MOMENT DIFFERENTIAL EQUATIONS FOR FLOW IN HIGHLY HETEROGENEOUS POROUS MEDIA

C.L. WINTER
Mathematical Modeling & Analysis Group and Center for Nonlinear Studies, Los Alamos National Laboratory, Los Alamos, NM 87545, USA; E-mail: winter@anl.gov

D.M. TARTAKOVSKY
Mathematical Modeling & Analysis Group, Los Alamos National Laboratory, Los Alamos, NM 87545, USA; E-mail: dmt@lanl.gov

A. GUADAGNINI
D.I.I.A.R. - Politecnico di Milano, Piazza Leonardo da Vinci, 32, 20133 Milano, Italy; E-mail: alberto.guadagnini@polimi.it

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Abstract. Quantitative descriptions of flow and transport in subsurface environments are often hampered by uncertainty in the input parameters. Treating such parameters as random fields represents a useful tool for dealing with uncertainty. We review the state of the art of stochastic description of hydrogeology with an emphasis on statistically inhomogeneous (nonstationary) models. Our focus is on composite media models that allow one to estimate uncertainties both in geometrical structure of geological media consisting of various materials and in physical properties of these materials.

Keywords: composite media, nonstationary, statistical, statistically inhomogeneous, stochastic

Abbreviations: GPR – Ground Penetrating Radar; MCS – Monte Carlo simulations; MDE – Moment differential equation; PDE – Partial differential equation; PDF – Probability density function; REV – Representative elementary volume

1. Introduction

Although the hydrogeologic properties of aquifers and other natural porous media are deterministic in principle, our knowledge of them is usually incomplete. Hydrogeologic properties, for instance hydraulic conductivity and specific storage, are ordinarily observed at only a few locations despite the fact that they exhibit a high degree of spatial variability at multiple length scales. The combination of significant spatial variability, or heterogeneity, with a relatively small number of observations leads to uncertainty about the values of aquifer properties and, thus, to uncertainty in estimates of groundwater flow and pressure distribution. While uncertainty in the values of properties can be reduced by improved aquifer characterization techniques, it can never be entirely eliminated. When computational models of flow are used to assess water supply and quality in aquifers, the degree

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of uncertainty in predicted production must be quantified in terms of uncertainty in hydraulic parameters.

The theory of random, or *stochastic*, processes provides a natural framework for evaluating aquifer uncertainties. In the stochastic formalism, uncertainty is represented by probability or by related quantities like statistical moments. Boundary conditions, initial conditions, and parameters can be treated as random fields whose values are determined by probability distributions. For instance, rather than demand that the value of hydraulic conductivity, $K(\mathbf{x})$, at point $\mathbf{x}$ be determined at every point in a reservoir, the stochastic approach only requires the information needed to characterize $P[K_{\mathrm{Low}} < K(\mathbf{x}) < K_{\mathrm{Up}})]$, the probability that K at any given point $\mathbf{x}$ within an aquifer lies in an interval $[K_{\mathrm{Low}}, K_{\mathrm{Up}}]$. Only the relatively weak information obtained from sampling is needed to characterize P. In turn, dependent variables like pressure head, $h(\mathbf{x})$, and flux, $\mathbf{q}(\mathbf{x})$, are also random fields, and the equations governing flow become stochastic differential equations whose solutions are probability distributions like $P[h_{\mathrm{Low}} < h(\mathbf{x}) < h_{\mathrm{Up}}]$ or their moments.

In this paper we review continuum-scale, or Darcy-type, stochastic models of groundwater flow through saturated highly variable porous media. Material properties may be *heterogeneous* or *homogeneous*, depending on the uniformity of the medium. A porous medium is homogeneous in a volume if it was formed by basically the same physical processes throughout the volume and takes on similar values within it. At small scales groundwater flows through a pore space embedded in a solid phase, and flow depends on factors defined only within the pore space like pore length, radius, orientation and tortuosity. At larger continuum scales, hydrogeologic system properties are represented by variables like $K(\mathbf{x})$, that are aggregated over relatively small volumes of space and time. The aggregated property is defined throughout the entire flow domain – pore space plus solid – so that there is effectively only one phase with variables like $K(\mathbf{x})$ defined everywhere.

On the other hand, a stochastic model of flow is *stationary* if its statistical properties are uniform throughout the flow domain and is *nonstationary* otherwise. A random field is *strictly stationary* if the probability of obtaining an arbitrary set of values of the field for an arbitrary configuration of points depends only on the relative positions of the points and not on their exact location. More precisely, a random field $K(\mathbf{x})$ is strictly stationary if its finite-dimensional probability distributions are not affected by translation from one point to another within the volume

$$P(K(\mathbf{x}_1) < k_1, \ldots, K(\mathbf{x}_n) < k_n) = P(K(\mathbf{x}_1 + \triangle) < k_1, \ldots, K(\mathbf{x}_n + \triangle) < k_n) \tag{1}$$

for arbitrary sets, $\{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$, and numbers, n, of points and for arbitrary translations $\triangle$. Note that strict stationarity implies that the one-dimensional density $P(K(\mathbf{x}) < k) = P(K < k)$ is the same at every point. A process is *weakly* *stationary* if its mean and variance are the same at every point and if its covariance depends on just the distance between two points. In general a stationary assumption imposes a high degree of uniformity on a model since every point, and configuration of points, is statistically the same, no matter where it is located.

Stochastic continuum-scale models suppose that groundwater obeys Darcy's law by flowing down gradients of hydraulic head, $h(\mathbf{x})$, as modified by conductivity, $K(\mathbf{x})$. When combined with conservation of mass, Darcy's law leads to a flow equation depending on $K(\mathbf{x})$, boundary and initial conditions, sources and sinks and specific storage, $S(\mathbf{x})$. Any or all of these may be random (Tartakovsky and Neuman, 1998, 1999), although we concentrate on models in which only $K(\mathbf{x})$ is random to keep the discussion simple. The flow equation, and Darcy's law too, is a stochastic partial differential equation. It can be used to estimate $\overline{h}(\mathbf{x})$, mean head and other statistics, for instance $\sigma_h^2(\mathbf{x})$, the variance of head. We emphasize estimates based on *moment differential equations* or *MDE*s. MDEs are deterministic partial differential equations for the moments of $h(\mathbf{x})$ and other random hydraulic state variables. They are obtained directly from the stochastic flow equation by averaging. Although the first few moments of $h(\mathbf{x})$ are usually all that is needed to statistically characterize flow, the system of moment equations is almost never closed, so reasonable closure approximations must be obtained. These are ordinarily based on perturbation expansions, and one of the challenges of dealing with moment equations in highly heterogeneous porous media is satisfying closure requirements. The MDE approach has important computational and analytical advantages over Monte Carlo simulations (MCS). To capture heterogeneity, MCS require numerically solving the flow equation in many detailed realizations of $K(\mathbf{x})$. This can be computationally expensive, especially when the flow system has transient terms. Sometimes the closed MDE system can be solved analytically. Even when it cannot, the grids used to solve for moments can be coarser than those used for MCS. Each Monte Carlo grid must capture the detailed heterogeneity of parameter fields, while moment equations are based on smoothed, ensemble mean parameters. Additionally, MDE can be analyzed qualitatively, which is impossible with MCS.

