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Closed-loop identification of a distillation column

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Abstract

To improve the closed-loop operation of a distillation column, a model is needed for control design. To obtain data, a closed-loop experiment is designed and executed. Models are obtained by applying the Direct Identification method and the Two-Step method to the closed-loop data. Both Output Error (OE) models and linear regression schemes based on orthonormal basis functions (ORTFIR) are used. It appears that a combination of the Two-Step method and an ORTFIR model in the first step gives good results.
The results are compared with the currently implemented model, and show considerable improvement in prediction capability. The new model is currently used to redesign the control loop.

1 Introduction

In practice we are often confronted with the problem of closed-loop identification: we need a model of a plant, but the identification experiment cannot take place in open loop; a controller must be present during the experiment. This situation occurs for example when it is dangerous for the environment to let the plant run unattended, or when it is too expensive to run in open loop, because of a loss of yield during the experiment. Consequently, if we want to identify a plant, the model must be built from closed-loop data.

In this paper we will focus on closed-loop identification of a distillation column: given a controlled distillation column, design and perform an identification experiment to identify a model from closed-loop data.
The distillation column and its control loops are discussed in section 2.

In general, the direct application of standard open-loop identification techniques does not give satisfactory models, mainly due to a bias, that is introduced by the correlation between inputs and disturbances [1].

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In the past several identification methods for closed-loop situations have been proposed [1, 6, 8]. Of these methods we will apply the *Direct Identification* (DI) method and the *Two-Step* (TS) method in this paper. DI [1] is very straightforward, since a Prediction Error Method (PEM,[4]) is directly applied to the data, without taking into account the closed-loop operation. An advantage of DI is its simplicity, a disadvantage is that the resulting models are biased in general, and the bias depends on the specific noise contribution [10]. As a consequence, we do not have a tunable expression to control the bias of the model.
The TS method [8] consists of two consecutive open-loop identification steps, in which a PEM is used to obtain estimates of the loop sensitivity function and the plant, respectively. The resulting models can be biased, but the bias does not depend on the noise contribution. Hence we can directly influence the bias of the model by applying appropriate filters, independent of the noise. More about closed-loop identification will be said in section 3, where we will also discuss the different model parametrizations: Output Error (OE,[4]) and Orthogonal Finite Impulse Response (ORTFIR,[2, 9]).

In section 4 we discuss the experiment design. Considerations include the choice of sampling time, input signal and data pretreatment. The results obtained from the measured closed-loop data are presented in section 5. A comparison will be made between the DI, the TS and the current model. Finally we draw some conclusions in section 6.

2 Problem description

The plant under consideration is a controlled two-input two-output distillation column, as depicted schematically in figure 1. The outputs, which are to be controlled, are the impurity of the top-product or distillate y_{tq} and the impurity of the bottom-product or residue y_{bq}. These are measured by analyzers, and expressed in [kg/ton] of the bottom-product, present in the top, and of the top-product, present in the bottom, respectively. Clearly they are a measure of the quality of the separation in the dis-

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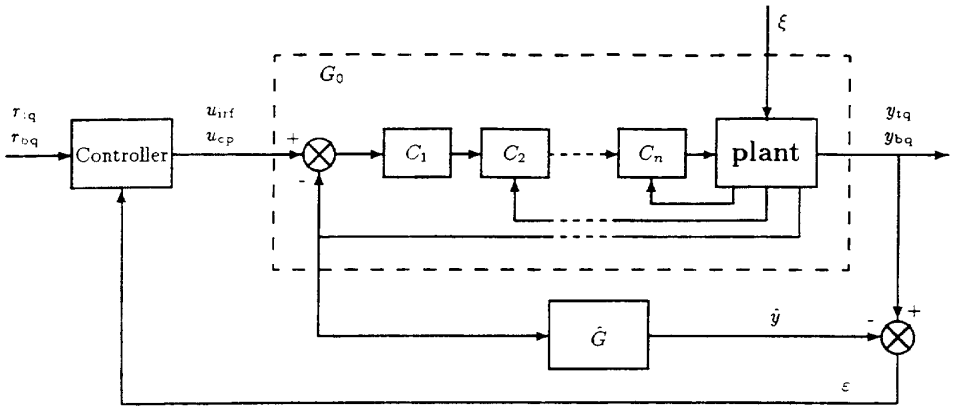

