$ will be stable (Desoer and Chan, 1975; Schrama, 1992*b*; Bongers, 1994) which implies $T(P, C) \in \mathbb{R}\mathcal{H}_\infty$, where $\mathbb{R}\mathcal{H}_\infty$ denotes the set of all rational stable transfer functions.

The controller C will be applied to both the real plant P_o and the model $\hat{P}$, according to the feedback connection structure given in Figure 1. The corresponding closed loop dynamics of the two different feedback configurations will be described respectively by the two transfer function matrices $T(P_o, C)$ and $T(\hat{P}, C)$.

2.2 Coprimeness and stability

Using the theory of fractional representations, an arbitrary plant P is expressed as a ratio of two stable mappings N and D. Following Vidyasagar (1985) the following definitions for coprimeness and coprime factorization will be used, where $\mathbb{R}\mathcal{H}_\infty$ denotes the set of all rational stable transfer functions.

Definition 2.1 *Let $N, D \in \mathbb{R}\mathcal{H}_\infty$, then the pair (N, D) is called right coprime over $\mathbb{R}\mathcal{H}_\infty$ if there exist right Bezout factors $X, Y \in \mathbb{R}\mathcal{H}_\infty$ such that*

$$XN + YD = I.$$

The pair (N, D) is a right coprime factorization (rcf) of P if $\det\{D\} \not\equiv 0$ and $P = ND^{-1}$ and (N, D) is right coprime over $\mathbb{R}\mathcal{H}_\infty$.

Based on the theory of fractional representations and the usage of left and right coprime factorizations given in definition 2.1 the following result for internal stability of a closed loop system $\mathcal{T}(P, C)$ can be derived.

Theorem 2.2 *Let $P = ND^{-1} = \tilde{D}^{-1}\tilde{N}$ where (N, D) is a rcf and $(\tilde{D}, \tilde{N})$ a lcf of P. Let $C = N_c D_c^{-1} = \tilde{D}_c^{-1}\tilde{N}_c$ where (N_c, D_c) is a rcf and $(\tilde{D}_c, \tilde{N}_c)$ a lcf of C. Now the following statements are equivalent*

i. *the feedback system $\mathcal{T}(P, C)$ given in Figure 1 is internally stable*

ii. $T(P, C) \in \mathbb{R}\mathcal{H}_\infty$

iii. $\Lambda^{-1} \in \mathbb{R}\mathcal{H}_\infty$, *with* $\Lambda := \begin{bmatrix} \tilde{D}_c & \tilde{N}_c \end{bmatrix} \begin{bmatrix} D \\ N \end{bmatrix}$

iv. $\tilde{\Lambda}^{-1} \in \mathbb{R}\mathcal{H}_\infty$, *with* $\tilde{\Lambda} := \begin{bmatrix} \tilde{D} & \tilde{N} \end{bmatrix} \begin{bmatrix} D_c \\ N_c \end{bmatrix}$

Proof: Vidyasagar (1985) and Bongers (1994). □

3 Merging identification and control

3.1 Norm based control design

In the analysis of feedback relevant identification, the characterization of a closed loop performance criterion plays an important role. This criterion is usually characterized by an objective function, which depends on a plant P and the controller C that assemble the closed loop configuration (Van den Hof and Schrama, 1994).

Definition 3.1 *Let $\mathcal{X}$ denote a complete normed space, where $\|\cdot\|_\mathcal{X}$ is the norm function defined on $\mathcal{X}$. Let a plant P and a controller C form a well posed feedback connection structure $\mathcal{T}(P, C)$ according to Figure 1, and let $J(P, C)$ be a any function depending on a plant P and the controller C. Then $\forall\ P, C$ with $J(P, C) \in \mathcal{X}$ the objective function is defined as a map from $\mathcal{X}$ onto $\mathbb{R}\mathcal{H}_\infty$ given by $\|J(P, C)\|_\mathcal{X}$*