A variety of studies suggest the importance of honoring geological features in hydrogeologic modeling. Webb (1995), Scheibe and Freyberg (1995), and Webb and Anderson (1996) simulated the geometrical distribution of facies from various depositional environments. Among others, Johnson and Dreiss (1989), Ritzi et al. (1994, 1995, 1996), Johnson (1995), and Langsholt et al. (1996) used ground penetrating radar data to condition predictions from statistically generated three-dimensional facies-based models and noted an improvement in the quality of flow and transport predictions. Adams and Gelhar (1992), Boggs and Adams (1992), Boggs et al. (1992) and Rehfeldt et al. (1992) pointed out that proper characterization of a buried channel was crucial in modeling the rate of solute spreading at the macro Dispersion site in Mississippi. Recently, Scheibe and Murray (1996) compared a series of stochastic simulation techniques for predicting flow and transport behavior in the subsurface and concluded that hydrogeologic models that preserve spatial distribution of geologic facies usually perform better. At the same time, it was noted that incorporating lithologic core information (Zhang and Brusseau, 1998) or using inverse techniques to extract relationships among sedimentary facies, gamma-ray values and hydraulic conductivities (Fisher et al., 1998) resulted in improved estimates of hydraulic conductivity. Along the same lines, Miller et al. (2000) incorporated multiple types of data (among which gamma-ray geophysical log data and percent clay data from cores) to recreate the stratigraphic sequence of materials in an aquifer in South Carolina.

After providing additional background for stochastic groundwater flow models in Section 1.1 and for their statistical solutions through MDE in Section 1.2, we classify stochastic models of flow according to how they represent highly variable media (Section 1.3). We review nonstationary models in Section 2, introduce a general model of heterogeneous media in Section 2.1 and consider two special cases in Sections 2.1.1 and 2.1.2. Of course, nonstationary models of heterogeneous media also demand more data than do those based on more uniform assumptions. Fortunately increasingly powerful aquifer characterization techniques are available or under development, so the approximate boundaries of different material blocks can often be characterized by geophysical surveying techniques. Errors for the boundary locations can usually be derived through geostatistics. We review the state of aquifer characterization in Section 4. Models with a deterministic trend in the mean fall approximately midway between stationary and nonstationary models. They are technically nonstationary because they allow mean log conductivity to vary from point to point, but they are stationary once the mean trend is removed. We review them in Section 2.2. In Section 3 we review stationary models that are appropriate for a unit composed of a single material. We discuss stationary models that account for material heterogeneity in Section 3.2.

1.1. Basic concepts of stochastic hydrogeology

Macroscopic (continuum-scale) description of fluid flow through porous media is based on *Darcy's Law*,

$$\mathbf{q} = -K\nabla h, \tag{2}$$

according to which the Darcian flux $\mathbf{q}(\mathbf{x}, t)$ of a fluid is down gradients of hydraulic head $h(\mathbf{x}, t)$, subject to constraints imposed by the medium's (scalar or tensor) conductivity $K(\mathbf{x})$. Coupled with conservation of mass, Darcy's law yields the groundwater *flow equation*,

$$S\frac{\partial h}{\partial t} = \nabla \cdot (K\nabla h) + f, \tag{3}$$

which, given appropriate initial and boundary conditions, uniquely describes the hydraulic head distribution. The specific storage, $S(\mathbf{x})$, is the volume of water released under a unit decline of h, and $f(\mathbf{x}, t)$ is a source function. The input parameters K, S, and f vary from point-to-point and their values may be relatively uncertain. In most cases, boundary and initial conditions are also highly uncertain. All these parameters and functions are conveniently described as random fields whose statistics, such as mean, variance, and two-point covariance can be inferred from field measurements. As already noted, we will concentrate on random $K(\mathbf{x})$ and fix other terms deterministically to simplify our discussion.

A random process is *ergodic* if ensemble and spatial averages can be interchanged. Ergodicity is required to estimate ensemble parameters from spatial measurements. It can be shown (Yaglom, 1987a, p. 216) that "in any application, non-ergodicity usually just means that the random function concerned is, in fact, an artificial union of a number of distinct ergodic stationary [statistically homogeneous] functions". It is possible for a process to be ergodic for some parameters and not for others. For the purposes of our survey it is enough to assume ergodicity of the mean and second statistical moments of the random function involved.

The need to estimate ensemble moments by spatial averages introduces the notion of scale. We review papers that assume (sometimes implicitly) fixed scales λ, for measurement, A, for averaging, and R, of the flow domain. The *measurement scale*, λ, is the size of the domain examined by an instrument or experiment designed to aggregate $K(\mathbf{x})$ from pore-scale properties. Variables like K are integrated over the *averaging scale*, A, to estimate macroscopic statistics. This is the scale on which ergodicity assumptions are applied. In general $\lambda \ll A \ll R$. Two other important scales are the *scale of material heterogeneity*, L, and the *correlation length* of a random field, l_K. The scale L is the characteristic length of a material inhomogeneity, for instance, the thickness of a layer in a stratified medium. Values of a random field are approximately independent when they are more than a distance l_K apart. If $A \approx \lambda$ or $A \approx l_K$, statistical estimates will be based on too small a sample to be meaningful. Since discussing the effects of scale on hydraulic parameters is outside the scope of this review, we refer the interested reader to Neuman (1994), Cushman (1997), Di Federico et al. (1999), and Winter and Tartakovsky (2001), among many others.

1.2. Solving stochastic flow equation

A complete solution of (3) is given by the finite-dimensional probability distributions, $P(h(\mathbf{x}_1) < h_1, \ldots, h(\mathbf{x}_n) < h_n)$, for hydraulic head h, and related distributions for the flux vector $\mathbf{q}$. In practice we can usually obtain sufficient data to estimate only the first two moments of h, specifically its mean $\overline{h}(\mathbf{x}, t)$ and variance $\sigma_h^2(\mathbf{x}, t)$. Another important statistic, cross-covariance between hydraulic conductivity and hydraulic head, C_{Kh}, is also of practical interest, since it is used in inverse modeling (McLaughlin and Townley, 1996).

