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Control relevant identification of a compact disc pick–up mechanism[‡]

Raymond A. de Callafon[§♯], **Paul M.J. Van den Hof** [♯] **and Maarten Steinbuch**[♭]

[♯]*Mechanical Engineering Systems and Control Group*
Delft University of Technology, Mekelweg 2, 2628 CD Delft, The Netherlands.
[♭]*Philips Research Laboratories, P.O. Box 80.000, 5600 JA Eindhoven, The Netherlands.*

Abstract. This paper discusses the control relevant parametric identification of a servo system present in a Compact Disc player. In this application an approximate closed loop identification problem is solved in order to come up with a linear multivariable discrete time model, suitable for control design. This identification problem is handled by a recently introduced two stage method. It yields an explicit and tunable expression for the bias distribution of the model being estimated, clearly showing the dynamics of the closed loop system in the (asymptotic) approximation criterion. This result will be exploited to identify the model in a control relevant way by additional data filtering. The recently introduced method in de Vries and Van den Hof (1993) for model uncertainty quantification is used to construct an upper bound for the corresponding model error.

Keywords. compact disc player; closed loop identification; two stage method; control relevant identification; model uncertainty.

1 Introduction

Compact Disc players use an optical decoding device to reproduce high quality audio from a digitally coded signal, recorded as a spiral track on a reflective disc, see also Bouwhuis *et al.* (1985). An increasing amount of equivalent optical devices will be used in portable applications, having severe shock disturbances. The track following properties of a CD player, operating in these conditions, could be improved by designing an enhanced multivariable controller. The intention of this paper is to estimate a (nominal) multivariable FDLTI (Finite Dimensional Linear Time Invariant) dynamical model, obtained from closed loop experiments, which can be used for control design. Additionally, the procedure presented in de Vries and Van den Hof (1993) is used for a quantification of the resulting model error by estimating a non–parametric additive model uncertainty.

There is a growing interest in merging the problems of control design and identification. On the one hand this is caused by the fact that from a robust control design point of view we require expressions for model uncertainty that have to be used in robust control design procedures. On the other hand the (nominal) models used to design control systems very often will have to be gathered by experimental methods.

Practically it is impossible to exactly characterize all phenomena that describe the dynamical behaviour of a physical system and the corresponding models will necessarily be approximative. Furthermore, control design methods can get unmanageable if they are applied to models of high complexity. Since the validity of any approximate model hinges on its intended use, the identification procedure being applied will be subjected to several requirements, in order to provide estimated models that are suitable for control design. These considerations have resulted in the statement that the best model for control design cannot be derived from

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open loop experiments alone, Bitmead *et al.* (1990), Schrama (1990).

A control relevant identification requires that the relevant dynamical behaviour of the system is estimated while it operates in a closed loop configuration with the controller to be designed. Since the controller obtained from the control design is (yet) unknown, this will generally lead to an iterative scheme of identification and control design, using the controller of step $i-1$ to estimate a model for step i. This has led to study several different types of iterative schemes of identification and control design, see Hakvoort *et al.* (1992), Lee *et al.* (1992), Liu and Skelton (1990), Schrama (1992), Schrama and Van den Hof (1992), Zang *et al.* (1991).

In this paper we concentrate on one identification step in such an iterative procedure. Within the framework of prediction error identification (Ljung (1987)) we will identify a multivariable control relevant approximate model, employing a number of recently introduced methods. An indirect (two-stage) method (Van den Hof and Schrama (1993)) will be employed to perform the approximate closed loop identification. The basic advantage of this approach is that an overall approximate identification results, in which the asymptotic bias distribution of the identified model becomes an explicit and tunable expression that is independent of the (unknown) noise disturbance on the data. Additional data filtering is applied to tune the approximation criterion to become a control relevant criterion.

The outline of this paper is as follows. First a concise description of the Compact Disc pick–up mechanism and the experimental set up is given in section 2. Next some preliminary notation is discussed. In section 4 we pay attention to the specific two-stage identification procedure, while in section 5 we discuss the use of orthonormal basis functions that are employed in the first stage of the procedure. Next, the control relevance of the identification approach is given attention and in section 6 we will present the experimental results.

2 Compact Disc Mechanism

The CD mechanism consists of a turn table DC–motor for the rotation of the Compact Disc and a radial arm in order to follow the track of the disc. Furthermore, an OPU (Optical Pick–up Unit) is mounted on the end of the balanced radial arm to read the digitally coded signal, recorded on the disc. Schematically the CD mechanism is given in figure 1.