Unfortunately the plant P_o under consideration is unknown and the control design method will be based on minimization of a closed loop performance criterion $\|J(\hat{P}, C)\|_\mathcal{X}$ using a model $\hat{P}$. In this way the control design will be model based and can be interpreted by the computation of a so called model based controller, denoted with $C_{\hat{P}}$, such that

$$C_{\hat{P}} = \arg \min_C \|J(\hat{P}, C)\|_\mathcal{X}. \tag{3}$$

This minimization gives rise to a value of the objective function given by $\|J(\hat{P}, C_{\hat{P}})\|_\mathcal{X}$ and can be classified as the *design cost* (Gevers, 1993). Applying the model based controller $C_{\hat{P}}$ to the real plant P_o gives rise to the value $\|J(P_o, C_{\hat{P}})\|_\mathcal{X}$ which is characterized as the *achieved cost*. In this perspective the controller $C_{\hat{P}}$ is said to *satisfy* the design objective for the corresponding model $\hat{P}$ if

$$\|J(\hat{P}, C_{\hat{P}})\|_\mathcal{X} \leq \gamma, \text{ with } \gamma > 0 \tag{4}$$

holds, which is a nominal performance specification. Related examples can for example be found in Bitmead (1993), Gevers (1993) or Van den Hof and Schrama (1994) for ∞- or 2-norm based minimization.

In this paper the normed space $\mathcal{X}$ is chosen to be the space $\mathbb{R}\mathcal{H}_\infty$. The function $J(P, C) \in \mathbb{R}\mathcal{H}_\infty$ is taken to be a weighted form of the closed loop dynamics described by the transfer function matrix $T(P, C) \in \mathbb{R}\mathcal{H}_\infty$ given in (1). In this way $J(P, C) = W_o T(P, C) W_i \in \mathbb{R}\mathcal{H}_\infty$ if W_o and W_i in (5) are weighting filters satisfying W_o, $W_i \in \mathbb{R}\mathcal{H}_\infty$, making

$$\|J(P, C)\|_\infty := \|W_o T(P, C) W_i\|_\infty \tag{5}$$

The objective function given in (5) represents a large class of ∞-norm based control design schemes and the usage of the weightings is inspired by the ability to create a trade off between conflicting requirements and constraints always present (Horowitz, 1963; Boyd and Barrat, 1991). In case of diagonal weighting filters, the weighting can be seen as an additional loop-shaping in the control design (Bongers, 1994).

3.2 A feedback relevant criterion

From an identification point of view, a model $\hat{P}$ can only be an approximation of the real plant P_o. The quality of any approximative model depends on the intended use of the model. In this perspective, the question wether a model $\hat{P}$ is good for model based control design gives rise to a so called feedback relevant identification, since the quality of the model $\hat{P}$ should be evaluated under feedback or closed loop conditions (Schrama, 1992*b*).

A successful controller $C_{\hat{P}}$, found by the norm based minimization given in (3) and based on a model $\hat{P}$, gives rise to a value of objective function $\|J(\hat{P},C_{\hat{P}})\|_{\mathcal{X}}$, which is said to satisfy the control objective (4) for the nominal model $\hat{P}$. From this perspective, the quality of the model $\hat{P}$ can be evaluated by considering the value of the objective function $\|J(P_o,C_{\hat{P}})\|_{\mathcal{X}}$ when applying the controller $C_{\hat{P}}$, to the real plant P_o.

Unfortunately, the real plant P_o is unknown and the following triangular inequalities (Schrama, 1992*b*) can be used to lower and upper bound $\|J(P_o,C_{\hat{P}})\|_{\mathcal{X}}$.

$$\|J(P_o,C_{\hat{P}})\|_{\mathcal{X}} \leq \|J(\hat{P},C_{\hat{P}})\|_{\mathcal{X}} + \|J(P_o,C_{\hat{P}}) - J(\hat{P},C_{\hat{P}})\|_{\mathcal{X}}$$

$$\|J(P_o,C_{\hat{P}})\|_{\mathcal{X}} \geq \left| \|J(\hat{P},C_{\hat{P}})\|_{\mathcal{X}} - \|J(P_o,C_{\hat{P}}) - J(\hat{P},C_{\hat{P}})\|_{\mathcal{X}} \right|$$

From the first inequality it can be seen that

$$\|J(\hat{P},C_{\hat{P}})\|_{\mathcal{X}} + \|J(P_o,C_{\hat{P}}) - J(\hat{P},C_{\hat{P}})\|_{\mathcal{X}} \leq \gamma \quad (6)$$

is a sufficient condition in order to have a model based controller $C_{\hat{P}}$ which satisfies the control objective (4) on the real plant P_o. From an identification point of view the performance degradation $\|J(P_o,C_{\hat{P}}) - J(\hat{P},C_{\hat{P}})\|_{\mathcal{X}}$ for the controller $C_{\hat{P}}$ should be minimized in order to find a model $\hat{P} = P(\hat{\theta})$ such that (6) holds and can be seen as a feedback relevant identification of the plant P_o.