Figure 1: Distillation control loop with quality controller

tillation column.
The reference values r_{tq} and r_{bq} give the desired or target impurities in the distillate and in the residue, respectively. These reference values should be attained with as small a variance as possible. They should not be exceeded.
To control the outputs we have access to two input variables: the internal reflux-to-feed ratio u_{irf} and the cutpoint u_{cp}. The signal u_{irf} is a set point of a ratio-controller, that controls the actual reflux-to-feed ratio. The signal u_{cp} is the set point of a temperature controller, that controls the cutpoint temperature. These, and other, low-level controllers are represented by $C_1, \ldots, C_n$ in figure 1. They are mostly of the PID type.

The plant and the low-level controllers $C_1 \ldots C_n$ are considered to be the system to be identified, denoted by G_0 and the dashed box in figure 1. It is controlled using the Internal Model Control (IMC,[5]) principle.
A model $\hat{G}$ runs parallel to the process. This model predicts the process behaviour. The predicted output $\hat{y}$ is compared to the actual output y, and the *bias* signal $\varepsilon = y - \hat{y}$ is fed back into a controller, containing the DC-gain of the model $\hat{G}$, and several constraints. At present the model consists of four first order SISO transfer functions.
External disturbances influence the process behaviour. Examples are the external feed, which comes from other distillation columns, and various other types of non-controllable inputs, such as feed quality, feed temperature and environment temperature. The disturbances are represented by ξ.
The goal of our study is to provide a model $\hat{G}$, that is a better description of the process G_0, to improve the performance of the outer loop: keep the outputs as close as possible to their reference values, using the internal reflux-to-feed ratio set point and the cutpoint temperature set point as control variables.

For identification we make a few simplifying assumptions. During normal operation the controller can be approximated by a linear controller C. We introduce an excitation signal η, added to the output of the controller. Also, since the reference signals r_{tq} and r_{bq} are constant, they are not relevant for the dynamic behaviour of the loop, and they are therefore neglected. The resulting simplified scheme is shown in figure 2.

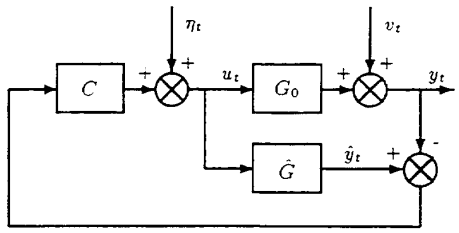

Figure 2: Simplified identification loop

In figure 2 $\eta_t, u_t \in \mathbb{R}^m$, $y_t, \hat{y}_t, v_t \in \mathbb{R}^p$, and C, G_0 and $\hat{G}$ are transfer function matrices of appropriate dimensions, depending on the forward shift operator q: $qu_t = u_{t+1}$. For readability this dependence is omitted in the equations.
Note that in our case $m = p = 2$.
The equations that govern the loop are:

$$\begin{pmatrix} y_t \\ u_t \end{pmatrix} = \begin{bmatrix} G_0 S_0 \\ S_0 \end{bmatrix} \eta_t + \begin{bmatrix} \{I_p - \hat{G}C\}\overline{S}_0 \\ -C\overline{S}_0 \end{bmatrix} v_t \quad (1)$$

where I_p is the $p \times p$-identity matrix, S_0 are the *input loop dynamics*

$$S_0 = \left[I_m + C\{G_0 - \hat{G}\}\right]^{-1} \quad (2)$$

and $\overline{S}_0$ are the *output loop dynamics*

$$\overline{S}_0 = \left[I_p + \{G_0 - \hat{G}\}C\right]^{-1} \quad (3)$$

The controller C is designed on the basis of the model $\hat{G}$. This model should describe the relation between the inputs u and the outputs y. The noise effects are not taken into account in the IMC design. Therefore we will not identify a noise model. Moreover, it is explicitly assumed that the plant is not in the model set, and hence a model is an *approximation* of the true plant.

3 Closed-loop identification

We will use the Direct Identification and the Two-Step methods to obtain models from closed-loop data. Before discussing these methods, we will give a brief review of the most important ingredients of the Prediction Error Method, to define notation. The last part of this section is devoted to the parametrization problem.