A diode generates a laser beam that passes through a series of optical lenses in the OPU to give a spot on the disc surface. The light reflected from the disc is measured on an array of photo diodes,

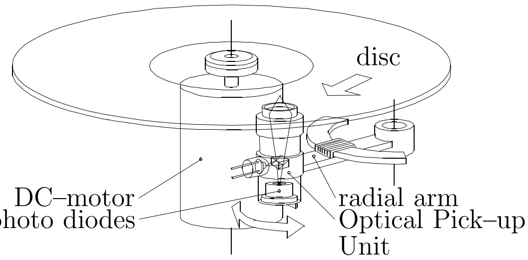

Fig. 1: Schematic view of CD mechanism

mounted in the bottom of the OPU, yielding the signals required for position error information of the laser spot on the Compact Disc, see also Draijer *et al.* (1992).

Following the track on the Compact Disc involves basically two control loops. First a radial control loop using a permanent magnet/coil system mounted on the radial arm, in order to position the laser spot in the direction orthogonal to the track. Secondly a focus control loop using an objective lens suspended by two parallel leaf springs and a permanent magnet/coil system, with the coil mounted in the top of the OPU to focus the laser spot on the disc. In the present configuration, both the radial and focus control loops have been realized by a SISO (Single Input Single Output) controller, which consists of a lead–lag element and a proportional and integrating action. The closed loop bandwidth is approximately 500 Hz, which is a compromise between several conflicting factors, see Draijer *et al.* (1992) and Steinbuch *et al.* (1992).

In figure 2 a block diagram of the two control loops is shown. In here $P_a(q)$ denotes the transfer function of radial and focus actuator, C_{opu} the OPU, $C(q)$ the controller and $P_0(q) = -C_{opu}P_a(q)$. The variable q is the forward shift operator, yielding $x(t+1) = qx(t)$.

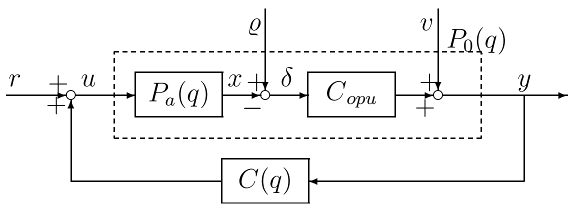

Fig. 2: Block diagram of the Compact Disc mechanism

The signals have the following interpretation. The spot position error $\delta(t)$, which is the difference between the track position $\varrho(t)$ and actuator position $x(t)$ in radial and focus direction, generates a (disturbed) error signal $y(t)$ via C_{opu}. This error signal $y(t)$ is led into the controller $C(q)$ and feeds the system $P_a(q)$ with the input $u(t)$. The signal $v(t)$ reflects the disturbance on the error signal $y(t)$.

The absolute track position $\varrho(t)$ and actuator position $x(t)$ cannot be measured directly and used for identification. Only the error signal $y(t)$ and the input $u(t)$ are available. Therefore an additional and known reference signal $r(t)$, uncorrelated with the additive noise $v(t)$ will be injected into the control loops, as illustrated in figure 2.

3 Preliminaries

Given figure 2, the system $P_0(q)$ will be described by the following FDLTI data generating system $\mathbf{S}$ throughout this paper.

$$\mathbf{S}: \begin{array}{rcl} y(t) &=& P_0(q)u(t) + H_0(q)e(t) \\ u(t) &=& r(t) + C(q)y(t) \end{array} \tag{1}$$

In (1) the disturbance $v(t) + C_{opu}\varrho(t)$ is described by a filtered white noise signal $H_0(q)e(t)$. Using the input sensitivity $S_0(q)$ and output sensitivity $W_0(q)$ of the closed loop system

$$\begin{array}{rcl} S_0(q) &=& [I - C(q)P_0(q)]^{-1} \\ W_0(q) &=& [I - P_0(q)C(q)]^{-1} \end{array} \tag{2}$$

we can rewrite (1) into the following equations.

$$u(t) = S_0(q)r(t) + C(q)W_0(q)H_0(q)e(t) \tag{3}$$
$$y(t) = P_0(q)S_0(q)r(t) + W_0(q)H_0(q)e(t) \tag{4}$$