Monte Carlo Simulation (MCS). Flow statistics can be obtained in a straightforward fashion by MCS. Early examples of such calculations are due to Warren and

Price (1961) in the petroleum literature and Freeze (1975) in groundwater. Multiple examples, or *realizations*, of the K field are generated via a pseudo-random procedure, flow is simulated for each realization by numerically solving (3), and the results analyzed statistically to estimate $\overline{h}(\mathbf{x}, t)$ and variance $\sigma_h^2(\mathbf{x}, t)$. Finely resolved numerical grids must be employed to properly resolve high-frequency space-time fluctuations in the random parameters. To avoid artificial boundary effects, these grids must span large space-time domains. Each sample calculation may therefore place a heavy demand on computer time and storage. To ensure that the sample moments converge to their (generally unknown) theoretical ensemble values, a very large number of Monte Carlo runs is often required. Even if some sample moments appear to stabilize after a sufficiently large number of runs, there is generally no guarantee that they have in fact converged.

Moment Differential Equations (MDEs). An alternative approach, the one we review here, is to solve directly for the moments of h by developing deterministic equations for the moments from (3). In general this involves taking the expected value of (3) and similar equations for higher order moments, closing the system of moment equations (usually through perturbation approximations), and solving the approximate system either analytically or, in most cases, numerically.

Consider steady-state flows without sources to keep the discussion simple. The ensemble averaged MDE for steady-state flow becomes

$$\nabla \cdot (\overline{K} \nabla \overline{h}) + \nabla \cdot \overline{\mathbf{r}} = 0 \tag{4}$$

which depends on a deterministic mean flux, $\overline{K}(\mathbf{x}) \nabla \overline{h}(\mathbf{x})$, and the mean of a random residual flux, $\overline{\mathbf{r}}(\mathbf{x}) = \overline{K'(\mathbf{x}) \nabla h'(\mathbf{x})}$. Solutions of (4) require the mean conductivity, $\overline{K}(\mathbf{x})$, and in most cases, a method for closing an expansion of $\overline{\mathbf{r}}(\mathbf{x})$. Usually $\overline{\mathbf{r}}$ is approximated through perturbation expansions based on σ_Y^2, the variance of $Y = \ln K$ (e.g., Dagan (1989) and references therein). This approach works well so long as σ_Y^2 is small, i.e., the conductivity field is mildly heterogeneous.

Numerical solutions for moment equations are typically computationally more efficient than Monte Carlo simulations. In the first place, taking expected values smoothes parameters in the moment equations, which in turn allows low-resolution grids for numerical solutions. Furthermore, the number of moment equations is much smaller than the number of realizations required by Monte Carlo simulations. Additionally, the moment equations lend themselves to qualitative analysis.

1.3. Classification of Models

Stochastic models of saturated groundwater flows can be classified according to a number of criteria. We show the loose taxonomy used to organize this paper in Figure 1. Our taxonomy breaks models down first according to statistical uniformity (stationary/nonstationary), then according to material uniformity (homogeneous/heterogeneous), and finally according to specific classes of models. Note that the actual models, not their taxonomy, is the focus of our discussion.

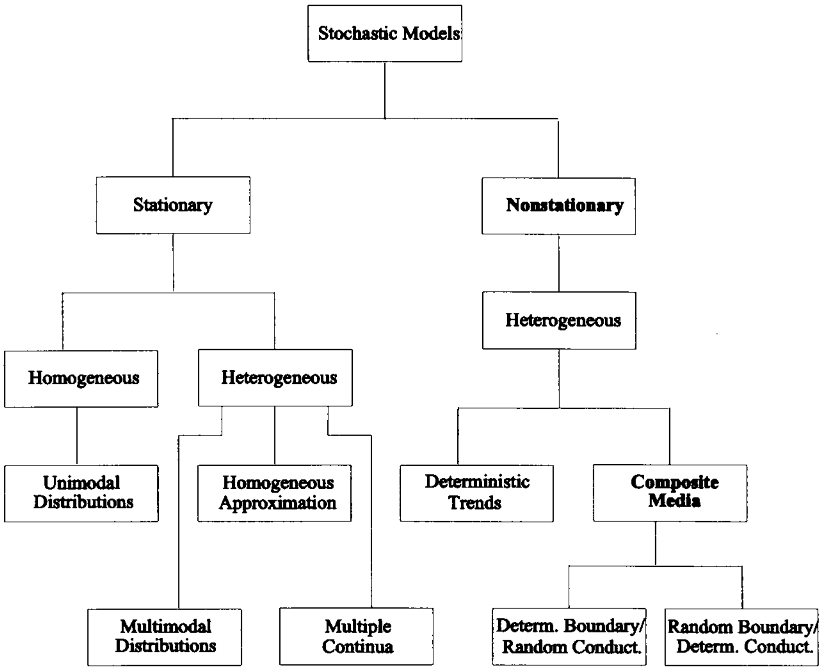

Figure 1. Model Taxonomy. Bold font indicates the emphasis of this review.

Nonstationary models are appropriate for heterogeneous media when sufficient data are available. Composite medium models explicitly take the spatial distribution of multiple materials into account. Technically a composite medium is a statistically inhomogeneous doubly stochastic process depending on: (i) the random geometry of the blocks, and (ii) the random spatial distribution of hydraulic conductivity within a block. The composite medium model substitutes the relatively tractable problem of determining the spatial distribution of disjoint blocks of homogeneous material for the difficult problem of dealing with large perturbation variances. Perturbation expansions for system states, such as hydraulic head $h(\mathbf{x}, t)$, can be sharpened by developing expressions for $\overline{K}(\mathbf{x})$ and $\overline{\mathbf{r}}(\mathbf{x}, t)$ that reflect material heterogeneity at both the within-block scale and especially, the larger, across-block scale. Errors in estimates of $\overline{K}$ and $\overline{\mathbf{r}}$ can be significantly reduced by accounting for the uncertain geometry of various geological units. In particular, perturbation expansions based on the composite medium approach rely only on small within-block variances. We survey such models in Section 2.1.

When the difference between materials is the result of a gradual change, for instance delta-forming sedimentation processes, it may suffice to model hydrologic variability as a statistically homogeneous process superimposed on a deterministic trend. We review models with a trend in Section 2.2. Since these models assume that the covariance structure of Y is the same throughout a porous medium, they do not apply to highly heterogeneous media composed of blocks of different materials.

Statistically homogeneous porous media can obviously be represented as stationary random fields. In such models a statistically homogeneous aquifer is assumed to consist of a single material with a unimodal, usually log-normal, distribution for $K(\mathbf{x})$. A considerable literature, which we discuss in Section 3.1, has grown up that analyzes Darcy flows in such media. Highly heterogeneous media can also be represented as stationary, but at the cost of large σ_Y^2 and therefore, relatively inaccurate perturbation expansions for $\bar{\mathbf{r}}$. In this case, the medium is composed of two or more materials, but sufficient information to specify the geometry of their spatial distribution is lacking. This has led to a class of models – reviewed in Section 3.2 – that represents the probability structure of conductivity as a multi-modal stationary distribution. From the point of view of estimating flow and pressure, the main problem with these approaches is the large variance of log conductivity, σ_Y^2, which violates the requirements of perturbation expansions. In dual-continuum models (Section 3.2.2) a porous medium is imagined to consist of two overlapping continua, or phases. Since the distribution of phases is the same throughout the medium, these models are intrinsically statistically homogeneous, and usually multi-modal.