However, the model $\hat{P}$ and thus the controller $C_{\hat{P}}$ is not available (yet), which give rise to an iterative scheme wherein the controller C (from the previous iteration) is used to evaluate $\|J(P_o,C)\|_{\mathcal{X}}$. With the choice of the objective function given in (5), the minimization of the performance degradation $\|J(P_o,C) - J(\hat{P},C)\|_{\mathcal{X}}$ then becomes

$$\min_{\theta} \|W_o[T(P_o,C) - T(P(\theta),C)]W_i\|_{\infty}. \quad (7)$$

By minimizing (7) such that (6) holds, the current controller C, applied to the plant P_o, is guaranteed to give a similar performance when applying it to the model $P(\hat{\theta})$ found by the minimization and the model $P(\hat{\theta})$ can be used for subsequent control design.

4 Closed loop identification

4.1 Identification of stable factorizations

Approximate identification on the basis of closed loop experiments could easily be defective due to the correlation between noise v and input u, (Ljung, 1987). Moreover, an explicit expression for the approximation of the plant P_o, independent of the noise contribution during the experiments, is needed to tune the bias of the model $\hat{P}$ in a feedback relevant way (7). Additionally, an unified approach to handle the identification of both stable and unstable plants P_o, that are stabilized during the closed loop experiments, is preferred. These demands can be handled by using the algebraic theory of fractional representations and to estimate stable (coprime) factorization of the plant P_o. Several authors have worked on this topic, see for example Hansen (1989), Van den Hof *et al.* (1993) or Lee *et al.* (1993).

To have access to a factorization of the plant P_o, the following approach can be followed. Consider the closed loop data generating system given in Figure 1 and define $r := r_1 + Cr_2$. With (2) this yields

$$r = r_1 + Cr_2 = u + Cy \quad (8)$$

and (2) reduces to

$$\begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} P_oS_i \\ S_i \end{bmatrix} r + \begin{bmatrix} S_o \\ -CS_o \end{bmatrix} He \quad (9)$$

where $S_i := [I + CP_o]^{-1}$ is the input sensitivity function and $S_o := [I + P_oC]^{-1}$ is the output sensitivity function. Since the controller C is used for the closed loop experiments, the closed loop system $\mathcal{T}(P_o,C)$ is assumed to be internally stable. With theorem 2.2 this yields $T(P_o,C) \in \mathbb{R}\mathcal{H}_{\infty}$ making both P_oS_i, $S_i \in \mathbb{R}\mathcal{H}_{\infty}$ in (9) but not necessarily coprime, which is summarized in the following corollary.

Corollary 4.1 *Let a plant P and a controller C create an internally stable feedback system $\mathcal{T}(P,C)$ then (PS_i, S_i) is a rcf of P if and only if $C \in \mathbb{R}\mathcal{H}_{\infty}$.*

Proof: See de Callafon (1994). □

Hence P_oS_i, S_i can be considered to be a stable right, but not necessarily coprime, factorization (N_o, D_o) of the plant P_o, with $N_o := P_oS_i$ and $D_o := S_i$.

4.2 Identification of coprime factorizations

To avoid the presence and estimation of unstable zeros in the factorization (PS_i, S_i), which gives rise to hidden unstable modes in the representation of the plant P_o, the factorization needs to be coprime. For an unstable controller C, the factorization (PS_i, S_i) is not coprime, as mentioned in corollary 4.1, while the operation given in (8) yields an unbounded signal. Furthermore, a rcf is not unique and access to factorizations different from (P_oS_i, S_i) would be preferable. In order to fulfil these requirements, an additional filtering of the signal r is introduced with $x := Fr$, similar as in Van den Hof *et al.* (1993) or de Callafon *et al.* (1994). With (2) and (8) this yields

$$x = F \left[\begin{array}{cc} C & I \end{array}\right] \left[\begin{array}{c} r_2 \\ r_1 \end{array}\right] = F \left[\begin{array}{cc} C & I \end{array}\right] \left[\begin{array}{c} y \\ u \end{array}\right] \tag{10}$$

and (2) now reduces to

$$\left[\begin{array}{c} y \\ u \end{array}\right] = \left[\begin{array}{c} P_o S_i F^{-1} \\ S_i F^{-1} \end{array}\right] x + \left[\begin{array}{c} S_o \\ -CS_o \end{array}\right] He \tag{11}$$

where (PS_iF^{-1}, S_iF^{-1}) again is a (right) factorization of the plant P_o.