Throughout this paper we will consider model sets $\mathcal{M}$ that are parametrized in an OE (Output Error) structure, Ljung (1987). For a general input/output system with input u and output y this model structure is reflected by the equation:

$$\mathcal{M}: \quad y(t) = P(q,\rho)u(t) + \varepsilon(t), \quad \rho \in D_{\mathcal{M}} \tag{5}$$

where $\varepsilon(t)$ is the one step ahead prediction error. The parameter ρ will be estimated by employing a least squares criterion, see also Ljung (1987),

$$\begin{array}{rcl} \hat{\rho} &=& \arg\min\limits_{\rho} V_N(\rho, Z^N), \quad \rho \in D_{\mathcal{M}} \\ V_N(\rho, Z^N) &=& \dfrac{1}{2N} \sum\limits_{t=0}^{N-1} \mathrm{tr}\left\{\varepsilon_l^T(t,\rho)\, Q\, \varepsilon_l(t,\rho)\right\} \\ \varepsilon_l(t,\rho) &=& L(q)\varepsilon(t,\rho) \end{array} \tag{6}$$

where Q is a symmetric weighting matrix, Z^N reflects the observed data of length N and $L(q)$ is an additional filter on the prediction error $\varepsilon(t,\rho)$.

4 Two Stage Method

The major problem arising from an approximate identification using closed loop experiments, is the correlation of the additive noise with the input of the system, see also figure 2. Most important in identification for control design is to estimate $P_0(q)$ given in (1). Furthermore, an explicit expression of the approximation of $P_0(q)$ is needed, to tune the bias distribution of the model $P(q,\hat{\rho}_N)$ being estimated in a feedback relevant way. The method to handle the closed loop situation in this paper, is based on the two stage identification method given in Van den Hof and Schrama (1993). The two steps are recapitulated in the following.

The external reference signal $r(t)$ given in (3) is uncorrelated with the additive noise $v(t)$ acting on the closed loop system. By using an OE model structure, similar as in (5)

$$u(t) = S(q,\alpha)r(t) + \varepsilon(t) \tag{7}$$

and the least squares criterion given in (6) to estimate α, it is possible to identify the input sensitivity $S_0(q)$ in an open loop way. In this step we take $L(q) = 1$. This is the *first* step in the two stage identification strategy. It is even possible to consistently estimate $S_0(q)$, provided a sufficiently high model order has been selected.

Given the estimate $S(q,\hat{\alpha}_N)$ of the input sensitivity $S_0(q)$, a noise free input signal $\hat{u}_r(t)$ can be simulated from the observations of the reference signal $r(t)$.

$$\hat{u}_r(t) = S(q,\hat{\alpha}_N)r(t) \tag{8}$$

which in the *second* step of the procedure is employed, again using an OE model structure

$$y(t) = P(q,\rho)\hat{u}_r(t) + \varepsilon(t) \tag{9}$$

and the least squares criterion given in (6) to estimate the parameter $\hat{\rho}_N$ in $P(q,\hat{\rho}_N)$.

A result for the asymptotic bias distribution of the estimate $P(q,\hat{\rho}_N)$ in the SISO case is given in the following theorem (Van den Hof and Schrama (1993)):

Theorem 4.1 *Consider the two-stage identification discussed above, resulting in a parameter estimate $\hat{\rho}_N$. Then, under weak conditions,*

$$\begin{array}{c} \hat{\rho}_N \to \rho^\star = \arg\min\limits_{\rho} \dfrac{1}{4\pi}\displaystyle\int_{-\pi}^{\pi} \Big|[P_0(e^{i\omega}) - P(e^{i\omega},\rho)]\cdot \\ \cdot S_0(e^{i\omega}) + P(e^{i\omega},\rho)[S_0(e^{i\omega}) - S(e^{i\omega},\alpha^\star)]\Big|^2 \cdot \\ \cdot \Phi_r(\omega)|L(e^{i\omega})|^2\, d\omega, \quad w.p.\ 1\ as\ N \to \infty \end{array} \tag{10}$$

and

$$\alpha^\star = \arg\min_{\alpha} \frac{1}{4\pi}\int_{-\pi}^{\pi} |S_0(e^{i\omega}) - S(e^{i\omega},\alpha)|^2 \Phi_r(\omega)\, d\omega \tag{11}$$

where $L(q)$ denotes the filter on the prediction error $\varepsilon(t)$, used in the second step and $\Phi_r(\omega)$ denotes the (auto)spectrum of the reference signal $r(t)$.

