

Stream Depletion by Groundwater Pumping in Leaky Aquifers

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Abstract: We present a simple approach to assess stream depletion by groundwater pumping in aquifers with leakage from an underlying source bed. The hydrogeological setting consists of a leaky aquifer that is hydraulically connected to a stream of shallow penetration. Under such conditions, the pumping rate is partially supported by the depletion of an adjacent stream. We quantify this phenomenon by deriving explicit analytical solutions that elucidate the interplay among the streambed, aquifer, aquitard, and well parameters. In particular, our solutions demonstrate how both hydraulic stream-aquifer connection and hydrostratigraphic conditions determine the possible fraction of the pumping rate supplied by the stream depletion. In leaky aquifers, this fraction decreases exponentially with the distance between a stream and a pumping well and is attenuated by a factor that depends on the streambed, aquifer, and aquitard parameters. The balance of pumping rate is supplied from recharge zone of the source bed.

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Introduction

The importance of accurate and reliable predictions of stream depletion rates (SDR) has been highlighted by such recent events as droughts, proliferation of large-capacity irrigation wells, and disruption of ecological equilibrium of streams caused by groundwater pumping. In the United States, tens of thousands of high-capacity wells are located in alluvial valleys. Vast water withdrawals have dramatically changed local and regional water budgets of aquifers and streams. For example, maps comparing perennial streams in Kansas in the 1960s to those of the 1990s show a marked decrease in the length of stream flow (Sophocleous 1997).

A typical detailed evaluation of SDR relies on an extensive database comprised of various characteristics of aquifers, streams, climate, land usage, etc., and employs a decision support system using numerical large-scale aquifer models (e.g., Ramireddygari et al. 2000; Sophocleous 2005). Ideally, such evaluation is required to obtain permits for construction and operation of new wells. However, many basins, including those in most developed countries, lack the appropriate databases and decision support tools, as a result of which the experts and the decision-makers often resort to less data- and computationally intensive approaches that are based on analytical solutions by Jenkins (1968). Certain tradeoffs between the accuracy of simulations and the

data availability are sometimes used in adjudication of water rights even in contentious cases (Bouwer and Maddock 1997).

Classic studies and recommendations on SDR primarily focused on the hydraulic connection between a fully penetrating stream and an aquifer in alluvial valleys (Theis 1941; Glover and Balmer 1954; Jenkins 1968; Hantush 1965). Only recently, such important factors as the width of a stream and a stream's partial penetration were incorporated into the analyses of SDR (Hunt 1999; Zlotnik et al. 1999; Zlotnik and Huang 1999; Butler et al. 2001), and were implemented in practice, e.g., in New Zealand (Guidelines 2001). Hunt (2003) investigated a special case of an aquifer that is hydraulically connected with a stream by an aquitard. All of these approaches predict that after extended pumping, 100% of pumping rate originates from the depletion of the adjacent stream.

However, Hantush (1955, 1964) and Zlotnik (2004) showed that, under common realistic hydrostratigraphic conditions in leaky aquifers, an adjacent stream might supply only a fraction of the pumping rate, which might vary from 0 to 100% depending on the aquifer, aquitard, and well properties. The fraction of the pumping rate supplied by depletion of the adjacent stream is important for water resources management, because it allows one to facilitate the adjudication of water rights for each well near the stream. As the above-noted leaky aquifer models do not consider either streams with shallow penetration of alluvial aquifers or aquitard storage, they overestimate the depletion rates from the adjacent stream by ignoring the constraints imposed on water fluxes between the aquifer and the stream.

Shallow stream penetration significantly complicates the analysis. In the case of nonleaky aquifers this complexity was resolved by using both analytical solutions for narrow streams (Hunt 1999; Zlotnik and Huang 1999) and numerical Laplace-Fourier transforms for a more realistic stream geometry (Butler et al. 2001). Kollet and Zlotnik (2003) found that Hunt's solution is effective in inverse modeling and more conducive to data interpretation than the solution of Butler et al. (2001). Hunt's solution is also easier to use in sensitivity analyses (Christensen 2000, 2005). At the same time, the numerical approach of Butler et al.

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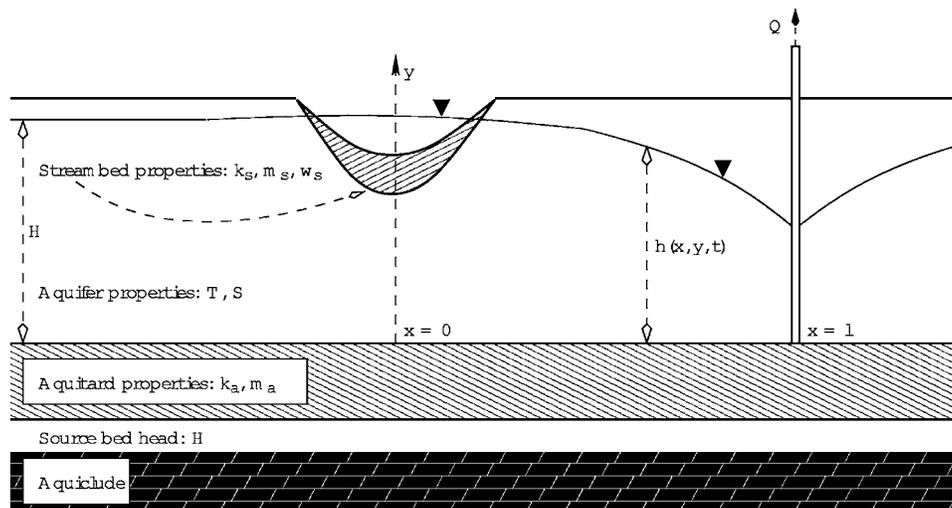


Fig. 1. A schematic representation of a stream-aquifer-aquitard system and major hydrological parameters

(2001) proved to be useful in the case study of alluvial with structured heterogeneity, where reliable data from an aquifer characterization program were available (Kollet and Zlotnik 2003).