The Prediction Error Method (PEM) [4] is a well-known open-loop identification method. Given measurements of a system's inputs u and outputs y, a loss function is minimized that depends on the difference between real outputs y and predicted outputs $\hat{y}$.
Let the model set be parametrized by a parameter vector θ and let an arbitrary model be denoted by $G(\theta)$. We will assume that no noise model is identified, and hence the prediction $\hat{y}$ can be written as

$$\hat{y}_t(\theta) = G(\theta)u_t \tag{4}$$

The parameter estimate $\hat{\theta}$, and accordingly the model $G(\hat{\theta})$, is obtained as

$$\hat{\theta} = \arg\min_{\theta} \frac{1}{N}\sum_{t=1}^{N} \varepsilon_t'(\theta)\varepsilon_t(\theta) \tag{5a}$$

$$\varepsilon_t(\theta) = y_t - \hat{y}_t(\theta) \tag{5b}$$

where $'$ denotes the transpose of a matrix, and N is the number of measurements of inputs and outputs.
It is interesting to see what happens if $N \to \infty$. We then obtain the asymptotic parameter estimate θ_∞ [4]. Using Parseval's relationship, a frequency-domain expression for θ_∞ is obtained:

$$\theta_\infty = \arg\min_{\theta} \operatorname{tr} \int_{-\pi}^{\pi} \Phi_\varepsilon(\omega;\theta)\, d\omega \tag{6}$$

where $\Phi_\varepsilon(\omega;\theta)$ is the spectrum of the prediction error ε, and tr is the trace of a matrix.
For notational convenience we introduce the induced 2-norm $\|X\|_2$ of a system $X(q)$ as the 2-norm of the output of X, when the input is white noise with unit variance:

$$\|X\|_2 \triangleq \left(\operatorname{tr}\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})X^*(e^{j\omega})\, d\omega\right)^{1/2} \tag{7}$$

where * denotes the conjugated transpose of a complex matrix.
Any quasi-stationary signal ε can be written as the result of filtering white noise with unit variance by a filter with transfer function $\Phi_\varepsilon^{1/2}(q)$. Hence, (6) can be rewritten as

$$\theta_\infty = \arg\min_{\theta} \left\|\Phi_\varepsilon^{1/2}(\theta)\right\|_2 \tag{8}$$

A key requirement for PEM to produce meaningful results is that inputs and noise are uncorrelated. Therefore it is an *open-loop* identification method.

When the experiment is carried out in closed loop we can encounter identifiability problems [1]: it is not guaranteed that there is a unique solution of (5a) and (8). By applying an external excitation signal η_t (see figure 2), in general these problems are solved. Moreover, the signal-to-noise ratio is improved, and the loop is guaranteed to be sufficiently excited. However, correlation between inputs and noise cannot be avoided, possibly resulting in biased estimates. Special care must be taken to handle this.

3.1 Direct Identification (DI)

In Direct Identification (DI) the correlation is neglected. The inputs u and outputs y are processed just as if they were obtained in open loop. When a PEM is applied, the asymptotic estimate θ_∞ is given by [10]:

$$\theta_\infty = \arg\min_{\theta} \left\| \{G_0 - G(\theta)\}S_0\Phi_\eta^{1/2} \quad \overline{S}^{-1}(\theta)\overline{S}_0\Phi_v^{1/2} \right\|_2 \tag{9}$$

where $\overline{S}(\theta)$ is defined as

$$\overline{S}(\theta) = \left[I_p + \{G(\theta) - \hat{G}\}C\right]^{-1} \tag{10}$$

The η-dependent term in (9) shows that the difference between G_0 and the model $G(\theta_\infty)$, the bias, is minimized in a frequency-weighted norm. The v-dependent term shows that the result is influenced by the noise term $\Phi_v(\omega)$, in an unknown way. Therefore the bias is not directly tunable.