The frequency representation (10) in theorem 4.1 shows the influence of a model error in the estimated sensitivity function on the final result of the identification. If in the first step of the procedure a very accurate (high order) model of the sensitivity function is identified, then the second term in the integrand expression in (10) will vanish. For the multivariable case, this will result in the following expression, where $\Delta P(e^{i\omega},\rho)$ is used to denote the difference $P_0(e^{i\omega}) - P(e^{i\omega},\rho)$.

$$\begin{aligned} \rho^\star = \arg\min_{\rho} \frac{1}{4\pi}\int_{-\pi}^{\pi} & \mathrm{tr}\{L(e^{-i\omega})^T Q L(e^{i\omega})\cdot \\ & \cdot \Delta P(e^{i\omega},\rho) S_0(e^{i\omega}) \Phi_r(\omega) \cdot \\ & \cdot S_0(e^{-i\omega})^T \Delta P(e^{-i\omega},\rho)^T\}\, d\omega \end{aligned} \tag{12}$$

Clearly, (12) is an explicit and tunable expression for the bias distribution of the asymptotic model $P(q,\rho^*)$. In this expression the prediction error filter $L(q)$, the input spectrum $\Phi_r(\omega)$ and the weighting matrix Q can been seen as design variables, see also Hakvoort *et al.* (1992) and Wahlberg and Ljung (1986). Therefore, we define the design variables $\mathcal{D}_c$ to be:

$$\mathcal{D}_c \overset{\text{def}}{=} \{L(q), \Phi_r(\omega), Q\}. \tag{13}$$

The usage of the design variables $\mathcal{D}_c$ will be scrutinized in section 6.

5 Linear Regression using Orthonormal Basis Functions

In the first step of the identification procedure we need an output error type algorithm in order to arrive at the results as presented in theorem 4.1. Moreover the identified sensitivity $S(q,\hat{\alpha}_N)$ has to be very accurate, which asks for high model orders to be applied. Since OE model structures in general require non-linear optimization algorithms to solve the least squares identification problem given in (6), high model orders are very unattractive from a computational point of view. Moreover the occurrence of local minima in the optimization may heavily influence the parameter estimate that is obtained.

In our procedure we will apply a linear regression identification that also has an output error structure, and that exploits the recently obtained results on system-based orthonormal basis functions as presented in Heuberger (1990) and Heuberger *et al.* (1992). This model structure is given by:

$$\varepsilon(t,\alpha) = u(t) - \sum_{k=0}^{n} L_k(\alpha) V_k(q) r(t-1) \tag{14}$$

where $\{L_k(\alpha)\}_{k=1,\cdots,n}$ is a sequence of expansion coefficients of the parametrized model of the sensitivity function $S(q,\alpha)$ with respect to the basis functions $\{V_k(z)\}_{k=1,\cdots,\infty}$. It is based on the fact that any stable, strictly proper FDLTI system $S(z)$ has a unique expansion

$$S(z) = \sum_{k=0}^{\infty} L_k V_k(z) \tag{15}$$

In the case $V_k(z) = z^{-k}$, this model structure matches a Finite Impulse Representation (FIR), while in that case L_k represent the impulse response coefficients of the model.

By choosing appropriate basis functions $V_k(z)$, the convergence rate of a series expansion as in (15) can become very fast, which means that a very accurate model can be identified by only incorporating a restricted number of coefficients $L_k(\alpha)$.

In Heuberger (1990), Heuberger *et al.* (1992) it is shown how dynamical systems themselves can induce orthonormal basis functions $V_k(z)$, pointing to an iterative scheme of identifying expansion coefficients and rebuilding basis functions. In our application we have iteratively constructed such basis functions that were found from the estimated model in the previous iteration step. For more details the reader is referred to the references.

6 Control Relevant Identification

6.1 Finding the right weight

The validity of any approximate model hinges on its intended use and therefore the identification procedure being applied will be subjected to several requirements to estimate a model suitable for control design. Since the "quality" of a model actually is dependent on the controller that is designed on the basis of the model, this future controller actually should be incorporated in a control relevant identification criterion.

Since the controller obtained from the control design is (yet) unknown, a minimization of the model error using the *current* feedback, provided by the present controller, is generally used to estimate a model for subsequent control design. In the literature a number of many techniques can be found to perform such an identification, see for example Bitmead *et al.* (1990), Hakvoort *et al.* (1992), Liu and Skelton (1990), Schrama (1992). In this paper a 2–norm minimization will be used, see (6), which is related to a LQG control paradigm, see also Hakvoort *et al.* (1992), Zang *et al.* (1991).

The (input) sensitivity $S_0(q)$ given in (2) is found to be of considerable importance in posing performance requirements of the closed loop system. The sensitivity, based on the (nominal) model $P(q,\hat{\rho}_N)$ being estimated will be denoted as

$$S(q,\hat{\rho}_N) = [I - C(q)P(q,\hat{\rho}_N)]^{-1} \tag{16}$$

Clearly, the difference between the sensitivities $S_0(q)$ and $S(q,\hat{\rho}_N)$ reflects a *feedback–relevant mismatch*, caused by the difference between the nominal model $P(q,\hat{\rho}_N)$ and the system $P_0(q)$. Considering any norm or distance function $\|\cdot\|$ and applying the triangle inequality to $\|S_0(q) - S(q,\hat{\rho}_N)\|$ yields:

$$\|S_0(q)\| \;\leq\; \|S(q,\hat{\rho}_N)\| + \|S_0(q) - S(q,\hat{\rho}_N)\| \tag{17}$$

$$\|S_0(q)\| \;\geq\; \Big| \|S(q,\hat{\rho}_N)\| - \|S_0(q) - S(q,\hat{\rho}_N)\| \Big| \tag{18}$$