Recently Butler et al. (2007) applied the streambed model of Zlotnik et al. (1999) and updated the code of Butler et al. (2001) to account for the combined effects of leakage and partial penetration. These numerical simulations confirmed the earlier conclusions of Hantush (1955, 1964) and Zlotnik (2004) according to which the stream-to-well distance, aquitard's hydraulic conductivity, and the aquifer's thickness all play important roles in predicting leakage and stream depletion, whereas the aquitard's storage can generally be safely neglected. However, the relative effects of streambed conductance, aquitard's properties, and the distance between a pumping well and a stream have not been quantified in an explicit functional form. This is because the very nature of numerical simulations renders their interpretation dependent on a particular combination of input parameters. Additionally, the solutions of Butler et al. (2007) are complex and require parameters that are rarely available (for example, shape of and magnitude of heterogeneity of aquifer parameter and effective streambed properties). Finally, the analysis of Butler et al. (2007) requires numerical inversion of double Laplace-Fourier transforms.

The main goal of this study is to augment the analysis of Butler et al. (2007) by focusing on the effects of major parameters, including streambed properties and hydraulic aquifer connection with the underlying source bed. Assuming aquifer's homogeneity and adopting the realistic, yet simple, streambed description of Hunt (1999)—instead of the more complex descriptions employed by Zlotnik et al. (1999) and Butler et al. (2001)—we derive closed-form analytical solutions that extend Hantush's (1955, 1964) results for SDR and drawdown. Because of their transparent form, our results dramatically simplify the operational properties of the solution. For example, it can be easily incorporated into geographic information system (GIS)-based procedures for assessment of stream depletion in watersheds.

Problem Formulation

Consider a well operating with a constant pumping rate Q in a leaky aquifer at a distance l from a shallow stream (Fig. 1). Our

assumptions for problem formulation are as follows:

- The Dupuit assumptions are valid, and hydraulic head $h(x, y, t)$ is a function of Cartesian coordinates x and y and time t .
- An alluvial aquifer with hydraulic conductivity k , transmissivity T , and storativity S is homogeneous and isotropic, and has infinite extent.
- Relative to the thickness of an unsaturated aquifer, drawdowns are small enough to warrant the use of linearized flow equations.
- Drawdowns are small enough to provide a permanent stream-aquifer hydraulic connection.
- Both the horizontal and vertical dimensions of a streambed's cross section are smaller than the thickness of the aquifer.
- A stream is located along the y axis and is of infinite extent ($-\infty < y < \infty$).
- Seepage flow rates between the stream and the aquifer are proportional to the difference in piezometric head across the streambed.
- The alluvial aquifer is separated from the source bed with constant head by an incompressible aquitard whose hydraulic conductivity is k_a ($k_a \ll k$) and thickness is m_a .
- Changes in both hydraulic head in the source bed and stream stage are negligible.
- The hydrologic system (i.e., the aquifer, the stream, and the source) is in the state of equilibrium before the commencement of pumping.

Under these assumptions, the flow problem can be described by (Hantush 1964; Zlotnik et al. 1999; Hunt 1999; Butler et al. 2001; Zlotnik 2004)

$$T \left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right) = S \frac{\partial h}{\partial t} + w \quad (1a)$$

where

$$w = Q\delta(x-l)\delta(y) - \lambda(H-h)\delta(x) - \frac{k_a}{m_a}(H-h) - R \quad (1b)$$

In Eq. (1b), the four terms represent ground water pumping, streambed seepage, aquitard leakage, and aquifer recharge R , respectively; $\delta(x)$ =Dirac delta function, H =hydraulic head in the aquifer, stream, and source bed at time $t=0$; and λ =streambed characteristic. For streambeds with small horizontal and

vertical dimensions, the latter can be approximated by $\lambda = k_s w_s / m_s$, where k_s , w_s , and m_s represents hydraulic conductivity, width, and thickness of the streambed, respectively (Hunt et al. 2001).

Flow Eq. (1) is subject to the initial and boundary conditions

$$h(x, y, 0) = H, \quad \lim_{x^2 + y^2 \rightarrow \infty} h = H \quad (2)$$

respectively.

The stream depletion rate q is obtained by integrating flux w over the streambed area [e.g., Hunt (1999), Eq. (6)]

$$q = - \lim_{\epsilon \rightarrow 0} \int_{-\infty}^{\infty} \int_{-\epsilon}^{\epsilon} w dx dy = \lambda \int_{-\infty}^{\infty} [H - h(0, y, t)] dy \quad (3)$$

In terms of drawdown $\phi(x, y, t) = H - h(x, y, t)$ and the dimensionless parameters

$$\phi_d = \frac{\phi T}{Q}, \quad t_d = \frac{Tt}{S l^2}, \quad x_d = \frac{x}{l}, \quad y_d = \frac{y}{l}, \quad B_d^2 = \frac{m_a T}{k_a l^2}, \quad \lambda_d = \frac{\lambda l}{T} \quad (4)$$

Eqs. (1)–(3) can be rewritten as

$$\nabla_d^2 \phi_d = \frac{\partial \phi_d}{\partial t_d} - \delta(x_d - 1) \delta(y_d) + \lambda_d \phi_d \delta(x_d) + B_d^{-2} \phi_d \quad (5)$$

subject to the initial and boundary conditions

$$\phi_d(x_d, y_d, 0) = 0, \quad \lim_{x_d^2 + y_d^2 \rightarrow \infty} \phi_d = 0 \quad (6)$$

respectively. The stream depletion rate is given by

$$\frac{q}{Q} = \lambda_d \int_{-\infty}^{\infty} \phi_d(0, y_d, t_d) dy_d \quad (7)$$

