

An analytical solution for two-dimensional contaminant transport during groundwater extraction

Daniel M. Tartakovsky *

Scientific Computing Group (CIC-19), MS B256, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

Received 9 May 1999; received in revised form 24 August 1999; accepted 16 September 1999

Abstract

We obtain an analytical solution for two-dimensional steady-state transport of conservative contaminant between injecting and pumping wells. Flow and transport are considered in the vertical cross-section. The Dupuit approximation and conformal mapping onto the complex potential domain are employed to determine the velocity and concentration distributions, respectively. We use this solution to derive a priori conditions under which widely used 1-D analytical solutions with constant velocity and dispersion coefficients provide accurate approximations. These conditions are formulated in terms of aquifer parameters, such as hydraulic conductivity, porosity and dispersivities, and remediation strategy, e.g., well spacing and pumping regimes. Published by Elsevier Science B.V.

Keywords: Convection; Dispersion; Variable velocity; Analytical

1. Introduction

Groundwater extraction and in situ bioremediation have become popular tools for the restoration of contaminated aquifers. Corresponding contaminant transport models must include various physico-chemical processes, such as contaminant adsorption and desorption and biological degradation. Accounting for these phenomena makes the advection–dispersion equation (ADE) hard to solve even under very restrictive flow conditions.

* Fax: +1-505-667-1126; e-mail: dmt@lanl.gov

While in many practical applications the geometry of flow domains is complex and fluid velocity is nonuniform, most analytical solutions of ADE deal with one-dimensional (1-D) flows and constant velocities (e.g., van Genuchten and Wagenet, 1989; Fry et al., 1993; Fry and Istok, 1994). A brief overview of the existing analytical solutions is given by Tartakovsky and Di Federico (1997).

When injection and extraction wells are used to remediate an aquifer, the resulting flow field is clearly multi-dimensional. To complicate matters further, the vertical cross-section of the flow domain (Fig. 1) is bounded from above by a phreatic surface whose shape remains unknown until the flow problem is solved. While averaging the velocity field in the horizontal and vertical directions is essential for the applicability of 1-D analytical solutions, the complex shape of the flow domain makes it hard to ascertain the resulting errors.

For many two-dimensional (2-D) problems, changing the Euclidean (x, y) coordinate system to the curvilinear coordinate system of equipotentials, $\varphi(x, y)$, and streamlines, $\psi(x, y)$, simplifies the geometry of flow domains. Bear (1972, p. 233) called the (φ, ψ) coordinate system natural or intrinsic. In this coordinate system, a second-rank dispersivity tensor retains only two non-zero (diagonal) components, the longitudinal and transverse dispersivities. Also, it is more convenient to average velocity along streamlines than along Euclidean coordinates, since the former has a clear physical meaning. In particular, one can define such an averaged velocity as the distance between the injection and extraction wells divided by the time it takes for a particle to travel this distance (Dillon, 1989). Since both quantities are easily measured, the velocity averaged along streamlines can be readily obtained. Conformal mappings, $(x, y) \rightarrow (\varphi, \psi)$, were used to solve a variety of 2-D flow (e.g., Polubarinova-Kochina, 1962; Bear, 1972) and transport (Nikolaevskij, 1990, p. 438; and Tartakovsky and Di Federico, 1997) problems.

Dillon (1989) used numerical simulations to demonstrate that, on a regional scale, ADE with the averaged-along-streamlines velocity provides a good prediction of the contaminant fate. In this paper, we derive rigorously the conditions under which it becomes possible to use the averaged velocity and dispersion coefficients. We analyze

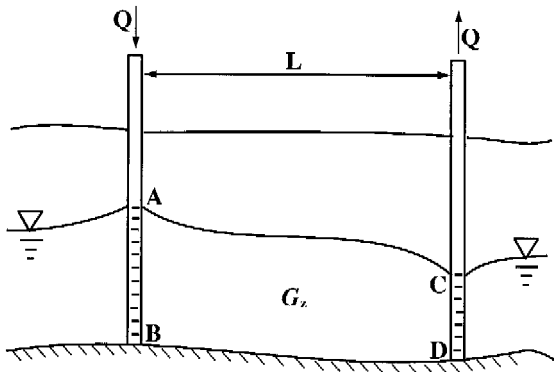


Fig. 1. Conceptual model of contaminant transport during groundwater extraction (after Fry et al. (1993)).