From (17) and (18) we see that by posing the following requirement

$$\|S_0(q) - S(q,\hat{\rho}_N)\| \ll \|S(q,\hat{\rho}_N)\| \tag{19}$$

similar performances of the controller $C(q)$ applied to the model $P(q,\hat{\rho}_N)$ and the system $P_0(q)$ can be derived, see also (Schrama (1992)). Therefore, minimizing the difference $\|S_0(q) - S(q,\rho)\|$ on the basis of measurement data can be seen as a control relevant identification. By rewriting the difference between $S_0(q)$ and $S(q,\hat{\rho}_N)$, omitting the use of the forward shift operator q for ease of notation, we may write

$$\begin{gathered} \|[I - CP_0]^{-1} - [I - CP(\rho)]^{-1}\| = \\ \|[I - CP(\rho)]^{-1} C \,[P_0 - P(\rho)]\, [I - CP_0]^{-1}\| \end{gathered} \tag{20}$$

From (20) it can been seen that minimizing the difference between $S_0(q)$ and $S(q,\hat{\rho}_N)$ is equal to a weighted norm applied to $[P_0(q) - P(q,\rho)]$, where $S_0(q)$ is used as input weighting and $S(q,\rho)C(q)$ as an output weighting. By replacing the norm operator $\|\cdot\|$ in (20) by the H_2–norm, see (Maciejowski (1989), pp. 99), the difference term in (20) matches the following closed loop performance criterion $J_c(\lambda)$

$$\begin{gathered} J_c(\lambda) \overset{\text{def}}{=} \frac{1}{4\pi}\int_{-\pi}^{\pi} \mathrm{tr}\{[S(e^{-i\omega},\rho)C(e^{-i\omega})]^T \cdot \\ \cdot [S(e^{i\omega},\rho)C(e^{i\omega})][P_0(e^{i\omega}) - P(e^{i\omega},\rho)] \cdot \\ \cdot S_0(e^{i\omega})S_0(e^{-i\omega})^T [P_0(e^{-i\omega}) - P(e^{-i\omega},\rho)]^T\}\, d\omega \end{gathered} \tag{21}$$

The way this minimization will be carried out for the identification of the Compact Disc pick–up mechanism, is discussed in the following section.

6.2 Prefiltering

The weighted minimization of $\|P_0(q) - P(q,\rho)\|_2$ given in (21) can be accomplished during the identification, by modifying the design variables $\mathcal{D}_c$ given in (13). The prediction error filter $L(q)$, the symmetric weighting matrix Q and the spectrum Φ_r can be exploited to 'shape' the model $P(q,\hat{\rho}_N)$ being estimated in the approximate identification. To achieve a minimization of the closed loop performance criterion given in (21) the design variables have to be chosen as follows.

Proposition 6.1 *Given a consistent estimate of the input sensitivity $S_0(q) = [I - C(q)P_0(q)]^{-1}$ used to simulate the noise free input $\hat{u}_r(t)$ given in (8), then with the choice of the design variables,*

$$\mathcal{D}_c = \left\{ \begin{aligned} L(q,\rho) \;&=\; [I - C(q)P(q,\rho)]^{-1}C(q) \\ \Phi_r(\omega) \;&=\; c_1 I \\ Q \;&=\; c_2 I \end{aligned} \right.$$

where c_1, c_2 are arbitrarily chosen real constants, the least squares criterion given in (6) will converge to the closed loop performance criterion defined in (21), under weak conditions as $N \to \infty$.

A proof of proposition 6.1 can be found in Hakvoort *et al.* (1992), since basically an equivalent closed loop performance criterion is used in this paper. The choice of the design variables given in proposition 6.1 can also be seen directly, by comparing a constant c_1c_2 times the closed loop performance criterion defined in (21) with the equivalent frequency domain representation of the least square identification algorithm given in (12).

Clearly, the consistent estimate of the input sensitivity used to simulate the noise free input signal $\hat{u}_r(t)$, given in (8) can be a strong requirement. An approximate identification of $S_0(q)$ can lead to a biased closed loop performance criterion, see theorem 4.1. As stated before, linear regression models using system based orthonormal functions are used to model the input sensitivity and can be used to substantially reduce this effect.