Note that, analogous to Jenkins (1968), the dimensionless time t_d is scaled with the “stream depletion factor” $S l^2 / T$. The dimensionless parameter B_d accounts for the effects of leakage, such that $B_d = \infty$ corresponds to a nonleaky aquifer and $B_d = 0$ corresponds to a perfect connection with a source bed. The dimensionless parameter λ_d accounts for water exchange between the stream and the aquifer, such that $\lambda_d = 0$ indicates the absence of a stream and $\lambda_d = \infty$ corresponds to a perfect stream-aquifer connection, i.e., to the full aquifer penetration by a stream. Dimensional counterparts of B_d and λ_d were used by Hantush (1964) and Hunt (1999), respectively.

Drawdown

Let $\Phi(x_d, \alpha, p)$ be the Laplace–Fourier transform of ϕ_d defined as

$$\begin{aligned} \bar{\Phi}(x_d, y_d, p) &= \int_0^{\infty} \phi(x_d, y_d, t_d) e^{-pt_d} dt_d, \Phi(x_d, \alpha, p) \\ &= \int_{-\infty}^{\infty} \bar{\Phi}(x_d, y_d, p) e^{i\alpha y_d} dy_d \end{aligned} \quad (8)$$

Taking the Laplace–Fourier transform of Eqs. (5) and (6) yields an ordinary differential equation

$$\frac{d^2 \Phi}{dx_d^2} - \beta^2 \Phi = -\frac{1}{p} \delta(x_d - 1) + \lambda_d \Phi \delta(x_d), \quad \beta^2 = \alpha^2 + p + B_d^{-2} \quad (9)$$

subject to the boundary condition

$$\lim_{x_d \rightarrow \pm \infty} \Phi = 0 \quad (10)$$

Following Hunt (1999), the solution of Eqs. (9) and (10) can be written as

$$\Phi = \Phi_1 - \Phi_2, \quad -\infty < x_d < \infty \quad (11)$$

where

$$\Phi_1 = \frac{1}{2\beta p} e^{-\beta |x_d - 1|} \quad (12)$$

and

$$\Phi_2 = \frac{\lambda_d}{2\beta p(2\beta + \lambda_d)} e^{-\beta(1 + |x_d|)} \quad (13)$$

Following Hunt [(1999), Eq. (25)], we note that in the absence of a stream ($\lambda = \lambda_d = 0$), our solution must reduce to the well-known solution by Hantush and Jacob (1955). Hence Φ_1 in Eq. (12) is a Laplace–Fourier image of the Hantush–Jacob solution for an observation well located at dimensionless distance $r_d^2 = (x_d - 1)^2 + y_d^2$,

$$\Phi_{d1}(x_d, y_d, t_d) = \frac{1}{4\pi} W\left(u, \frac{r_d}{B_d}\right) \quad (14)$$

where $u = r_d^2 / (4t_d)$ and W = well function defined by Hantush (1964) as

$$W(u, x) = \int_u^{\infty} \frac{1}{y} \exp\left(-y - \frac{x^2}{4y}\right) dy \quad (15a)$$

The relevant properties of the well function W are (e.g., Hantush 1964)

$$W(u, \infty) = 0, \quad W(u, 0) = E_1(u), \quad W(0, x) = 2K_0(x) \quad (15b)$$

where E_1 = exponential integral, and K_0 = modified Bessel function of the second kind.

The Laplace–Fourier transform of Eq. (13) is inverted using Hunt’s [(1999, Eq. 29)] technique

$$\phi_{d2}(x_d, y_d, t_d) = \frac{1}{4\pi} \int_0^{\infty} e^{-\theta} W\left(u_{\lambda}, \frac{r_{\lambda}}{B_d}\right) d\theta \quad (16)$$

where $r_{\lambda}^2 = (1 + |x_d| + 2\theta/\lambda_d)^2 + y_d^2$ and $u_{\lambda} = r_{\lambda}^2 / (4t_d)$. Hence the dimensionless drawdown $\phi_d = \phi_{d1} - \phi_{d2}$, or its dimensional counterpart $\phi = \phi_1 - \phi_2$, is given by

$$\phi(x, y, t) = \frac{Q}{4\pi T} W\left(u, \frac{r_d}{B_d}\right) - \frac{Q}{4\pi T} \int_0^{\infty} e^{-\theta} W\left(u_{\lambda}, \frac{r_{\lambda}}{B_d}\right) d\theta \quad (17)$$

This compact solution for drawdown unifies and generalizes the results by Theis (1941), Hantush (1955), Hantush and Jacob (1955), and Hunt (1999) for drawdown and extends these to the case of groundwater withdrawals from a leaky aquifer near a shallow stream.