flow and transport in the cross-section between the injection and extraction wells (Fig. 1) in the following steps

1. An analytical solution for 2-D steady-state transport of conservative contaminant is obtained. The velocity distribution and velocity-dependent dispersion coefficients are found by solving the corresponding flow problem;
2. The same problem is solved for constant velocity and dispersion coefficients. Comparing the two solutions provides the conditions for operating the injection and extraction wells;
3. We demonstrate that, under these conditions, one can adopt the 1-D analytical solutions of Fry et al. (1993) and Fry and Istok (1994) for modeling remediation of the 2-D portion of an unconfined aquifer situated between the injection and extraction wells.

To obtain the velocity distribution, we employ the Dupuit approximation, i.e., we assume the flow to be essentially horizontal. Youngs (1965) demonstrated that the discharge towards a pumping well, calculated by means of the Dupuit assumption, is equal to the exact discharge. Dagan and Zeitoun (1998) suggested that the Dupuit assumption leads to a shape of the water-table which differs from its exact counterpart only in the vicinity of the well. Tartakovsky and Di Federico (1997) explored the effects of the Dupuit approximation on the contaminant transport.

2. Statement of the problem

Steady-state transport of a conservative contaminant in the cross-section between the injection and extraction wells (Fig. 1) is described by the 2-D steady-state ADE,

$$\nabla \cdot (\mathbf{D}\nabla C) - \mathbf{V} \cdot \nabla C = 0 \quad \mathbf{x} \in ABCD, \quad (1)$$

subject to the boundary conditions

$$-D_{11} \frac{\partial C(\mathbf{x})}{\partial x} + uC(\mathbf{x}) = 0 \quad \mathbf{x} \in AB$$

$$C(\mathbf{x}) = C_e \quad \mathbf{x} \in CD$$

$$\frac{\partial C(\mathbf{x})}{\partial y} = 0 \quad \mathbf{x} \in BC$$

$$\mathbf{n} \cdot \nabla C(\mathbf{x}) = 0 \quad \mathbf{x} \in AD. \quad (2)$$

Here $\mathbf{x} = [x, y]^T$ are the Euclidean space coordinates (L); $\nabla = [\partial/\partial x, \partial/\partial y]^T$; $\mathbf{n}(\mathbf{x})$ the outward unit normal vector of the phreatic surface, AD ; $C(\mathbf{x})$ the contaminant concentration ($M L^{-3}$); $\mathbf{V}(\mathbf{x}) = [u(\mathbf{x}), v(\mathbf{x})]^T$ the seepage velocity vector (LT^{-1}); and $\mathbf{D}(|\mathbf{V}|)$ the hydrodynamic dispersion coefficient, second-rank tensor with four non-zero components, D_{ij} , (L^2T^{-1}).

Zero contaminant flux (due to advection and dispersion) across the injection well, AB , satisfies conservation of mass within the control volume, $ABCD$, (van Genuchten

and Alves, 1982). It is assumed that concentration, C_e , at the extraction well, CD , remains constant and that there is no mass flux across either the impermeable base, BC , or the water-table, AD .

Steady-state seepage velocity distribution, $\mathbf{V}(\mathbf{x}) = \mathbf{q}(\mathbf{x})/n$ where \mathbf{q} is the Darcy’s flux (LT^{-1}) and n the porosity, is determined from Darcy’s law and mass conservation,

$$\mathbf{q}(\mathbf{x}) = -K\nabla h(\mathbf{x}) = \varphi(\mathbf{x}) \nabla \cdot \mathbf{q}(\mathbf{x}) = 0 \quad \mathbf{x} \in ABCD \tag{3}$$

subject to the boundary conditions (Polubarinova-Kochina, 1962, p. 33)

$$\begin{aligned} \varphi(\mathbf{x}) &= -KH_i \quad \mathbf{x} \in AB \\ \varphi(\mathbf{x}) &= -KH_e \quad \mathbf{x} \in CD \\ \psi(\mathbf{x}) &= 0 \quad \mathbf{x} \in BC \\ \varphi(\mathbf{x}) + Ky &= 0 \quad \psi(\mathbf{x}) = Q \quad \mathbf{x} \in AD. \end{aligned} \tag{4}$$