Furthermore, the following notes on proposition 6.1 should be given.

- Firstly, it should be noted that the input weighting with the 'real' sensitivity $S_0(q)$ can *only* be achieved when performing closed loop experiments. Note that this weighting factor is already present in the asymptotic identification criterion (12).

- Instead of $L(q,\rho) = [I - C(q)P(q,\rho)]C(q)^{-1}$ given in proposition 6.1, a fixed filter will generally be used to filter the prediction error, as to avoid very complicatedly parametrized nonlinear optimization problems. An iterative scheme using the model $P(q, \hat{\rho}_N)$ from step $i-1$, for constructing a filter $L(q, \hat{\rho}_N)$ used in step i to filter the prediction error can be used to overcome this problem. The control relevant model $P(q, \hat{\rho}_N)$ and the matching filter $L(q, \hat{\rho}_N)$ will be found when the iterative scheme converges.

- Finally it should be noted that the iterative scheme mentioned above, is performed in a SISO configuration. In this way the filtering of the prediction error $\varepsilon(t)$ can be replaced by filtering the input and output of the system to be identified.

7 Application to the CD Player

7.1 Data acquisition

Measurements of the CD mechanism have been obtained from an experimental set up of a Compact Disc player at Philips' Research Laboratories. This experimental set up is used to gather time sequences of $r(t)$, $u(t)$ and $y(t)$, see figure 2, in radial and focus control loops simultaneously. Matching software is used to control the sample frequency, anti aliasing filter, data storage and input generation.

The signals have been sampled at 25 kHz and the reference signal $r(t)$ injected in the closed loop was chosen to be a white noise signal, to fulfil the choice of the second design variable $\Phi_r(\omega)$ given in proposition 6.1. The white noise reference signal was chosen to be bandlimited in the frequency domain of interest (100 Hz – 10 kHz). A 5^{th} order Butterworth filter, with a cut off frequency at 9.5 kHz was used to reduce the effects of aliasing.

The two-stage identification procedure previously discussed is applied to this experimental data. Furthermore, a non parametric estimate of the input sensitivity $S_0(\omega)$ and the system $P_0(\omega)$ is obtained by a spectral analysis (Priestly (1981)) and given by

$$\hat{S}^T(\omega) = \hat{\Phi}_r(\omega)^{-1}\hat{\Phi}_{ru}(\omega), \ \det\{\hat{\Phi}_r(\omega)\} \not\equiv 0 \quad (22)$$

$$\hat{P}^T(\omega) = \hat{\Phi}_{ru}(\omega)^{-1}\hat{\Phi}_{ry}(\omega), \ \det\{\hat{\Phi}_{ru}(\omega)\} \not\equiv 0. \quad (23)$$

The estimates of the spectra in (22) and (23) have been carried out by using 100 averages over 409600 time samples. The results will be used only as a (additional) validation tool for the parametric models $S(q, \hat{\alpha}_N)$ and $P(q, \hat{\rho}_N)$ being estimated, which is based only on 2000 time samples.

7.2 Estimate of sensitivity function

As mentioned in section 5, a linear regression scheme based on orthonormal functions has been used. Firstly, a relatively rough (low order) estimate is computed by a multivariable Output Error minimization using the DUMSI[1]–package. Secondly, an iterative scheme using the model from step $i-1$ for constructing a set of orthonormal functions $V_k(z)$ used in step i will be utilized. The results of this identification procedure can be found in figure 3 and 4. The model $S(q, \hat{\alpha}_N)$ is constructed by estimating 4 coefficients $L_k(\alpha)$ based on an 12-th order model inducing the basis functions. This results in a model with state space dimension 48.

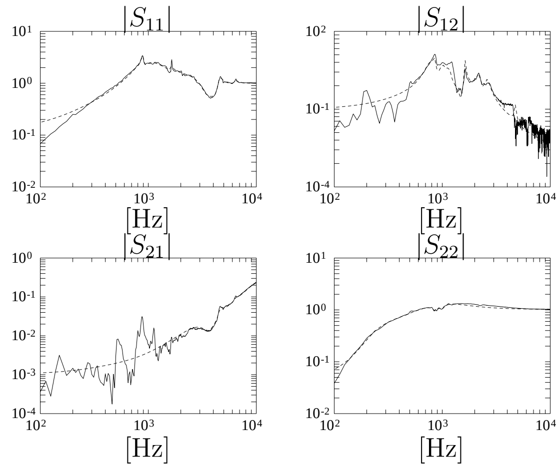

Fig. 3: Amplitude of spectral estimate $\hat{S}(\omega)$ (—) and parametric model $S(e^{i\omega}, \hat{\alpha}_N)$ (- -)

Figure 3 presents the amplitude plots of the spectral estimate $\hat{S}(\omega)$ and the parametric model $S(e^{i\omega}, \hat{\alpha}_N)$. The input sensitivity has been estimated reasonably well, which has been emphasized by comparing a part of the simulation of the input $\hat{u}_r(t)$ and the actual input $u(t)$ measured in closed loop, given in figure 4. This data is taken from a data set, not used for identification. Furthermore, it can been seen from figure 4 that the amount of noise on the input $u(t)$ in closed loop is relatively small.