Stream Depletion

The Laplace image of the stream depletion rate in Eq. (7) can be obtained from the Laplace–Fourier transform of the drawdown in Eq. (8) as

$$\frac{\bar{q}}{Q} = \lambda_d \int_{-\infty}^{\infty} \bar{\phi}(0, y_d, t_d) dy_d = \lambda_d \Phi(0, 0, p) \quad (18)$$

Substituting Eq. (11) into (18) gives a Laplace image of the stream depletion rate

$$\frac{\bar{q}}{Q} = \frac{\lambda_d}{p(2\beta_0 + \lambda_d)} e^{-\beta_0}, \quad \beta_0^2 = p + B_d^{-2} \quad (19)$$

Using an expansion

$$\frac{2\lambda_d}{(p - 1/B_d^2)(2\sqrt{p} + \lambda_d)} = \frac{B_d a_1}{\sqrt{p} - 1/B_d} + \frac{B_d a_2}{\sqrt{p} + 1/B_d} - \frac{4}{\lambda_d} \frac{a_1 a_2}{\sqrt{p} + \lambda_d/2} \quad (20)$$

in Eq. (19), where

$$a_1 = \frac{B_d}{2/\lambda_d + B_d}, \quad a_2 = \frac{B_d}{2/\lambda_d - B_d} \quad (21)$$

and taking the inverse Laplace transform of each term separately [Carslaw and Jaeger (1959), Appendix V, Eq. (12)] gives a new solution for the stream depletion rate

$$\begin{aligned} \frac{q}{Q} = & \frac{a_1}{2} e^{-1/B_d} \operatorname{erfc}\left(\frac{1}{2\sqrt{t_d}} - \frac{\sqrt{t_d}}{B_d}\right) - \frac{a_2}{2} e^{1/B_d} \operatorname{erfc}\left(\frac{1}{2\sqrt{t_d}} + \frac{\sqrt{t_d}}{B_d}\right) \\ & + a_1 a_2 e^{\lambda_d/2 + \lambda_d^2 t_d/4 - t_d/B_d^2} \operatorname{erfc}\left(\frac{1}{2\sqrt{t_d}} + \frac{\lambda_d \sqrt{t_d}}{2}\right) \end{aligned} \quad (22)$$

Analysis of the Solutions

To explore the general properties of the derived solutions and to demonstrate that they contain a plethora of classical expressions (e.g., Theis curves) as special cases, it is worthwhile to recall the physical meaning of the dimensionless parameters B_d and λ_d . The dimensionless parameter B_d accounts for the effects of leakage, such that $B_d = \infty$ (small k_a and/or large m_a) corresponds to a nonleaky aquifer and $B_d = 0$ (large k_a and/or small m_a) corresponds to a perfect connection with a source bed. The dimensionless parameter λ_d accounts for water exchange between the stream and the aquifer, such that $\lambda_d = 0$ (small k_s and/or large m_s) indicates the absence of a stream and $\lambda_d = \infty$ (large k_s and/or small m_s) corresponds to a perfect stream-aquifer connection, i.e., to the full aquifer penetration by a stream.

Analysis of Drawdown

A new expression for *steady-state drawdown* ϕ_{st} is obtained from Eq. (17) by taking the limit as dimensionless time $t_d \rightarrow \infty$, and recalling Eq. (15b)

$$\phi_{st}(x, y) = \frac{Q}{2\pi T} K_0\left(\frac{r_d}{B_d}\right) - \frac{Q}{2\pi T} \int_0^\infty e^{-\theta} K_0\left(\frac{r_\lambda}{B_d}\right) d\theta \quad (23)$$

The *Theis solution* for flow to a pumping well in a nonleaky aquifer without a stream is obtained from Eq. (17) by taking the limit as $B_d \rightarrow \infty$ and $\lambda_d \rightarrow 0$

$$\phi^T(x, y, t) = \frac{Q}{4\pi T} E_1(u) \quad (24)$$

The *Theis solution* for flow to a pumping well in a nonleaky aquifer with a perfect stream-aquifer connection is obtained from Eq. (17) by taking the limit as $B_d \rightarrow \infty$ and $\lambda_d \rightarrow \infty$

$$\begin{aligned} \phi^{MT}(x, y, t) = & \frac{Q}{4\pi T} E_1(u) - \frac{Q}{4\pi T} E_1(u_M), \quad u_M = \frac{r_{Md}^2}{4t_d}, \\ r_{Md}^2 = & (1 + |x_d|)^2 + y_d^2 \end{aligned} \quad (25)$$

where r_{Md} = dimensionless distance between an observation point (x_d, y_d) and a mirror image of the pumping well with respect to the stream.

The *Hantush–Jacob (1955) solution* for flow to a pumping well in a leaky aquifer without a stream is obtained from Eq. (17) by taking the limit as $\lambda_d \rightarrow 0$ (or $r_\lambda \rightarrow \infty$) and recalling Eq. (15b)

$$\phi^{HJ}(x, y, t) = \frac{Q}{4\pi T} W\left(u, \frac{r_d}{B_d}\right) \quad (26)$$

The *modified Hantush–Jacob solution* (e.g., Hantush 1964) for flow to a pumping well near a fully penetrating stream is obtained from Eq. (17) by taking the limit as $\lambda_d \rightarrow \infty$. As

$$\lim_{\lambda_d \rightarrow \infty} \phi_{d2}(x_d, y_d, t_d) = \frac{1}{4\pi} W\left(u_M, \frac{r_{Md}}{B_d}\right) \quad (27)$$

the modified Hantush solution is given by

$$\phi^{MH}(x, y, t) = \frac{Q}{4\pi T} W\left(u, \frac{r_d}{B_d}\right) - \frac{Q}{4\pi T} W\left(u_M, \frac{r_{Md}}{B_d}\right) \quad (28)$$