2. Non-Stationary Models

At given scales of observation and averaging, most porous media are composed of distinct facies, or blocks, of internally uniform materials. A general stochastic model for a heterogeneous medium should ideally be non-stationary since accounting for the spatial distribution of material properties requires location-dependent probabilities and statistics. Most natural porous media are composite at some scales, and some porous media may be composite at all scales. In this paper we assume the given medium is heterogeneous at a fixed scale and the problem is to estimate the statistics of conductivity and of flow for the given scale.

The physical blocks of a heterogeneous medium suggest a probability model whose components are (i) a random geometry that defines the probable locations of blocks, and (ii) the random distribution of conductivity and other hydrogeologic parameters within a block. The general model also includes those in which a deterministic mean trend is superimposed on an otherwise stationary hydrogeologic process. However, we discuss trend models separately in this section because they have generated a sub-literature of their own. We also discuss the state of aquifer characterization techniques in this section because the data requirements of a non-stationary model are greater than those of stationary models. Nonstationary models yield more accurate estimates of head statistics and of the statistics of conductivity when the required data are available.

2.1. Composite media

Winter and Tartakovsky (2000) have proposed a composite stochastic model for steady-state flow through composite media that includes both geometric uncertainty about the distribution of blocks in space and uncertainty about the distribution of hydraulic conductivity within blocks. The model is based on a bivariate process consisting of (i) a subprocess, β, to define the spatial distribution of blocks, and (ii) a within-block subprocess to specify $K_i(\mathbf{x})$ for each of the $i = 1, \ldots, n$ blocks. Since the geological processes that form the material of a given block are almost always uniform, Winter and Tartakovsky (2000) assume $K_i(\mathbf{x})$, the within-block conductivity process, to be stationary within the ith block, or at worst to be some simple transformation of a stationary process. The conductivity process is assumed to be uncorrelated across blocks. This is physically reasonable when blocks are formed independently of each other. Winter and Tartakovsky (2000) follow the usual assumptions of stochastic hydrology in supposing that, for all i $\sigma^2_{Y_i}$, the variance of log conductivity, $Y_i = \ln K_i$, within the ith individual block, is small.

The boundary process, $\beta = \{\beta_1, \ldots, \beta_{n-1}\}$ consists of a set of $n-1$ surfaces, β_j, delineating the n blocks. The number of blocks, their boundary surface probabilities and the within-block distribution of conductivity must be determined from aquifer characterization studies. Winter and Tartakovsky (2002) derive $P_i(\mathbf{x})$, the block membership probability that the point $\mathbf{x}$ is in material i, from β. Thus, the boundary process implies the kinds of indicator functions used in geostatistics to describe the spatial distribution of geologic materials, and the data required to apply MDE to composite media are about the same as the requirements for MCS of heterogeneous media. Of course, the computational requirements of MDE are much less.

Approximations of MDE require estimates for both mean conductivity, $\overline{K}(\mathbf{x})$, and the mean residual flux, $\bar{\mathbf{r}}(\mathbf{x})$. We discuss the statistics of $K(\mathbf{x})$ in some detail because $\overline{K}(\mathbf{x})$ appears in all moment equation solutions of the flow equation and because the total variance of log conductivity, σ^2_Y, enters directly into perturbation approximations of mean residual flux in stationary models. In heterogeneous media σ^2_Y is generally quite large, especially near block boundaries, which renders perturbation expansions based on it inaccurate. On the other hand, perturbation approximations for flow through composite media only depend on the smaller parameters, $\sigma^2_{Y_i}$. Hence, they are usually much sharper than those based on stationary assumptions.

Mean conductivity,

$$\overline{K}(\mathbf{x}) = \sum_{i=1}^{n} P_i(\mathbf{x})\overline{K}_i \tag{5}$$

varies with location, $\mathbf{x}$, because $P_i(\mathbf{x})$ depends on the spatial distribution of materials. The block mean, $\overline{K}_i$, also depends on the material but is constant within a block. When the point $\mathbf{x}$ is deep in the ith material, $\overline{K}(\mathbf{x}) \approx \overline{K}_i$ since then $P_i(\mathbf{x}) \approx 1$ and $P_j(\mathbf{x}) \approx 0$ for $j \neq i$. Otherwise $\mathbf{x}$ is near a boundary, and $\overline{K}(\mathbf{x})$ is a location-dependent mixture of the (constant) mean conductivities near the boundary. The total variance of log conductivity,

$$\sigma_Y^2 = \sum_{i=1}^{n} P_i(\mathbf{x})\sigma_{Y_i}^2 + \sum_{i,j=1}^{n} P_i(\mathbf{x})P_j(\mathbf{x})\left(\overline{Y}_i - \overline{Y}_j\right)^2, \tag{6}$$

is approximately $\sigma_{Y_i}^2$ in the middle of the ith block. On the other hand, small $\sigma_{Y_i}^2$ does not guarantee small σ_Y^2 near block boundaries. In most cases, in fact, the sum including $(\overline{Y}_i - \overline{Y}_j)^2$ will dominate because mean conductivities of different materials can vary by orders of magnitude. The composite model does not use (6) to approximate $\overline{\mathbf{r}}(\mathbf{x})$. Instead, Winter and Tartakovsky (2000) calculate mean statistics in two steps, first by conditioning on block location and then by averaging over all possible block configurations. The conditioning step bases perturbation expansions on small variances, $\sigma_{Y_i}^2$, of individual block conductivities, rather than on the potentially large σ_Y^2. Furthermore, perturbations are based on the constants, $\sigma_{Y_i}^2$, instead of the variable σ_Y^2.

Winter and Tartakovsky (2002) develop the basic theory of composite media and compare it to stationary models for heterogeneous materials. Although the composite model usually leads to sharper perturbation approximations than stationary models, stationary models do perform better than the composite when one material occupies most of the flow domain. In that case the medium is nearly homogeneous anyway. Winter and Tartakovsky (2002) also evaluate the accuracy of perturbation approximations for moment differential equations by comparing approximate solutions for $\overline{K}$ and $\overline{\mathbf{r}}(\mathbf{x})$ to exact one-dimensional solutions. That paper considers flow in a bounded domain driven by two different boundary conditions. In each case flow is from right to left and head at the right boundary is held at zero. In one case, head is fixed at the left boundary while, in the other case, flux is fixed on the left.