In Van den Hof *et al.* (1993) the freedom in choosing the filter F is found by restricting both the factorization (PS_iF^{-1}, S_iF^{-1}) and the map $F[C\ I]$ in (10) to be stable. However, stability of the map $F[C\ I]$ is not necessary in general. In the case that $r_2(t) = 0\ \forall t$, $x = Fr_1$, hence stability of F is required only. By restricting (PS_iF^{-1}, S_iF^{-1}) to be a rcf, stability of $F[C\ I]$ is implied directly and is summarized in the following lemma.

Lemma 4.2 *Let a plant P and a controller $C := \tilde{D}_c^{-1}\tilde{N}_c$, where $(\tilde{D}_c, \tilde{N}_c)$ is a lcf of C, form an internally stable feedback system $\mathcal{T}(P,C)$ then the following conditions are equivalent*

(i) (PS_iF^{-1}, S_iF^{-1}) *is a rcf.*

(ii) $F = W\tilde{D}_c$ *with* $W,\ W^{-1} \in \mathbb{R}\mathcal{H}_\infty$

and imply $F \left[\begin{array}{cc} C & I \end{array}\right] \in \mathbb{R}\mathcal{H}_\infty$.

Proof: See Van den Hof *et al.* (1993) or de Callafon (1994). □

Lemma 4.2 is a generalisation of corollary 4.1 and characterizes the freedom in choosing the filter F by the choice of any stable and stably invertible filter W. The choice of W however can be related to the choice of an auxiliary model P_x and an auxiliary controller C_x with $T(P_x, C_x) \in \mathbb{R}\mathcal{H}_\infty$ (de Callafon, 1994). Since C_x can be any controller, it can be chosen to be equal to the controller C that the controls the plant P_o under consideration. In this way the filter F in lemma 4.2 can be characterized as follows.

Corollary 4.3 *Let a plant P and a controller C create an internally stable feedback system $\mathcal{T}(P,C)$ and let (N_x, D_x) be any rcf of any auxiliary model P_x, then*

$$F = [D_x + CN_x]^{-1} \tag{12}$$

satisfies the conditions of lemma 4.2 if and only if $T(P_x, C) \in \mathbb{R}\mathcal{H}_\infty$.

Proof: See Van den Hof *et al.* (1993). □

With the result of lemma 4.2 the following proposition for the open loop identification of a right coprime factor can be given.

Proposition 4.4 *Let the plant P_o and a controller C create a stable feedback system $\mathcal{T}(P_o, C)$, then the closed loop data $[y\ u]^T$ in (2) can be rewritten into*

$$\left[\begin{array}{c} y \\ u \end{array}\right] = \left[\begin{array}{c} N_o \\ D_o \end{array}\right] x + \left[\begin{array}{c} I \\ -C \end{array}\right] [I + P_oC]^{-1} v$$

where x is given in (10), F is any filter satisfying lemma 4.2 and (N_o, D_o) is a rcf of the plant P_o given by

$$\begin{aligned} \left[\begin{array}{c} N_o \\ D_o \end{array}\right] &= \left[\begin{array}{c} P_o \\ I \end{array}\right] S_i F^{-1} = \\ &= \left[\begin{array}{c} P_o \\ I \end{array}\right] [I + CP_o]^{-1}[I + CP_x]D_x \end{aligned} \tag{13}$$

Proof: By use of (11) with $N_o := P_oS_iF^{-1}$ and $D_o := S_iF^{-1}$ and direct application of corollary 4.3. Equation (13) is found by substituting (12). □

The specific rcf (N_o, D_o) in (13) of the plant P_o to be identified is related to the filter F since $N_o = P_oS_iF^{-1}$ and $D_o = S_iF^{-1}$. With F given by (12) in corollary 4.3, the rcf (N_o, D_o) is related to the rcf (N_x, D_x) of the auxiliary model P_x, used to create the filter F and is summarized in the following corollary.

Corollary 4.5 *The rcf (N_o, D_o) of the plant P_o given in proposition 4.4 and based on the realization of F given in corollary 4.3, satisfies*

$$[D_o + CN_o] = F^{-1} = [D_x + CN_x]. \tag{14}$$

Proof: With $N_o = P_oS_iF^{-1}$ and $D_o = S_iF^{-1}$, $[D_o + CN_o] = [I + CP_o]S_iF^{-1} = F^{-1}$ proving equation (14), where F is given in (12). □

The transfer function matrix $[D_o + CN_o]$ is unknown, since it contains the specific rcf (N_o, D_o) of the unknown plant P_o, but (14) indicates that this can be replaced by the filter operation F^{-1}, which is completely known. From corollary 4.5 it can also be seen that (N_o, D_o) can be of high order, containing redundant dynamics. A sensible choice of the model P_x may lead to cancelling of redundant dynamics, which is used in Van den Hof *et al.* (1993) to estimate possibly low order (normalized) factorizations of the plant P_o.