The *transient Hunt (1999) solution* for flow to a pumping well in a nonleaky aquifer near a shallow partially penetrating stream is obtained from Eq. (17) by taking the limit as $B_d \rightarrow \infty$ and recalling Eq. (15b)

$$\phi^{Hu}(x, y, t) = \frac{Q}{4\pi T} E_1(u) - \frac{Q}{4\pi T} \int_0^\infty e^{-\theta} E_1(u_\lambda) d\theta \quad (29)$$

which is identical to Eqs. (25) and (29) of Hunt (1999). The steady-state counterpart of Eq. (29) exists for $\lambda_d > 0$ and is given by Eq. (5) of Kollet et al. (2002)

$$\begin{aligned} \phi_{st}^{Hu} = & \frac{Q}{4\pi T} \ln\left[\frac{(1 + |x_d|)^2 + y_d^2}{(1 - x_d)^2 + y_d^2}\right] + \frac{Q}{2\pi T} \int_{1+|x_d|}^\infty \frac{\eta}{\eta^2 + y_d^2} \\ & \times \exp\left(-\lambda_d \frac{\eta - 1 - |x_d|}{2}\right) d\eta \end{aligned} \quad (30)$$

Fig. 2 illustrates the effect of leakage ($B_d = 10$ and $B_d = 100$) on the cone of depression. Leakage (the smaller B_d , the larger the leakage and the ratio k_a/m_a) reduces the lateral extent of the cone of depression. For example, the equipotential $\phi_d = 0.3$ barely reaches the stream for a larger leakage $B_d = 10$. This effect can be explained by the presence of a source bed.

Fig. 3 compares the time behavior of the normalized local drawdown ϕ_d in the observation well located at $(x_d, y_d) = (0.2, 0.0)$ for $B_d = 10$ and $B_d = 100$. The latter case corresponds to an example presented by Hunt's (1999) Fig. 6. For smaller values of B_d (i.e., $B_d = 10$), drawdown practically reaches the steady state at earlier times, because of the increased leakage across the aquitard to the pumped aquifer at earlier times.

The *Hunt (2003) solution* considers a two-layer system (the pumped aquifer and an aquitard above) without a source bed.

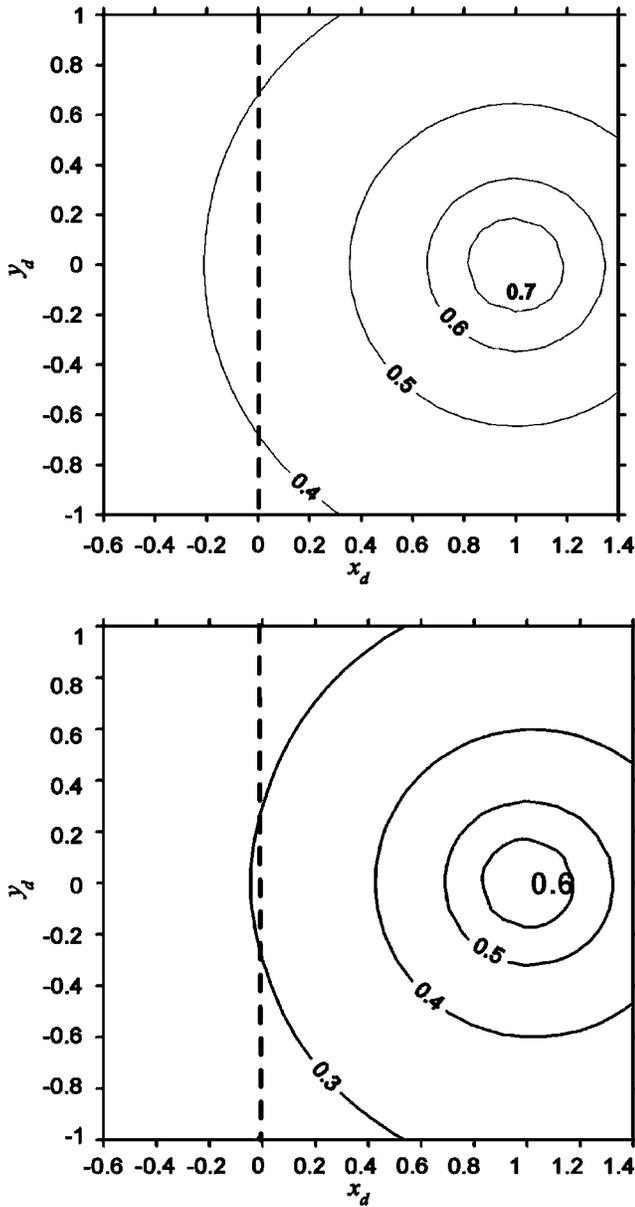


Fig. 2. Normalized drawdown $\phi_d = \phi T / Q$ at dimensionless time $t_d = 100$ for $\lambda_d = 0.1$ and (a) $B_d = 100$; (b) $B_d = 10$. The stream is located at $x_d = 0$ and is indicated by the dashed line.

Nevertheless, the Hunt solution [Hunt (2003), Eq. (40)] reduces to solution Eq. (17) if the aquitard's specific yield σ is very large (B. Hunt, private communication, 2005). In the latter case, the drawdown can be readily computed with Hunt's software, which is available on the website <http://www.civil.canterbury.ac.nz/staff/bhunt.asp>.

Analysis of SDR

Evaluation of the SDR is especially important for various applications. General expression Eq. (22) for the SDR accounts for both the streambed-aquifer water exchange and the leakage across the aquitard. This solution corresponds to the dimensionless streambed conductance $\lambda_d \geq 0$ and leakage $B_d \leq \infty$, and contains several important special cases.