3.2 Two-Step identification (TS)

A method that does not have this disadvantage is the Two-Step method (TS) [8]. It is based on the definition of a signal u_t^η by rewriting (1) as

$$u_t^\eta = S_0\eta_t \tag{11a}$$

$$u_t = u_t^\eta - S_0 C v_t \tag{11b}$$

$$y_t = G_0 u_t^\eta + [I - G_0 S_0 C]v_t \tag{11c}$$

The signal u_t^η is the noise-free part of u_t, and is therefore uncorrelated with the noise v_t. Hence (11c) shows that if we have u_t^η available, a model $\hat{G}$ of G_0 can be obtained by standard open-loop techniques, such as PEM.
How do we obtain u_t^η? From (11a)–(11b) it follows that this can be done by estimating a model $S(\hat{\beta})$ of S_0 by a PEM, with η_t as input and u_t as output, since η and the noise v are uncorrelated. The asymptotic estimate β_∞ is then given by

$$\beta_\infty = \arg\min_{\beta} \left\|(S_0 - S(\beta))\Phi_\eta^{1/2}\right\|_2 \tag{12}$$

We then reconstruct u_t^η according to

$$\hat{u}_t^\eta = S(\hat{\beta})\eta_t \tag{13}$$

In the second step we identify $\hat{\theta}$ from $\{y(t), \hat{u}^\eta(t)\}_N$. The asymptotic estimate θ_∞ is given by

$$\theta_\infty = \arg\min_{\theta} \left\|(G_0 S_0 - G(\theta)S(\beta_\infty))\Phi_\eta^{1/2}\right\|_2 \tag{14}$$

Clearly, as opposed to DI, the asymptotic model $G(\theta_\infty)$ is independent of the noise. If the error $S_0 - S(\hat{\beta})$ is sufficiently small, the bias $G_0 - G(\hat{\theta})$ is directly influenced by Φ_η, and possible weighting filters. We thus have a tunable expression for the bias distribution.

3.3 Parametrization

So far we have not discussed the parametrization of G and S (in the TS method). We will not identify noise models, so possible parametrizations are Output Error (OE) and Finite Impulse Response (FIR).

An OE-model is nonlinear in the parameters, so (5a) needs a nonlinear optimization procedure. The solution can then be a local minimum. A FIR-model is linear in the parameters, so (5a) has a unique global minimum, which can be calculated analytically. However, a very large number of parameters M needs to be estimated (typically around 50). For chemical processes we do not have a large number of measurements N. Since the variance of the estimated parameters is proportional to M/N, this implies that the variance of the estimated FIR-parameters will be high. This will not be the case if we use *orthonormal basis functions* as a basis in our FIR description [2, 9]. It is this parametrization that we are going to use in the estimation of the loop dynamics.

A classical FIR parametrization of S is a series expansion of $S(z;b)$:

$$S(z;b) = \sum_{k=1}^{M} b_k z^{-k} \tag{15}$$

where $M \in \mathbb{N}$ is large, and where z^{-k} are the *basis functions*.
In [2, 9] orthogonal basis functions $V_k(z)$ are used, containing dynamics. The series expansion then becomes:

$$S(z;\beta) = \sum_{k=1}^{d} \beta_k V_k(z) \tag{16}$$

The basis functions $V_k(z)$ are generated by a so-called *basis generating system*. It has been shown in [3] that if the dynamics of the basis generating system approach the dynamics of the plant to be modelled, the convergence rate of (16) becomes very high. As a consequence, the number of parameters d that need to be identified is smaller than M, resulting in a smaller variance of the estimated parameters. For a more detailed discussion of the orthogonal FIR representation we refer to [2, 3, 9].

We will use the standard and the orthogonal FIR (ORTFIR) parametrization in the first step of TS. In the second step of TS and in DI we will use an OE parametrization to estimate $G(\hat{\theta})$.

4 Experiment design

In the identification experiment design phase we choose the sample time for data acquisition, the sample time for identification, the input signals and the experiment length. Currently the controller calculates a new control action every T_c seconds. The output impurity is determined by analyzers, that give a sample every T_o seconds. Since there is no synchronization in the loop, the model, that runs parallel to the process, is updated every T_s seconds ($T_s < T_c < T_o$). These sample times cannot be chosen freely, since they are induced by the process.

Several free-run experiments (no external excitation) are done to determine the frequency contents of the signals. It is decided to choose a sample time for data acquisition equal to T_s. To have the model run at the same rate as the analyzers, a model will be estimated with a sample time for identification of T_o. This immediately shows some of the data preprocessing necessary: decimation of the measured signals, i.e. filtering the data through an anti-aliasing filter, and taking every T_o/T_s sample.