The same approach of filtering signals present during the closed loop experiments is also being used in the two stage method described in Van den Hof and Schrama (1993). In this method the filter F is given by an accurate estimate of the input sensitivity function $S_i = [I + CP_o]^{-1}$. The specific factorization (N_o, D_o) to be identified becomes approximately (P_o, I) and an estimate of P_o is found by estimating N_o only. It should be noted that $F = [I + CP_o]^{-1}$ does not satisfy the conditions mentioned in lemma 4.2 and clearly, the factorization (P_o, I) is not coprime over $\mathbb{R}\mathcal{H}_\infty$ for an unstable plant P_o. Moreover, if the filter F is given by an *approximation* of the input sensitivity function $[I + CP_o]^{-1}$, the situation can become even worse since both $N_o := P_o S_i F^{-1}$ and $D_o = S_i F^{-1}$ can become unstable. This is due to the fact that F^{-1}, which is the inverse of the estimated input sensitivity function, can be unstable and the unstable modes will not be cancelled completely in the operation $P_o S_i F^{-1}$ or $S_i F^{-1}$.

The estimate of the right coprime factorization (N_o, D_o) in Van den Hof *et al.* (1993) and de Callafon *et al.* (1994) is found by a 2-norm minimization based on a prediction error method with an OE (output error) model structure (Ljung, 1987). However, for sake of analysis and to maintain generality, it is assumed here that an identification procedure based on the data given in proposition 4.4 is able to come up with an estimate $\hat{\theta}$ given by

$$\hat{\theta} = \arg\min_{\theta} \left\| W_1 \left(\begin{bmatrix} N_o \\ D_o \end{bmatrix} - \begin{bmatrix} N(\theta) \\ D(\theta) \end{bmatrix} \right) W_2 \right\|_{\mathcal{X}} \quad (15)$$

where W_1, W_2 are arbitrary weighting functions and $\|\cdot\|_{\mathcal{X}}$ is a norm function to be specified. The role of the weighting functions W_1, W_2, the norm function $\|\cdot\|_{\mathcal{X}}$ to be used and the parametrization of the factorization $(N(\theta), D(\theta))$ will be scrutinized in the following section.

5 Estimation of coprime factors

5.1 Feedback relevant identification

In order to perform a feedback relevant identification, the norm of the difference $\Delta T(P_o, \hat{P}, C) := W_o[T(P_o, C) - T(\hat{P}, C)]W_i$ introduced in section 3.2, needs to be minimized for a *fixed order* model $\hat{P}$. Using the filter F of corollary 4.3 the mismatch $\Delta T(P_o, \hat{P}, C)$ can be expressed in terms of the weighted difference between the rcf (N_o, D_o) and $(\hat{N}, \hat{D})$ respectively of the plant P_o and the model $\hat{P}$, along with an *additional* constraint, depending on the filter F being used. This is summarized in the following lemma.

Lemma 5.1 *Let the plant P_o with rcf (N_o, D_o) given in corollary 4.5 and a controller C create an internally stable feedback system $\mathcal{T}(P_o, C)$. Consider a model $\hat{P}$ with rcf $(\hat{N}, \hat{D})$ and any filter F satisfying lemma 4.2 then*

$$\Delta T(P_o, \hat{P}, C) = W_o[T(P_o, C) - T(\hat{P}, C)]W_i$$

equals

$$W_o \left(\begin{bmatrix} N_o \\ D_o \end{bmatrix} - \begin{bmatrix} \hat{N} \\ \hat{D} \end{bmatrix} \right) F \begin{bmatrix} C & I \end{bmatrix} W_i \Bigg|_{\hat{D} + C\hat{N} = F^{-1}}$$

Proof: With (N_o, D_o) as rcf of P_o the matrix $T(P_o, C)$ can be rewritten as

$$T(P_o, C) = \begin{bmatrix} N_o \\ D_o \end{bmatrix} [D_o + CN_o]^{-1} \begin{bmatrix} C & I \end{bmatrix}$$

and using the fact $[D_o + CN_o] = F^{-1}$ from (14) in corollary 4.5, this can be rewritten into

$$T(P_o, C) = \begin{bmatrix} N_o \\ D_o \end{bmatrix} F \begin{bmatrix} C & I \end{bmatrix}.$$