Maximum stream depletion rate (MSDR) is the maximum fraction of the pumping rate supplied by the SDR from an adjacent

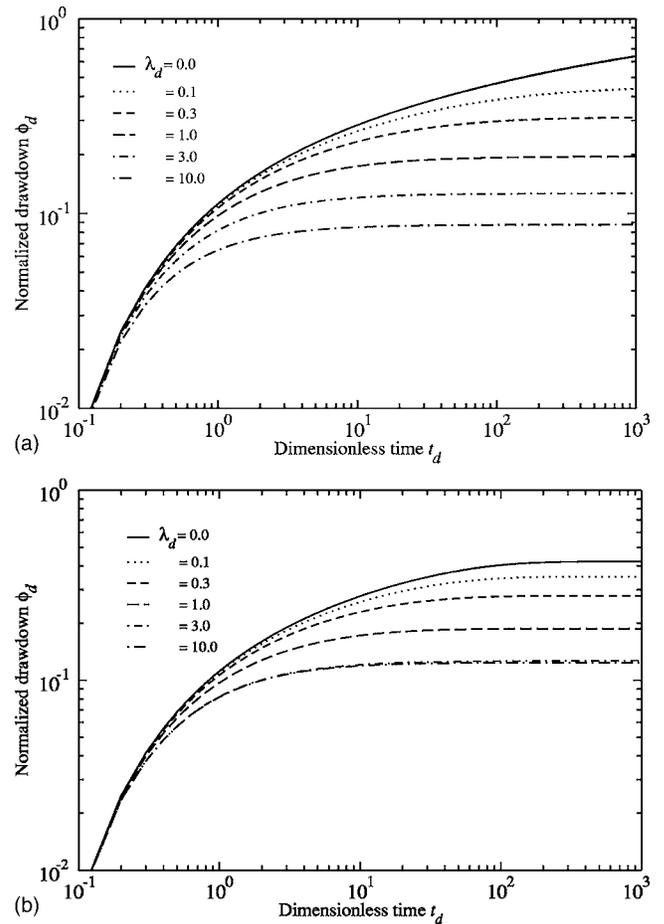


Fig. 3. Normalized drawdown $\phi_d = \phi T / Q$ at $(x_d, y_d) = (0.2, 0.0)$ as a function of dimensionless time $t_d = Tt / (S_l^2)$ for two values of the dimensionless leakage parameter: (a) $B_d = 100$; (b) $B_d = 10$, and several values of the dimensionless streambed characteristics λ_d

stream. It corresponds to the steady-state SDR (Zlotnik 2004). A new expression, which accounts for streambed properties and partial penetration of the aquifer, is obtained from Eq. (22) by taking the limit as $t_d \rightarrow \infty$ and recalling the definition of streambed parameter $\lambda = k_s w_s / m_s$

$$\text{MSDR} = \frac{q_{st}}{Q} = a_1 e^{-1/B_d} = \frac{1}{2\lambda^{-1} \sqrt{k_a T / m_a} + 1} \exp\left(-l \sqrt{\frac{k_a}{m_a T}}\right) \quad (31)$$

Eq. (31) demonstrates that both leakage from the source bed and the stream contribute to the MSDR. Leakage causes the MSDR to decrease exponentially with the distance between the well and the stream. This decrease is attenuated by a factor that depends on the complex interplay of the streambed ($k_s w_s / m_s$), aquifer (T), and aquitard (k_a / m_a) parameters. It is important to note that the MSDR is always less than one. In particular, the effect of increase of aquifer transmissivity would not be apparent without this explicit relationship.

The effectiveness of the presented solutions becomes apparent if one is interested in the sensitivity of the MSDR to various hydraulic properties. Butler et al. (2007) used a series of numerical simulations to arrive at a qualitative conclusion that the "knowledge of streambed properties is not required to assess impact of pumping wells located at large distance from a stream."

Eq. (31) yields quantitative understanding of this phenomenon. Specifically, the MSDR will not exceed 1% of the pumping rate, if the well-to-stream distance meets criterion $l > 4.6\sqrt{m_a T/k_a}$.

Eq. (31) allows us to elucidate more interesting effects occurring at practically important moderate and small well-to-stream distances, when $l < 4.6\sqrt{m_a T/k_a}$. This can be illustrated by using parameters from Butler et al. [(2007), Table 3], where $T = 4.1 \times 10^{-3}$ m²/s, $k_a = 2.4 \times 10^{-8}$ m/s, $m_a = 5.2$ m, $k_s = 1.2 \times 10^{-5}$ m/s, $m_s = 0.3$ m, and $w_s = 10$ m. If the well is located at distance $l = 4.6\sqrt{m_a T/k_a} = 942$ m, then the change in streambed properties to $k_s = 1.2 \times 10^{-6}$ m/s and $m_s = 1.0$ m reduces the MSDR from 36 to 21%. If the well-to-stream distance is reduced by half ($l = 471$ m), then the same change in the streambed properties k_s and m_s has more drastic effects on the MSDR, which decreases from 59 to 35%.

The Hantush (1955, 1964) solution for the SDR induced by a well pumping in a leaky aquifer near a fully penetrating stream is obtained by taking the limit of Eq. (22) as $\lambda_d \rightarrow \infty$

$$\frac{q^H}{Q} = \frac{1}{2} e^{-1/B_d} \operatorname{erfc}\left(\frac{1}{2\sqrt{t_d}} - \frac{\sqrt{t_d}}{B_d}\right) + \frac{1}{2} e^{1/B_d} \operatorname{erfc}\left(\frac{1}{2\sqrt{t_d}} + \frac{\sqrt{t_d}}{B_d}\right) \quad (32)$$

The MSDR for this case is obtained from Eq. (32) by taking the limit as $t_d \rightarrow \infty$

$$\text{MSDR}^H = e^{-1/B_d} = \exp\left(-l \sqrt{\frac{k_a}{m_a T}}\right) \quad (33)$$

The exponential term again indicates the strong attenuation of the MSDR with increase of distance between the well and the stream. Other factors that may affect the MSDR are alluvial valley width and availability of other sources of the aquifer recharge (Zlotnik 2004).