The model is built with a sample time of T_o, and hence the input signal η_t should be based on this sample time. Because of the easy tuning, we choose two independent Generalized Binary Noise (GBN) [7] sequences as input signals. A GBN signal has a basic switching time T_{sw}, where it switches between two values with a *non-switching probability* p. This p is the tuning parameter. Choosing p large will result in a larger spectrum at low frequencies. Choosing p small results in a larger spectrum at high frequencies. Note that if $p = 0.5$ we obtain a Pseudo Random Binary Noise (PRBN) sequence.
In [7] some rules-of-thumb are given for choosing p. For our distillation column we choose $p = 0.82$ for η_{irf} and $p = 0.87$ for η_{cp}, with a basic switching time (shortest period between two switches) of T_o, to match the sample time with which the model is built.

The experiment length is approximately 48 hours, resulting in a data set of $N = 360$ samples after decimation.

Besides decimating the data set, the following data preprocessing is performed:

- the *outliers* are removed;
- the *mean values* of the signals are subtracted;
- the signals are high-pass filtered to remove low-frequency trends;
- the signals are *scaled* to obtain equal variance of 1 for all signals.

With the preprocessed data set we start the identification of models of G_0.

5 Experimental results

5.1 Time delay estimation

First we estimate the time delays of the plant. Note that these are the same as the time delays from the excitation signals to the outputs. The estimated time delays are given in table 5.1. They are incorporated in the models.

	from u_{irf}	from u_{cp}
to y_{tq}	2	4
to y_{bq}	2	6

Table 1: Estimated time delays from inputs to outputs

5.2 Identification of loop dynamics

First we estimate a model $\hat{S}$ of the loop dynamics from the data set $\{u_t, \eta_t\}$. A non-parametric spectral estimate $\hat{S}_{\text{spec}}$ is obtained as

$$\hat{S}_{\text{spec}} = \Phi_{u\eta}\Phi_{\eta}^{-1}$$

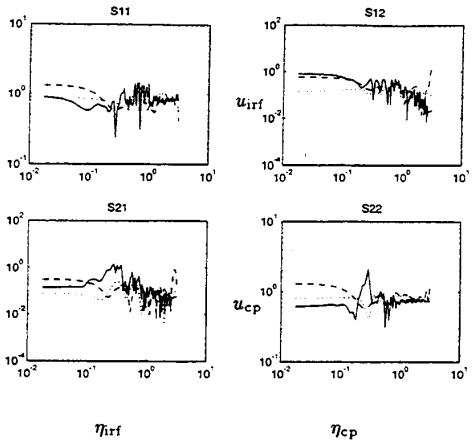

Figure 3: Amplitude vs. frequency of the loop dynamics $\hat{S}$: spectral estimate (solid), FIR estimate (dashed) and ORTFIR estimate (dotted)

In figure 3 the solid lines are the amplitude of $\hat{S}_{\text{spec}}$ of the different transfers from inputs η to outputs u. This estimate is used to compare the parametric estimates with: a MIMO FIR model (dashed lines) and a MIMO ORTFIR model (dotted lines).

From this figure we conclude that the ORTFIR model has smaller variance than the FIR model, as expected, because of the smaller number of parameters. Therefore we calculate $\hat{u}^{\eta}{}_t$ (13) with the ORTFIR model.

Note that for low frequencies the spectral estimate has a small number of points, and a high variance. Therefore a qualitative judgement of the models for low frequencies has little significance.

5.3 Identification of plant model

A plant model is obtained using three techniques. For comparison, a spectral estimate $\hat{G}_{\text{spec}}$ is calculated as

$$\hat{G}_{\text{spec}} = \Phi_{y\eta}\Phi_{u\eta}^{-1} \tag{17}$$

The solid lines in figure 4 represent the amplitude of the spectral estimate. Again the low-frequency part has little significance.
Parametric models are obtained with DI and TS. The DI-model $\hat{G}_{\text{DI}}$ is given by the dashed lines in figure 4. The dotted lines correspond to the TS-model $\hat{G}_{\text{TS}}$. From this figure we conclude that the model $\hat{G}_{\text{TS}}$ is closer to the spectral estimate than $\hat{G}_{\text{DI}}$ in the region of the dynamics of the plant.