With $(\hat{N}, \hat{D})$ as rcf of $\hat{P}$, the matrix $T(\hat{P}, C)$ equals

$$\begin{bmatrix} \hat{N} \\ \hat{D} \end{bmatrix} [\hat{D} + C\hat{N}]^{-1} \begin{bmatrix} C & I \end{bmatrix}$$

and under the constraint $[\hat{D} + C\hat{N}] = F^{-1}$ this yields

$$T(\hat{P}, C) = \begin{bmatrix} \hat{N} \\ \hat{D} \end{bmatrix} F \begin{bmatrix} C & I \end{bmatrix}$$

making $T(P_o, C) - T(\hat{P}, C)$ equal to

$$\left(\begin{bmatrix} N_o \\ D_o \end{bmatrix} - \begin{bmatrix} \hat{N} \\ \hat{D} \end{bmatrix} \right) F \begin{bmatrix} C & I \end{bmatrix} \Bigg|_{\hat{D} + C\hat{N} = F^{-1}}$$

□

Clearly, lemma 5.1 reflects an additional constraint on the parametrized coprime factorization $(N(\theta), D(\theta))$ of the model $\hat{P}$ to be identified. In case the choice of the filter F is replaced by the choice of a rcf (N_x, D_x) of an auxiliary model P_x as in corollary 4.3, the constraint equals

$$[D(\theta) + CN(\theta)] = F^{-1} = [D_x + CN_x] \quad (16)$$

which has to be incorporated in the feedback relevant identification of a model $\hat{P}$.

With the result of lemma 5.1 the following observations can be made for the weightings W_1, W_2 and the norm function $\|\cdot\|_{\mathcal{X}}$ in (15), in order to minimize the feedback relevant criterion given in (7).

Proposition 5.2 *The feedback relevant criterion of* (7) *and the estimation problem of* (15) *can be made compatible by taking* $W_1 = W_o$, $W_2 = F[C\ I]W_i$, $\|\cdot\|_{\mathcal{X}} = \|\cdot\|_\infty$ *and satisfying the constraint given in* (16), *which yields*

$$\hat{\theta} = \arg\ \min_\theta \left\| W_o \left(\begin{bmatrix} N_o \\ D_o \end{bmatrix} - \begin{bmatrix} N(\theta) \\ D(\theta) \end{bmatrix} \right) \cdot \right. \\ \left. \cdot F \begin{bmatrix} C & I \end{bmatrix} W_i \right\|_\infty \Bigg|_{\hat{D}+C\hat{N}=F^{-1}} \tag{17}$$

Proof: With $W_1 = W_o$, $W_2 = F[C\ I]W_i$ the argument of $\|\cdot\|_{\mathcal{X}}$ in (15) equals the argument of $\|\cdot\|_\infty$ in (7), by substituting the results of lemma 5.1. Since the argument $\Delta T(P_o, \hat{P}, C) \in \mathbb{R}\mathcal{H}_\infty$, the norm function $\|\cdot\|_{\mathcal{X}}$ in (15) can be chosen to be $\|\cdot\|_\infty$ and both (7) and (15) are equal. □

5.2 Minimization with constraint

According to proposition 5.2, the minimization of

$$\min_\theta \|W_o[T(P_o, C) - T(P(\theta), C)]W_i\|_{\mathcal{X}}$$

for any norm function $\|\cdot\|_{\mathcal{X}}$ can be replaced by the minimization given in (17) and involves basically a non-linear minimization for a model $P(\theta)$ with a specified McMillan degree, even if the model is parametrized linearly.

To avoid the use of the constraint (16) in the minimization, an iterative scheme of minimization without the constraint in step $i-1$ and updating the constraint in step i was proposed in de Callafon *et al.* (1994) and was based on the estimation of normalized coprime factors. However, updating the constraint involves only the update of the filter F, used to create the signal x in (10). In case the filter F is defined via a rcf (N_x, D_x) of any auxiliary model P_x as in corollary 4.3, (N_x, D_x) can be computed directly and is given in the following proposition.

Proposition 5.3 *Let the filter* F *in* (10) *be given by corollary 4.3 then the rcf* (N_x, D_x) *given by*

$$\begin{bmatrix} N_x \\ D_x \end{bmatrix} = T(P_x, C) \begin{bmatrix} N(\theta) \\ D(\theta) \end{bmatrix}$$

satisfies the constraint given in (16).