Winter et al. (2002) perform conditional flow simulations in a simple composite domain consisting of two materials with random properties separated by random boundaries. The model domain consists of an outer square with a square inner inclusion whose side is taken to be a random variable. Head statistics resulting from (i) uncertainty in both the inclusion size and hydraulic conductivities (the most realistic situation), (ii) uncertainty only in the inclusion size, and (iii) uncertainty only in hydraulic conductivities of the two materials are computed and compared. Winter et al. (2002) note that uncertainty in the internal geometry results in smoother mean head profiles, and captures the main trend of the head predictor of case (i). After computing the head variance/covariance, Winter et al. (2002) find that a direct comparison of the relative importance of the two contributions, i.e., randomness in aquifer properties or randomness in material boundaries, is extremely difficult. The paper interprets the model of a formation with a single material as the upper limit of the bivariate stochastic process based on the location of boundaries between materials and the distributions of hydraulic properties within each material. Winter et al. (2002) also observe that the spatial pattern of head variance caused only by uncertainty in domain internal geometry is strongly dependent on the particular boundary conditions and location of the inclusion within the flow domain, shape and orientation of the tested inclusion. The paper concludes by stating that more complex shapes of internal boundaries between materials could give rise to head variance patterns that are difficult to interpret. However, preliminary results in the paper make it clear that large-scale block variability can have a significant effect on the moments of head.

Two special cases of composite media are especially important. In the first, block boundaries are assumed known, while conductivities within a given block are stationary. Although this model is a little artificial, it is of interest as a limiting case. The exchange of mass and momentum between blocks is especially easy to assess in this case since the block geometry is assumed known. The second case, when boundaries are stochastic but conductivity is deterministic within a block, has considerable practical significance.

2.1.1. *Deterministic geometry and uncertain conductivity*

One of the simplest models of natural heterogeneity is that of perfectly layered formations, for which the hydraulic conductivity varies only in the vertical direction. Interest in this model has risen on the one hand from the recognition that layering is common in most sedimentary formations and on the other hand from its simplicity. In a few cases the vertical correlation scale of natural formations was found to be less than one tenth of the horizontal scale (e.g., Sudicky, 1986; Hess et al., 1992). Gelhar et al. (1979) represent the variability of the hydraulic conductivity within each deterministic layer as a spatial stochastic process with constant expectation. The fundamental work of Matheron and de Marsily (1980) considers flow and transport in a perfectly stratified formation, where the flow velocity (i) is a random function of the elevation of the layer, (ii) is always parallel to the bedding, and (iii) is constant within each layer. Matheron and de Marsily (1980) also investigate an additional type of flow, namely flow driven by a uniform mean gradient tilted with respect to aquifer stratification. This particular regime is of interest in that it can be found in practical situations when recharge is present, and is explored in more detail by Salandin et al. (1991).

More recently, Indelman et al. (1996) and Fiori et al. (1998) considered radial flow in perfectly stratified formations. These analyses do not rely on the implicit description of each layer. Instead, log conductivity is modeled as a three-dimensional, statistically homogeneous random field with anisotropic Gaussian autocovariance. The requirement that $e = l_v/l_h$, the anisotropy ratio between the vertical, l_v, and horizontal, l_h, correlation scales be small corresponds to perfect layering.

2.1.2. *Uncertain geometry and deterministic conductivity*

The practical importance of this model arises because it is often a fair approximation to a full composite model. In many cases $\sigma^2_{Y_i}$ are much smaller than the differences between means. When that is so, $K_i(\mathbf{x})$ is effectively a constant, k_i, within a given block. Nonetheless, geometric uncertainty can induce high total variance in log conductivity near boundaries since we still have $\sigma^2_Y \approx \sum_{i \neq j} P_i(\mathbf{x}) P_j(\mathbf{x})$.

Levermore et al. (1986) investigate flow through composite media in which the material type of a point is set by a Poisson process. Fontes et al. (1999) show that a random walk through a one-dimensional medium composed of sites with very long waiting times (low conductivity) and short waiting times (high conductivity) is dominated by the low conductivity sites. This is not the case for two or more dimensions. Effective properties of homogeneous media with randomly located spherical inclusions are studied in Batchelor (1974) and in Kohler and Papanicolaou (1981). MCS of Haldorsen and Lake (1982), Begg and King (1985), and Begg et al. (1985) extend their analytical results to incorporate inclusions of an arbitrary shape.

2.2. Models with deterministic trend

The problem of identifying and removing spatial or temporal trends from available datasets is an important subject in itself and can serve as a subject for a separate review. It is used to analyze data in such diverse fields as biology (Crone and Gehring, 1998), medicine (Mungiole et al., 1999), and agriculture (Stroup et al., 1994). The reader interested in hydrogeologic applications should consult comparative analyses by Neuman and Jacobsen (1984), Russo and Jury (1987), Crawford and Hergert (1997), and Eggleston and Rojstaczer (1998) among others. Here we concentrate on analyzing flow in porous media where deterministic trends have already been identified.

The advantages of de-trending conductivity data prior to analyzing flow and transport are two-fold. First, most stochastic analyses based on spectral representations are applicable only to statistically homogeneous random fields (Li and McLaughlin, 1995). Removing trends in conductivity makes these techniques workable so long as the residuals are stationary. Second, existing techniques for analyzing stochastic groundwater flow and transport equations require closure approximations, which are robust as long as variance of log conductivity is small. This limitation can be overcome by removing conductivity trends when they are the sole source of nonstationarity (McLaughlin and Wood, 1988).

Most theoretical analyses of flow in random porous media with trends assume a linear trend in log hydraulic conductivity, $Y = \ln K$. This implies an infinite exponential growth of hydraulic conductivity, an assumption that may not be justified in many applications. While linear trends are convenient in the MDE approach (e.g., Indelman and Rubin, 1995), they are essential for classical spectral representation techniques (Loaiciga et al., 1993) since the latter require constant mean hydraulic gradient, **J**. The presence of a trend violates the requirement for a constant gradient (Li and McLaughlin, 1995; Indelman and Rubin, 1995), but its effect can be eliminated if the residuals that remain after removing a trend are stationary. Analyzing steady-state flow with constant mean head gradient, **J**, parallel to the direction of the linear log conductivity trend, Li and McLaughlin (1995) demonstrate that the assumption of local stationarity fails to conserve mass in the mean. It also leads to improperly behaving effective conductivity and cross-covariance between log conductivity and hydraulic head. Hence, generalizations of the classical spectral analysis, such as nonstationary spectral analysis of Li and McLaughlin (1991), must be used. This technique, however, still requires the underlying source of uncertainty, such as hydraulic conductivity or its de-trended residuals, to be statistically homogeneous.