5.4 Model evaluation

The (scaled) step responses of the different models are shown in figure 5, where the solid lines correspond to the model that is currently used, the dashed lines correspond to the DI model, and the dotted lines are the TS model. From this figure we conclude that the current model does not contain enough dynamics; there are higher order dynamics that need to be taken into account. The DC-gain

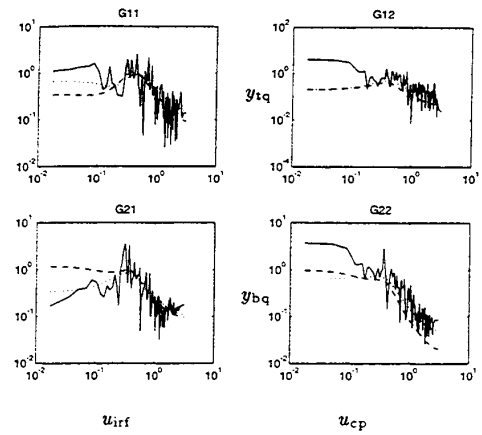

Figure 4: Amplitude vs. frequency of the model $\hat{G}$: spectral estimate (solid), DI model (dashed) and TS model (dotted)

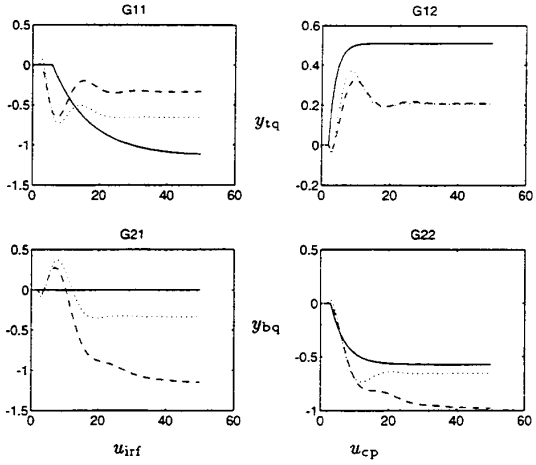

Figure 5: Step responses of the model $\hat{G}$: current model (solid), DI model (dashed) and TS model (dotted)

of the TS-model $\hat{G}_{\text{TS}}$ corresponds to values, obtained from other experiments. Therefore the TS-model seems reasonable, and is preferred to the DI-model $\hat{G}_{\text{DI}}$.

Finally, we compare the measured outputs with the model responses in figure 6. The upper graph corresponds to y_{tq}, and the lower graph corresponds to y_{bq}. The solid lines are the measured outputs, the dashed lines correspond to the DI-model, and the dotted lines correspond to the TS-model. We can hardly see any difference between the two models, although we have seen that there is a considerable difference in their frequency plots (figure 4) and their step responses (figure 5). The measured outputs are matched reasonably well by both the DI- and the TS-model.
The dash-dotted lines in figure 6 correspond to the currently implemented model. Clearly this model is not very good, and a considerable improvement can be expected when using the TS-model for control design, since this model is a better approximation of the real plant.

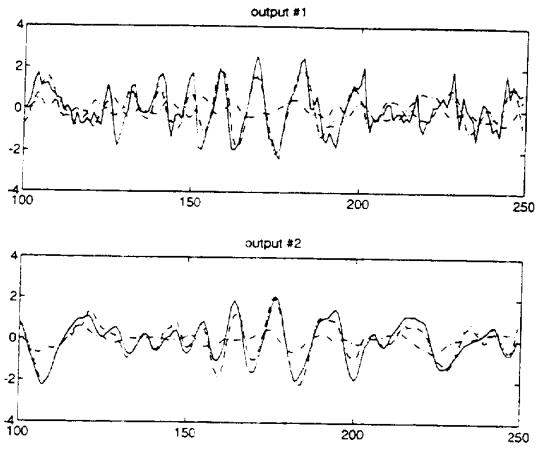

Figure 6: Measured outputs (solid) and predicted outputs with DI model (dashed), TS model (dotted) and current model (dash-dotted)

6 Conclusions

In this paper we have discussed a closed-loop identification experiment. Two closed-loop identification methods, Direct Identification (DI) and the Two-Step (TS) method, were discussed, and applied to closed-loop data of a distillation column.

The TS-model is better than the DI-model, following from both a frequency-domain and a time-domain (step response) analysis. It appears advantageous to use an OR-TFIR parametrization in the first step of the TS method, rather than a FIR parametrization, to obtain an accurate estimate of the loop dynamics.

The results were compared to the currently implemented model, and it appeared that a considerable improvement is made in the prediction capabilities of the model. At present, the insights gained in the experiment are used to redesign the control loop.

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