Proof: Similar as in corollary 4.5. □

Clearly, the estimate of the rcf $(N(\theta), D(\theta))$ in proposition 5.3 is not available (yet). Taking any rcf (N_x, D_x) such that $P_x := N_x D_x{}^{-1}$ satisfies $T(P_x, C) \in \mathbb{R}\mathcal{H}_\infty$, this gives rise to an update algorithm to handle the minimization given in (17) for performing a feedback relevant identification of the plant P_o as indicated in proposition 5.2 and can be summarized as follows.

1. In step i, create F_i from corollary 4.3.
2. Estimate a rcf $(N(\hat{\theta}_i), D(\hat{\theta}_i))$ based on a parametrization given in theorem 5.4 and the minimization given in proposition 5.2 *without* the constraint (16).
3. Update the rcf (N_x, D_x) with proposition 5.3 according to
$$\begin{bmatrix} N_{x\,i+1} \\ D_{x\,i+1} \end{bmatrix} = T(P_x, C) \begin{bmatrix} N(\hat{\theta}_i) \\ D(\hat{\theta}_i) \end{bmatrix}$$
making $D(\hat{\theta}_i) + CN(\hat{\theta}_i) = D_{x\,i+1} + CN_{x\,i+1}$ and $P_x = N_{x\,i+1} D_{x\,i+1}^{-1}$ with $T(P_x, C) \in \mathbb{R}\mathcal{H}_\infty$ remains fixed for all i.
4. $i := i + 1$ and go to 1.

If the iteration converges then $D(\hat{\theta}_i) + CN(\hat{\theta}_i) = D_{x\,i} + CN_{x\,i}$ is independent of i and the constraint (16) has been satisfied, thus a feedback relevant estimate $\hat{P}$ of the plant P_o has been obtained according to proposition 5.2. A rigorous proof of the convergence of the iteration is not available (yet) but extensive simulations reveal promising results.

5.3 Parametrization

To control the McMillan degree of the model $\hat{P} = P(\theta) = N(\theta)D(\theta)^{-1}$ being estimated, the factorization $(N(\theta), D(\theta))$ has to be parametrized in a special way and boils down to the fact that both $N(\theta)$ and $D(\theta)$ should have *common* stable modes. Furthermore, any common unstable zeros should be avoided to ensure coprimeness of the factorization $(N(\theta), D(\theta))$. The result has been stated in the following theorem.

Theorem 5.4 *Let* $(\hat{N}, \hat{D}) \in \mathbb{R}\mathcal{H}_\infty$ *be given by a minimal and stable state space representation*

$$\left(\bar{A}, \bar{B}, \begin{bmatrix} \bar{C}_N \\ \bar{C}_D \end{bmatrix}, \begin{bmatrix} \bar{E}_N \\ \bar{E}_D \end{bmatrix} \right)$$

such that $\det\{\bar{E}_D\} \neq 0$ *and*

$$\begin{bmatrix} \hat{N}(z) \\ \hat{D}(z) \end{bmatrix} = \begin{bmatrix} \bar{C}_N \\ \bar{C}_D \end{bmatrix} [zI - \bar{A}]^{-1} \bar{B} + \begin{bmatrix} \bar{E}_N \\ \bar{E}_D \end{bmatrix}$$

then

(i) $\det\{\hat{D}\} \not\equiv 0$

(ii) $\hat{P} := \hat{N}\hat{D}^{-1}$ *is given by the state space representation* $[A, B, C, E]$ *with*

$$\left\{ \begin{array}{rcl} A &=& \bar{A} - \bar{B}\bar{E}_D^{-1}\bar{C}_D \\ B &=& \bar{B}\bar{E}_D^{-1} \\ C &=& \bar{C}_N - \bar{E}_N\bar{E}_D^{-1}\bar{C}_D \\ E &=& \bar{E}_N\bar{E}_D^{-1} \end{array} \right. \tag{18}$$

(iii) $(\hat{N}, \hat{D})$ *is a rcf of* $\hat{P}$.