The MDE approach to trends is more general because it allows trends, in principle, that have an arbitrary functional form, and residual, that are statistically nonstationary. However, to date solutions of the moment equations have only been obtained for stationary residuals. For example, Rubin and Seong (1994) present and analyze moment equations for flow through porous media with linear trends, whose mean head gradient, $\mathbf{J}(\mathbf{x})$, is aligned with, or perpendicular to, the direction of the trend. This approach was further generalized by Indelman and Rubin (1995, 1996) to allow for an arbitrary angle between $\mathbf{J}(\mathbf{x})$ and the direction of the linear trend. Zhang (1998) numerically solves MDE for steady-state flow in media with trends given by a second degree polynomial and by a periodic function.

Analyzing hydraulic conductivity and head data from the Waste Isolation Pilot Plant (WIPP) in New Mexico, Seong and Rubin (1999) observed a linear trend in log conductivity perpendicular to the mean flow direction. Seong and Rubin (1999) further compared the head semivariograms obtained from the model accounting for a trend (Rubin and Seong, 1994) and a statistically homogeneous model with the semivariogram inferred from the actual data. This comparison demonstrates that incorporating the log conductivity trend leads to results that are far superior to those obtained from statistically homogeneous models (Seong and Rubin, 1999). Accounting for the conductivity trend still results in a quantitative discrepancy between the head variogram derived from the moment equations and from experimental data, despite providing a great improvement over statistically homogeneous models. Seong and Rubin (1999) attribute such a discrepancy to, among other factors, the assumption that the conductivity trend is linear. Analysis by Eggleston and Rojstaczer (1998) of a data set from Columbus Air Force Base in Mississippi sheds some light on this question. Eggleston and Rojstaczer (1998) conclude that, while plume prediction is sensitive to the choice of a method for trend delineation (polynomial regression, Kalman filtering and simple kriging were used), hydrofacies delineation provides the best overall prediction of flow and transport at the site. The hydrofacies delineation of Eggleston and Rojstaczer (1998), which models a medium as a collection of geological units whose hydraulic conductivities and boundaries are deterministic, is a degenerate case of the simple composite medium of Section 2.1.2.

3. Stationary Models

Stationary models assume a high degree of statistical uniformity throughout a porous medium. A particular configuration of hydraulic conductivity is as likely to be found in one part of a stationary medium as in another. Ensemble means of hydraulic properties are constant everywhere, while their covariances are invariant with respect to translation. Such a high degree of statistical uniformity is justified in media composed of a single material. Analyses of flow through heterogeneous media may resort to a stationary assumption if data are not available to characterize the hydrogeology with a nonstationary, composite model.

3.1. Homogeneous media

Many applications of Darcy's equation deal with horizontal flow in a single homogeneous layer of an aquifer. Stationarity and small variance σ_Y^2 are often reasonable assumptions within such an aquifer, since then the conductivity at each point is generated by basically one physical process whose statistics can be supposed to be the same everywhere. The classical approach to modeling flow and transport in random porous media treats hydraulic conductivity as a lognormally distributed stationary random field.

The first analysis of seepage processes carried out by statistical methods seems to have been in a seminal paper by B. B. Devison published in 1938 (Shvidler, 1964). There the concepts of seepage velocity, porosity, and hydraulic conductivity are re-interpreted as mathematical expectations of statistical ensembles defined within a volume of the porous medium containing sufficiently many irregular interstitial channels to be ergodic. Shvidler (1964), Dagan (1989), Gelhar (1993), Cushman (1997), and Dagan and Neuman (1997) paint a fascinating picture of the development of stochastic hydrogeology over the years.

The assumption of stationarity is basic to the spectral analysis of random flows (Baker et al., 1978; Gelhar, 1993). In general, stationarity is required for Fourier representation of random fields, such as hydraulic conductivity $K(\mathbf{x})$ and hydraulic head $h(\mathbf{x}, t)$. Since the presence of boundary conditions renders $h(\mathbf{x}, t)$ statistically inhomogeneous, this approach is strictly limited to infinite domains and homogeneous initial conditions. This limitation can sometimes be relaxed in practical applications by employing the so-called local stationarity hypothesis (e.g., Mizell et al., 1982; Gelhar, 1986). According to this hypothesis, hydraulic head fluctuations, $h'(\mathbf{x})$, can be treated as a statistically homogeneous random field if mean hydraulic head $\mathbf{J} \equiv \nabla \overline{h}$ varies on a scale much larger than a scale of the h' variation. Such an assumption may lead to erroneous results, as mentioned earlier when we discussed models with deterministic trends.

MDEs, which are based on perturbation analyses without spectral representation, are applicable equally well to infinite (Shvidler, 1962; Matheron, 1967) or bounded (Naff and Vecchia, 1986; Rubin and Dagan, 1988) domains. In fact, they are formally valid for nonstationary conductivity fields as well (Neuman and Orr, 1993). However, perturbation expansions carried out in the global log conductivity variance, σ_Y^2, limits applicability of the results to very mildly heterogeneous media.

3.2. Heterogeneous media

Often a medium's random block geometry cannot be characterized because adequate geophysical data are not available or else because λ, the scale of measurement, is about the same as L, the block size, so there is not sufficient resolution to determine boundaries. Then data cannot be classified according to the type of material they came from, and they must be lumped together in a single sample regardless of material type. In that case a composite model cannot be applied. Nonetheless the distribution of hydraulic conductivity will reflect the composite nature of the medium through a very high variance. In most cases, the log conductivity sample will have a multi-modal sample density with modes near the means of individual materials.

3.2.1. *Multi-modal distributions*

Consider a geologic system consisting of several materials characterized by constant and deterministic hydraulic conductivities K_i. Journel (1983) uses a statistically homogeneous random indicator function $I(\mathbf{x})$ to describe such a medium consisting of two materials, M_1 and M_2. This indicator function is defined such that $I(\mathbf{x}) = 1$ for $\mathbf{x} \in M_1$ and $I(\mathbf{x}) = 0$ for $\mathbf{x} \in M_2$. The ensemble average of I corresponds to the volumetric fraction of the material M_1 in the flow domain, i.e., $\overline{I} = Q_1$. (Clearly, the volumetric fraction of the second material is defined as $Q_2 = 1 - Q_1$.) This results in a log conductivity field, $Y(\mathbf{x}) = Y_2 + (Y_1 - Y_2)I(\mathbf{x})$, with mean, $\overline{Y} = Q_1 Y_1 + Q_2 Y_2$, and variance, $\sigma_Y^2 = Q_1 Q_2 (Y_1 - Y_2)^2$. For materials with highly contrasting hydraulic properties (e.g., $Y_1 \gg Y_2$) the variance σ_Y^2 will be large, rendering standard perturbation solutions invalid. MCS have been used to obtain effective hydraulic conductivity of bimodal media (Desbarats, 1987), and to simulate flow and transport through such media (Desbarats, 1990).