The classic Theis–Glover–Balmer solution for the SDR induced by a well pumping in a nonleaky aquifer near a fully penetrating stream is obtained from Eq. (22) by taking the limit as both $B_d \rightarrow \infty$ and $\lambda_d \rightarrow \infty$

$$\frac{q^{\text{TGB}}}{Q} = \operatorname{erfc}\left(\frac{1}{2\sqrt{t_d}}\right) \quad (34)$$

The comparison of Eqs. (32) and (34) reveals that the assumption of full penetration overestimates the magnitude of the MSDR.

The Hunt (1999) transient solution for the SDR induced by a pumping well in a nonleaky aquifer near a shallow partially penetrating stream is obtained by taking the limit of Eq. (22) as $B_d \rightarrow \infty$, i.e., by assuming an impermeable aquitard

$$\frac{Q^{\text{Hu}}}{Q} = \operatorname{erfc}\left(\frac{1}{2\sqrt{t_d}}\right) - e^{\lambda_d^2 t_d} \operatorname{erfc}\left(\frac{1}{2\sqrt{t_d}} + \frac{\lambda_d \sqrt{t_d}}{2}\right) \quad (35)$$

Both the Theis–Glover–Balmer and Hunt solutions for the SDR indicate that in nonleaky aquifers the pumping rate is fully supplied by the stream depletion after extended pumping time. Hence, the MSDR eventually reaches 1 (Zlotnik 2004).

Fig. 4 illustrates the combined influence of partial stream penetration and aquifer leakage on both the SDR and MSDR. It contains a family of the Hunt (1999) curves for the SDR in a nonleaky aquifer with a partially penetrating stream (the lines without circles), which is given by Eq. (35) and includes the Theis–Glover–Balmer solution for a case of perfect stream–aquifer connection ($\lambda_d = \infty$). While Hunt's SDR, Eq. (35), is obtained from Eq. (22) by taking the limit as $B_d \rightarrow \infty$, it is

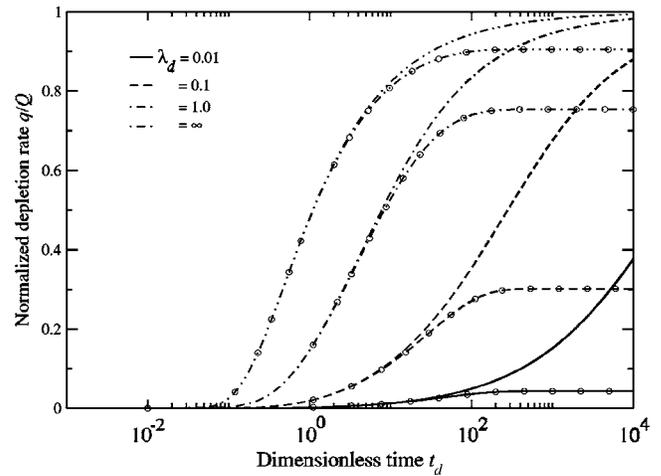


Fig. 4. Normalized stream depletion rate q/Q as a function of dimensionless time $t_d = Tt/(S_l^2)$ for two values of the dimensionless leakage parameter, $B_d = \infty$ and $B_d = 10$ (the corresponding lines with circles), and several values of the dimensionless streambed characteristics λ_d

within 9% of the SDR given by Eq. (22) at $B_d = 100$, and practically attains this limit at $B_d = 500$. By the same token, the Theis–Glover–Balmer SDR Eq. (34) is obtained from Eq. (22) with $B_d \rightarrow \infty$ (or $B_d = 500$) and $\lambda_d \rightarrow \infty$ ($\lambda_d = 10$). Needless to say, $\text{MSDR} = 1$ for all these curves.

Fig. 4 also contains a family of curves for the SDR in a leaky aquifer with $B_d = 10$ [lines of the same type but with circles correspond to the same values of λ as those used to compute the Hunt (1999) curves]. The comparison of these two families of curves reveals that the time it takes for the stream depletion to reach the steady state increases with B_d . Leakage across the aquitard causes the MSDR to decay exponentially with the distance between the well and the stream and prevents it from reaching 1. The leakage also leads to an earlier stabilization of the MSDR after the commencement of pumping. The time it takes for each SDR curve to reach the corresponding MSDR may differ, and sensitivity analysis to the stream–aquifer–aquitard parameters would be appropriate (Christensen 2000).

Implications for Streams in Leaky Aquifers

Alluvial valleys with aquitards and underlying source beds are encountered in many watersheds. Our solutions, especially Eq. (31) for the SDR, can be used to evaluate the impact of new wells on hydrological conditions from the aquifer, aquitard, and streambed properties. Often regional databases (e.g., Calver 2001) can provide data for such evaluations, but such data are poorly constrained. There are two approaches that enhance their reliability by incorporating site-specific information (Hunt 1999). The first approach is to use the drawdown inversion to infer the aquifer, aquitard, and streambed properties from the pumping test data. This is achieved by matching a drawdown solution, such as Eq. (17), with drawdown data from available piezometers. In various forms, Sophocleous et al. (1988), Hunt et al. (2001), Nyholm et al. (2002), and Kollet and Zlotnik (2003) used this technique to evaluate the SDR by means of analytical models. The second approach is to match the SDR solution, such as Eq. (22), with direct stream discharge measurements (Hunt et al.

2001; Nyholm et al. 2003). Both inversion techniques require evaluation of sensitivity coefficients, such as $\partial s/\partial T$, $\partial q/\partial T$, etc. (Christensen 2000). Christensen (2005) noted that numerical methods of evaluating such coefficients sometimes do not yield adequate accuracy and lead to errors in parameter estimation. This is precisely the situation where analytical expressions, such as Eqs. (17) and (22) are very efficient.