Proof: The factor $\hat{D}$ has a state space representation $(\bar{A}, \bar{B}, \bar{C}_D, \bar{E}_D)$ and due to the non-singular feedthrough matrix $\bar{E}_D$, $\hat{D}$ is always invertible having a state space representation given by $(\bar{A} - \bar{B}\bar{E}_D^{-1}\bar{C}_D, \bar{B}\bar{E}_D^{-1}, -\bar{E}_D^{-1}\bar{C}_D, \bar{E}_D^{-1})$ which proves (i). $\hat{N}$ has a state space representation $(\bar{A}, \bar{B}, \bar{C}_N, \bar{E}_N)$. Performing the series connection of $\hat{D}^{-1}$ and $\hat{N}$ in $\hat{P} = \hat{N}\hat{D}^{-1}$, basic matrix manipulation yields an extended state space representation, wherein n uncontrollable states can be omitted, where n is the dimension of $\bar{A}$. This leads to the state space representation given in (18), which proves (ii). From this, the matrices $\bar{A}$, $\bar{B}$, $\bar{C}_N$, $\bar{C}_D$ and $\bar{E}_N$ can be found from (18) leading to

$$\left\{ \begin{array}{rcl} \bar{A} &=& A - BK \\ \bar{B} &=& B\bar{E}_D \\ \bar{C}_N &=& C - EK \\ \bar{C}_D &=& -K \\ \bar{E}_N &=& E\bar{E}_D \end{array} \right.$$

making

$$\begin{array}{rcl} \hat{N}(z) &=& ([C - EK][zI - A + BK]^{-1}B + E)\bar{E}_D \\ &=& \bar{N}(z)\bar{E}_D \in \mathbb{R}\mathcal{H}_\infty \\ \hat{D}(z) &=& (-K[zI - A + BK]^{-1}B + I)\bar{E}_D \\ &=& \bar{D}(z)\bar{E}_D \in \mathbb{R}\mathcal{H}_\infty . \end{array} \qquad (19)$$

The factorization $(\bar{N}(z), \bar{D}(z))$ is proven to be a right coprime factorization in Nett *et al.* (1984). Since the factorization $(\bar{N}(z), \bar{D}(z))$ is post multiplied by a constant non-singular matrix $\bar{E}_D$ only, the factorization $(\hat{N}(z), \hat{D}(z))$ is a also a rcf, which proves (iii). □

The result of theorem 5.4 gives rise to a wide class of parametrizations needed to estimate a rcf $(N(\hat{\theta}), \hat{D}(\hat{\theta}))$, since it involves the parametrization of a stable, minimal state space representation $[\bar{A}, \bar{B}, \bar{C}]$ with $\bar{C}^T = [\bar{C}_N^T \; \bar{C}_D^T]$, wherein the direct feedthrough matrix of the factor $\hat{D}$ is restricted to be non-singular. Restricting the estimate to be stable and minimal can be enforced by using the specific parametrization of asymptotically stable systems as given in Ober (1991) and further elaborated in Chou (1994). This gives rise to an estimate of the factorization $(N(\hat{\theta}), \hat{D}(\hat{\theta}))$ which is guaranteed to be stable, minimal and balanced.

Using prediction error methods (Ljung, 1987) to estimate the state space matrices in theorem 5.4, a stable and minimal state space estimate with non-singular feedthrough matrix $\bar{E}_D$ is found in the generic case, which is due to the following facts. Firstly, the map from x onto $[y \; u]^T$ is defined to be stable, according to proposition 4.4. Secondly, the map from x onto u is given by $[I + CP_o]^{-1}[I + CP_x]D_x$ according to (13), which is non-singular by definition. In this way the matrices are parametrized by standard pseudo canonical (overlapping) forms (Gevers and Wertz, 1984) without stability or non-singularity condition. Finally it should be noted that the matrix operations given in (18) leads to model $\hat{P}$ with McMillan degree less than or equal to n, where n is simply the McMillan degree of the factorization $(\hat{N}, \hat{D})$ being estimated.

6 Conclusions

In this paper the filtering and parametrization issues involved in the usage of fractional representations in multivariable, approximate and feedback relevant identification of a possibly unstable plant operating under closed loop conditions have been discussed. It has been shown that any stable right coprime factorization of the plant can be accessed by the filtering of signals present in the closed loop system. The freedom in choosing the filter has been characterized by employing the knowledge of the controller present during the closed loop experiments.

Consequently, a stable right coprime fractional representation generated by the closed loop system and the filtering being used, can be estimated. In order to have a model with a prefixed McMillan degree, a specific class of parametrizations with the same McMillan degree can be used to estimate a stable right coprime factorization of the model.

Finally, the approximate and feedback relevant estimation of a fixed order linear time invariant model based on coprime factor identification leads to an additional constraint. This constraint is intrinsic in many schemes on feedback relevant identification but can be written down explicitly in case of the coprime factor identification. The constraint boils down to a relation between the filter used to gain access to the coprime factors of the plant and model being estimated. A possible solution to deal with the constraint by updating the filtering is presented here.

7 References

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