Rubin and Journel (1991) generalize such bimodal models by letting Y_1 and Y_2 be statistically homogeneous, mutually uncorrelated random fields with constant means $\overline{Y}_i$ and variances $\sigma_{Y_i}^2$ ($i = 1$ or 2). This results in a (log) hydraulic conductivity with mean,

$$\overline{Y} = Q_1 \overline{Y}_1 + Q_2 \overline{Y}_2, \tag{7}$$

and variance,

$$\sigma_Y^2 = Q_1 \sigma_{Y_1}^2 + Q_2 \sigma_{Y_2}^2 + Q_1 Q_2 (\overline{Y}_1 - \overline{Y}_2)^2. \tag{8}$$

Despite such a generalization, the total variance of hydraulic conductivity, σ_Y^2, will still be large if the means of the two materials are sufficiently different. In the context of MDE, this obvious observation was confirmed by Rubin (1995) for flow and transport in media with a generalized bimodal distribution of conductivity. Monte Carlo simulations of flow and transport in such systems were reported by Rubin and Journel (1991).

3.2.2. *Dual-continua*

The dual-continuum model of Shvidler (1986, 1988) is conceptually similar to stationary models with multi-modal distributions in that each relies on the random indicator function, $I_i(\mathbf{x})$, to designate the membership of a point $\mathbf{x}$ in the material M_i. The main difference is that materials (continua) M_i are allowed to overlap in dual-continuum models. The possibility of such a co-existence of various materials at the same point $\mathbf{x}$ might seem troubling unless it is remembered that every point in the continuum description of porous media represents a volume that can be comprised of several materials M_i. Their volumetric fractions are now given by $\overline{I}_i(\mathbf{x}) = Q_i$. Since Shvidler (1986, 1988) assumes that the volumetric fractions, Q_i, are constant over an entire flow domain, his dual-continuum model is bivariate but stationary.

The main goal of the dual-continuum model has been to quantify the exchange of mass and momentum between fractured and matrix phases in fractured media. Such phenomenological models commonly assume that the cross flow between the materials at a point $\mathbf{x}$ is proportional to the pressure difference between these materials. Shvidler (1998) demonstrates that this assumption is valid only under restricted conditions. Zhang and Sun (2000) explore an alternative route to analyzing flow in dual-continuum models. The paper uses the phenomenological dual permeability model as a starting point and treats the model parameters, including the transfer coefficient between the two materials, as statistically homogeneous random fields.

4. Aquifer Characterization

Quantitative analysis of heterogeneous aquifers is an extremely complex issue, mainly due to the fact that we are usually working under conditions of data scarcity. Parameters like hydraulic conductivity (or transmissivity) and porosity are of interest to hydrologists since they control groundwater and solutes' paths, and rate of dispersion of solutes on various scales. Inclusion of relevant geological features in a model is essential in a proper characterization of a natural aquifer. Location-dependent mixtures like those used in the composite model provide a natural framework for incorporating the results of aquifer characterization in stochastic models. First, the method includes the kinds of spatially distributed material heterogeneities that are found in most characterization studies; second, error models for characterization techniques can be explicitly included (in principle) in models of random block boundaries; and third, the outputs of different characterizations can be combined using standard techniques like Bayesian updating since the re-normalization model is probabilistic.

Jussel et al. (1994a) suggest a procedure for the synthetic numerical generation of heterogeneous aquifer models based upon statistical description of heterogeneous structures in gravel formations. The paper investigates a large number of outcrops in several gravel pits in Switzerland, and is able to specify distinct structures (lenses and/or layers) on the basis of sedimentological information. The probability density function of the geometrical features of the observed sedimentary structures is inferred from visual inspection of photographs of the outcrops. Statistical analysis of the hydraulic parameters of each structure is then performed to assess the spatial variability of hydraulic conductivity and porosity. As expected, the standard deviation of (log)hydraulic conductivity within each material is moderately low and the within-block random process may be considered second-order stationary. On this basis, a Monte Carlo based synthetic generation of random layers and lenses of different material can be performed using the volume fractions determined in situ and a correlated random field of hydraulic conductivity can be generated within each type of material. This procedure has been used by Jussel et al. (1994b) to numerically investigate the transport of a conservative tracer. Due to time constraints, ten different stochastic realizations of gravel aquifers were evaluated. Resulting effective conductivity and dispersivities are compared to hydraulic and transport parameters predicted Gelhar and Axness (1983) and Dagan (1989). Jussel et al. (1994b) state that discrepancies between numerical experiments and the theory are mainly due to the fact that the basic assumptions of the theories are not net in the investigated gravel deposits, since the deposits cannot be satisfactorily reproduced by a single, homogenized structure. According to de Marsily (1986), this procedure may sometimes be misleading in that it assumes that the structure observed in outcrops does not change underground. With regard to this point, White and Willis (2000) propose a procedure for estimating dimensions of shales and other geologic bodies from analogous deposits exposed in outcrops. This procedure relies on an Erlangian probability density function to eliminate bias associated with inferring shale lengths observed in outcrops. More recently, Rauber et al. (1998) developed a three-dimensional stochastic facies-based aquifer model using known facies information to condition the random generation process. Unconditioned facies in the domain outside the known profile are generated randomly on the basis of sedimentary information collected in gravel pits of the same formation (Jussel et al., 1994a). Numerical MCS transport simulations were then performed (i) conditional to the observed GPR profile, considered as hard data, and (ii) unconditional, i.e., without the use of the GPR profile. The simulations show that conditioning does not reduce the uncertainty associated with transport but even increased it. Jussel et al. (1994a) attribute this effect to a discrepancy in the mean volumetric fraction of the different facies in the unconditional and conditional case.

The importance of properly representing stratigraphic aquifer structures at different scales of observation is recognized both in oil-reservoir modeling (e.g., Flint et al., 1998) and in aquifer modeling (Koltermann and Gorelick, 1996, and references therein). The approximate boundaries of different material blocks must be characterized by geophysical surveying techniques or other means. In any event, the number of blocks must be known, although their precise locations need not. Errors for the inter-block boundary locations can be derived through geostatistics.

Recently Bi et al. (1999) implemented a randomized maximum likelihood method to condition a three-dimensional stochastic channel to pressure data and well observation of channel thickness and depth of the channel top. Bi et al. (1999) adopted a probabilistic approach according to which the *a posteriori* probability density function of the model is conditioned on (and therefore makes full use of) the available pressure data and geometry information inferred from a well. By generating multiple realizations of the model one can (in principle) evaluate the reduction in uncertainty associated with flow and/or transport scenarios conditioned on geometry and state variables data.