7.3 Towards a low order model

This section discusses the second step of the two stage identification algorithm, where an approximate identification will be performed, using the reconstructed input $\hat{u}_r(t)$ and output $y(t)$. For the sake of completeness it should be mentioned that the input $\hat{u}_r(t)$ cannot be used directly. This is

[1]Delft University Multivariable System Identification

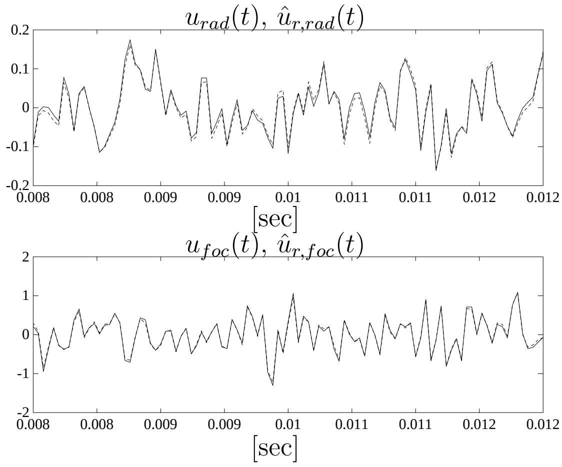

Fig. 4: Measured input $u(t)$ (—) and simulated input $\hat{u}_r(t)$ (- -) of *rad*ial and *foc*us loop

caused by the fact that the radial and focus actuators act like double integrators in the frequency domain of interest.

The properties of the prediction error methods, like the results given in proposition 6.1, are valid only for a stable prediction error mapping, see Ljung (1987), Van den Hof and Schrama (1993). Hence, identifying a double integrator will inevitably lead to undesirable results. In order to omit the identification of the (known) double integrator, the input $\hat{u}_r(t)$ will be put through a zero order hold equivalent of a continues time double integrator. In this way the *remaining* dynamics of the system $P_0(q)$ has to be identified only.

As mentioned before, the iterative scheme of filtering and identification, discussed in section 6.2, is performed on the radial $P_{0,11}(q)$ and focus $P_{0,22}(q)$ transfer functions in a SISO configuration. In this way filtering of the prediction error $\varepsilon(t)$ now can be replaced by filtering of input $\hat{u}_r(t)$ and output $y(t)$ of the system to be identified.

Finally, the filters $L_{11}(q)$ and $L_{22}(q)$ arising from the iterative scheme mentioned above, are used to estimate a multivariable Output Error model, using the DUMSI–package. This multivariable model has a 16^{th} order (without the double integrators) and is parametrized using a pseudo canonical (observability) form (Ljung (1987), pp. 119–123), with structure indices (7,9). It should be mentioned that the multivariable model being estimated now, will not be optimal in the sense of the closed loop criterion given in (21), since the choice of the filter $L(q, \rho)$ does not exactly meet the requirements of proposition 6.1. However, the results of this control relevant scheme can be quite illuminating. The results of the multivariable model being estimated can be found in figure 5 and 6.

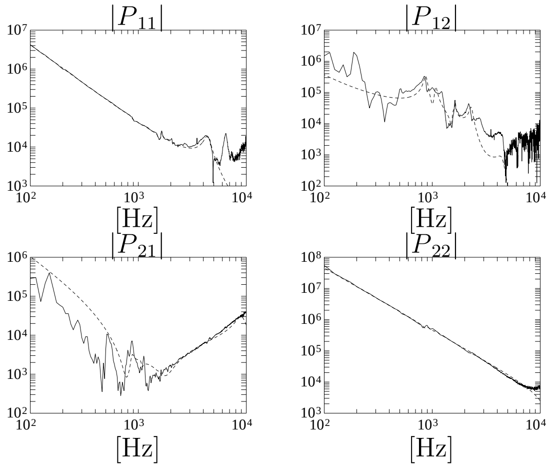

Fig. 5: Amplitude of spectral estimate $\hat{P}(\omega)$ (—) and parametric model $P(e^{i\omega}, \hat{\rho}_N)$ (- -)