The SDR evaluation strongly depends on the validity of the widely accepted assumptions by Hantush and Jacob (1955) and Hunt (1999). These include the assumptions that stream width and depth are small, an aquitard is incompressible, and a source bed is present. Hunt et al. (2001), Butler et al. (2001), and Kollet and Zlotnik (2003) explored the validity of the assumption of infinitely small width and depth on SDR. These studies showed, both quantitatively and qualitatively, that the accuracy of this approximation is adequate for the placement of pumping wells at distances larger than five stream widths, which is typically the case.

A hydraulic analysis indicates that this assumption of an incompressible aquitard slightly increases the stabilization time for the SDR compared to Eq. (22) but does not change the steady state SDR or MSDR in Eq. (31). The actual SDR will reach the MSDR later than predicted by our model, but will not exceed it. So, this value will be also the lower bound of SDR for various hydrogeological conditions at the site. The actual SDR will fall between the same bounds of Eqs. (22) and (35).

The Hantush (1965) concept of the source bed, which implies a constant head over the pumping time, is widely used in theory and practice, but he did not provide either recommendations or a discussion on conditions for applications of this concept. There are two necessary conditions that make this approximation appropriate for the lower aquifer. First, a source bed (lower aquifer) must have a very large transmissivity. Second, the proximity of a pumping location to a recharge zone for the lower aquifer is essential. For example, the source bed can be hydraulically connected with another stream. Note that this connection would supply the balance of pumping rate that complements the fraction of pumping rate supplied by the adjacent stream up to 100%. This is the case in some areas in the High Plains Aquifer, United States (Zlotnik 2004).

The most straightforward approach to validate this concept in each specific case is to analyze drawdown in both the upper and lower aquifers during pumping. Large drawdown in the lower aquifer indicates that the lower aquifer becomes depleted, more pumped water comes to the well from the upper aquifer, and the actual values of the SDR for the adjacent stream may exceed those obtained from Eqs. (22) or (31). We must emphasize that these equations represent the lowest bound on an estimate of depletion for the adjacent stream in the aquifer–aquitard–aquifer system. The actual SDR might fall into the range between those described by Eq. (22) and the equations developed by Hunt (1999) (see Sec. 5.2). In practical assessment of the SDR, information on hydrostratigraphy and aquifer recharge zone must be emphasized. When data are scarce, an assessment of the SDR must include both the lower bound from Eq. (22) and the upper bound from Eq. (35).

In our analysis, we assumed uniform properties of the aquifer, streambed, and aquitard and assessed their effects on the SDR and MSDR. It is natural to expand this analysis to hydrological systems where some or all of these parameters are spatially heterogeneous. An investigation of structured heterogeneity might address only exceptional cases of alluvial aquifers. In future studies, stochastic representation of alluvial aquifers, streambed, and

aquitard will be needed (e.g., Winter et al. 2003; Zappa et al. 2006).

Summary and Conclusions

We obtained steady-state and transient solutions for drawdown and the SDR, which allow one to elucidate the combined effect of streambed leakage, stream penetration, and aquifer leakage. Stream depletion reaches 100% of the pumping rate in one-unit aquifers, but it may only partially support the groundwater withdrawal from a pumping well in leaky aquifers. The balance of ground water withdrawals that is not supported by the stream depletion from an adjacent stream can be supplied from other sources. In leaky aquifers, the maximum possible SDR (or MSDR) from a given well decreases exponentially with the distance between the well and the stream. This decrease is attenuated by a factor that depends on the streambed, aquifer, and aquitard parameters.

The SDR for an adjacent stream induced by a given well can be assessed only with full consideration of hydrogeological conditions that include the hydraulic properties of the aquifer and streambed, geometry of recharge and discharge zones of the upper and lower aquifers, and location of the pumping well. In general this would require numerical modeling. The obtained solutions may be used for preliminary assessment of the SDR and will complement detailed numerical techniques that are applied for evaluation of stream–aquifer water budgets, when necessary parameters are available a priori. Also, solutions can be used for designing the aquifer testing programs similar to these by Sophocleous et al. (1988), Hunt et al. (2001), Nyholm et al. (2002, 2003), and Kollet and Zlotnik (2003, 2005). The pretesting designs can be significantly enhanced by a sensitivity analysis of the on-site conditions (Christensen 2000).

In the absence of all parameters and data confirming the model assumptions, the derived Eqs. (22) and (31) may serve as a lower bound of depletion values for an adjacent stream. An upper bound for the SDR for an adjacent stream will be constrained by a model of a single (upper) aquifer without connection with the lower aquifer as it was presented in the models of Glover and Balmer (1954) or Hunt (1999). These constraints provide useful tools for the SDR evaluation in leaky aquifer–aquitard–stream systems.

Notation

The following symbols are used in this paper:

- B = leakage parameter;
- d = subscript indicating dimensionless quantities;
- E_1 = exponential integral;
- H = initial hydraulic head;
- h = hydraulic head;
- k = hydraulic conductivity of the alluvial aquifer;
- k_a = hydraulic conductivity of the aquitard;
- k_s = hydraulic conductivity of the streambed;
- K_0 = modified Bessel function;
- l = distance between the stream and the well;
- m_a = thickness of the aquitard;
- m_s = thickness of the streambed;
- p = Laplace transformation variable;
- Q = pumping rate;
- q = stream depletion rate;
- R = aquifer recharge;

S = storativity;
 T = transmissivity;
 t = time;
 w_s = width of the streambed;
 W = well function;
 x, y = spatial coordinates;
 α = Fourier transformation variable; and
 δ = Dirac delta function;

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