Koltermann and Gorelick (1996) provide a review of numerous methods for interpolating between data values and then use geologic, hydrogeologic, and geophysical information to create images of aquifer properties. The paper focuses on methods for generating maps of spatial variations of hydraulic properties in clastic deposits at different scales of interest. The definition of scales is based on criteria taking into account (i) the effect of geologic features on heterogeneous spatial arrangement of hydraulic properties (e.g., Mast and Potter, 1963; Davis et al., 1993), (ii) the possibility of recognizing features in the field, (iii) the applicability to a variety of field situations, (iv) the measurability of hydraulic properties, and (v) diagenetic processes. Koltermann and Gorelick (1996) subdivide the existing approaches to create images of aquifers at the various scales into three general categories: structure-imitating, process-imitating, and descriptive.

Information used to designate zones includes geologic data and conceptual models. An extensive overview of geophysical methods for hydrogeological site characterization for both gross and detailed field studies is offered by Rubin et al. (1999). These geophysical techniques include:

- electrical
  - electrical resistivity (Ward, 1990; Van Nostrand and Cook, 1966),
  - electromagnetic induction (Frischknecht et al., 1991; Hoekstra and Bohm, 1990), and
  - ground-penetrating radar (Davis and Annan, 1989);
- seismic (Hyndman et al., 1994; Hyndman and Gorelick, 1996);
- gravimetric (Hinze, 1990; Butler, 1991);
- magnetic (Hinze, 1990);
- well logs (Keys, 1989).

Such data can be used as a complement to other available data (e.g., pumping tests) to obtain a clearer picture of an aquifer at different scales, with various levels of resolution. All geophysical methods measure a "surrogate" subsurface property, e.g., apparent resistivity, contrasts in dielectric properties, and seismic wave arrival times that must be either combined with (Hyndman et al., 1994), or transformed into (Rubin et al., 1999), hydrogeologic parameters by some kind of petrophysical model. Though these techniques are very flexible and sometimes easy to implement in the field, the interpretation of some of these techniques is not unique. Hence some results are subject to a certain amount of subjectivity, and very often it is advisable to couple observations resulting from different geophysical techniques.

5. Summary

Groundwater flows down gradients of pressure along paths of least resistance determined by spatial variations in the hydrogeologic properties of aquifers. This raises the question of how to include the effects of spatial variability in mathematical models of groundwater flow. Of course better information about the spatial distribution of properties can improve almost any model, and hydrogeologists have developed a number of aquifer characterization techniques to map the extent and degree of material heterogeneities. Nonetheless, it will never be possible to completely characterize all relevant details of the variability of an aquifer. "At the very least, we must recognize the uncertainties associated with our deterministic predictions due to the inherent nonuniformity of the porous media and to our uncertainty as to the exact nature of these nonuniformities" (Freeze, 1975). Thus hydrologists have also developed stochastic models to quantify the uncertainty that inevitably remains after even the most thorough characterization studies. The statistical moments of a flow system can be estimated by either Monte Carlo simulation (MCS) or by developing differential equations for the moments of head and other variables of flow. In this paper we have emphasized the method of moment differential equations (MDEs) because of its computational and analytical advantages.

The choice of an estimation method leaves open the question of which stochastic model to apply in a given setting. Stationary models assume a high degree of statistical uniformity in the spatial distribution of hydrogeological parameters. The initial success of stationary models can be attributed to the fact that most were applied to flow through a single homogeneous hydrogeologic unit, often a single layer of a stratified medium (see Dagan (1989), Gelhar (1993), and Dagan and Neuman (1997) for reviews of the early development of stochastic subsurface hydrology). The mathematical assumptions of stationary models break down when applied to highly heterogeneous porous media. In those circumstances, the distributions of hydrogeologic parameters usually become multi-modal, and stationary models greatly overestimate the variance of the hydrogeologic parameters, especially conductivity. This makes solving for the statistics of flow virtually impossible whether the method be MCS or moment differential equations (MDEs). MCS need a prohibitive number of realizations to sample accurately from the probability space, while solutions of MDE usually require perturbation parameters based on small variances.

When the structure and geometry of spatial variability can be inferred from characterization studies, an aquifer can usually be viewed at continuum scales as composed of disjoint blocks of homogeneous porous material. Such a composite medium corresponds to a bivariate stochastic process defined by both the uncertain locations of block boundaries and by spatial variations of hydrogeologic parameters within blocks. The latter is the kind of variability usually found in stationary models. Composite models lead to sharper closures for perturbation expansions of flow statistics. From a physical point of view, they allow evaluation of differential flow paths arising from structural variability, an effect that is washed out by stationary models. Composite models also lead to expressions for variable uncertainty near boundaries between blocks. Although it is possible to imagine more general non-stationary models than the composite model, it is not clear that more generality is necessary in hydrogeology. Furthermore, the composite model reduces to the traditional stationary model when there is only one block. Models based on assuming a deterministic trend superimposed on a stationary conductivity process are another special case of composite models.

A number of questions arise when flow models are extended to include additional sources of uncertainty and weaker assumptions. Foremost is the relative importance of the two sources of variation in composite models. That can be investigated by analyzing equation (6) in relation to expressions for $\overline{h}$, the mean pressure head. Equation (6) represents uncertainty in K as a function of within-block variability (the weighted sum of variances) and between block variations (the weighted squared differences between means). When the location probability weights are much greater than zero, between block variations clearly matter most. It remains to be seen how sensitive $\overline{h}$ is to these variations.

The relationship between scaling and heterogeneity is a major problem in hydrogeology that may be clarified by composite models. All of the models considered in this paper are defined for a specific (continuum) scale that is constrained by the measurement scale on the one hand and the scale of averaging on the other. It is clear, however, that many, perhaps most, porous media can be represented as composite on many scales. Using a composite model, it should be possible to investigate (i) the scaling of hydrologic variables as averaging volumes increase, and (ii) the relation of heterogeneities at various scales to the concept of a representative elementary volume over which parameters like conductivity are statistically uniform.

Thus far, analyses of composite models in multiple dimensions have been based on simple rectilinear block geometries. More realistic geometries may be represented by polynomials and other parametric curves whose parameters are random variables. It should be clear by now that composite models require a considerable amount of characterization data. In most applications, characterization data will come from many different kinds of observations and information, including expert opinion. This raises the closely related problems of fusing uncertain data from different sources into a single stochastic representation of a given medium and fusing new data into existing representations. Stochastic groundwater models in general, and composite models in particular, are compatible with the Bayesian techniques often used to combine data. Indeed, the confidence intervals that spring from data fusion are exactly what composite models require to represent the uncertain geometry. On the other hand, stationary multi-modal models are about the best that can be done when sufficient data are not available to characterize aquifer geometry. This raises the question of when stationary models are a reasonable approximation of heterogeneous media.

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