Figure 5 presents the amplitude Bode plots of the spectral estimate $\hat{P}(\omega)$, see (23), and the model $P(e^{i\omega}, \hat{\rho}_N)$ being estimated. It can be seen from this figure that there is some parasitic dynamics in the radial transfer function $P_{0,11}(e^{i\omega})$, around 0.9, 1.7, 4 and 6 kHz. Some of these parasitic dynamics only have a small contribution in the *open loop* behaviour of the system and therefore should not have to be estimated. On the other hand, in the closed loop behaviour of the system these parasitic dynamics play an significant role. This is illustrated in figure 3, where one can recognize peaks in the sensitivity function. Clearly, this discussion illustrates the use of a control relevant identification scheme. A part of the simulations, based on closed loop data that has not been used for identification, has been depicted in figure 6. It illustrates that the model $P(e^{i\omega}, \hat{\rho}_N)$ predicts the closed loop data very well.

Given the nominal model $P(q, \hat{\rho}_N)$, the procedure presented in de Vries and Van den Hof (1993) can be used to quantify an additive model error. Using a partly periodic input signal $\hat{u}_r(t)$ and additional information about the decay rate of the impulse response of the system under consideration, an additive model error can be estimated using an Empirical Transfer Function Estimate, see de Vries and Van den Hof (1993) for further details. The results of this procedure, applied to the radial transfer function only, can be found in figure 7.

In figure 7 a part of the Nyquist contour of $C_{11}(q)P_{11}(q, \hat{\rho}_N)$ is depicted, based on the given controller $C_{11}(q)$ of the radial servo loop and the nominal model $P_{11}(q, \hat{\rho}_N)$ of the radial transfer function being estimated. Furthermore, the addi-

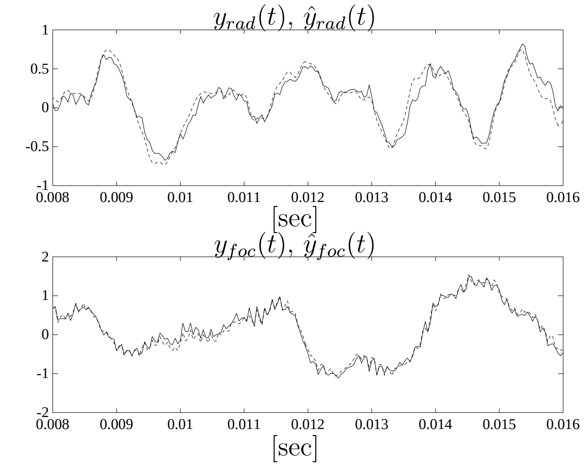

Fig. 6: Measured output $y(t)$ (—) and simulated output $\hat{y}(t)$ (- -) of *rad*ial and *foc*us loop

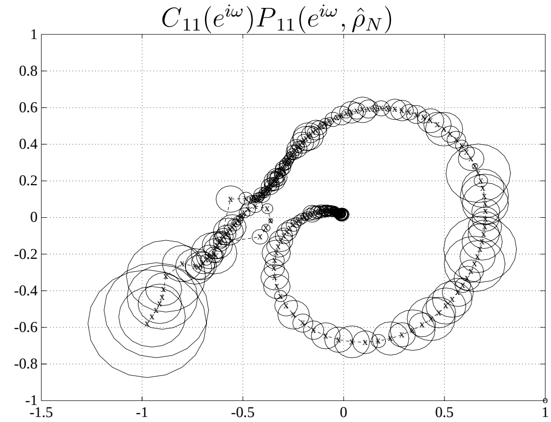

Fig. 7: Nyquist contour of radial servo loop, based on the nominal model $P_{11}(q, \hat{\rho}_N)$ (- -) and uncertainty bounds (—)

tive error bounds on the nominal model are characterized by circles in the complex plane for several frequency points. From figure 7 it can also be seen that the additive model error has been kept small in the closed loop frequency domain of interest (around the bandwidth).

8 Conclusions

In this paper a control relevant parametric identification scheme is applied to a Compact Disc servo system, using the well known Prediction Error methods, wherein the problems of approximate and closed loop identification have been merged. This is done by using a two stage identification algorithm, wherein a simulation of the input signal is used to estimate the system. The two stage algorithm requires an accurate estimate of the input sensitivity of the closed loop system. This can be achieved by employing a linear regression scheme using system based orthonormal functions. The resulting expression for the bias distribution of the model being estimated, is tuned in a control relevant way by choosing appropriate design variables. Using closed loop time domain observations of a Compact Disc pick–up mechanism, this has led to a multivariable discrete time model that can be used for designing an enhanced controller.

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