Here K is the uniform hydraulic conductivity (LT^{-1}); $h(\mathbf{x})$ the hydraulic head (L); H_i and H_e the hydraulic heads at the injection, AB , and extraction, CD , wells, respectively; Q the total water discharge per unit width (L^2T^{-1}); $\varphi(\mathbf{x}) = -Kh(\mathbf{x})$ the velocity potential function; and $\psi(\mathbf{x})$ the stream-function. Advantages and disadvantages of prescribing head rather than pumping rates at the wells are discussed by Indelman et al. (1996).

We now introduce complex variables $z = x + iy$ and $w = \varphi + i\psi$ ($i^2 = -1$) for the physical and complex potential planes, respectively. Then it follows from Eq. (4) that the flow region $ABCD$ of the physical plane corresponds to the rectangle G_w (Fig. 2) in the complex potential plane. In the complex potential plane, the boundary-value problem (1)–(2) takes the form (Bear, 1972, p. 620)

$$\frac{\partial}{\partial \varphi} \left(D_L \frac{\partial C}{\partial \varphi} \right) + \frac{\partial}{\partial \varphi} \left(D_T \frac{\partial C}{\partial \varphi} \right) - \frac{\partial C}{\partial \varphi} = 0 \quad (\varphi, \psi) \in G_w \tag{5}$$

subject to

$$\begin{aligned} -D_L \frac{\partial C(\varphi, \psi)}{\partial \varphi} + C(\varphi, \psi) &= 0 \quad \varphi = -KH_i \\ C(\varphi, \psi) &= C_e \quad \varphi = -KH_e \\ \frac{\partial C(\varphi, \psi)}{\partial \psi} &= 0 \quad \psi = 0 \\ \frac{\partial C(\varphi, \psi)}{\partial \psi} &= 0 \quad \psi = Q. \end{aligned} \tag{6}$$

Here $D_L(\mathbf{x})$ and $D_T(\mathbf{x})$ are the longitudinal and transverse dispersion coefficients, respectively ($L^2 T^{-1}$). Assuming that effects of molecular diffusion are smaller than those caused by convection, the dispersion coefficients are given by

$$D_L(\mathbf{x}) = \lambda_L V(\mathbf{x}) \quad D_T(\mathbf{x}) = \lambda_T V(\mathbf{x}) \tag{7}$$

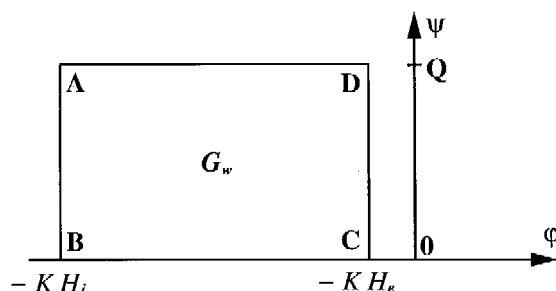


Fig. 2. Complex potential domain.

where $V(\mathbf{x}) = |\mathbf{V}(\mathbf{x})|$, and λ_L and λ_T are the longitudinal and transverse dispersivities, respectively (L). It follows from Eqs. (3) and (4) that V is not constant, and neither are D_L and D_T .

3. An analytical solution

We now assume that aquifer thickness is small compared with the distance, L , between the injection, AB , and extraction, CD , wells, so that the Dupuit assumption remains valid. As was discussed earlier, the Dupuit approximation affects the accuracy of the solutions only in the vicinity of the well. Since during aquifer remediation one is mostly concerned with the amount of contaminant removed from the ground, we feel comfortable to employ this approximation.

Under the Dupuit assumption, the boundary value problem (3)–(4) has the solution known as the Dupuit–Forchheimer discharge formula (Bear, 1972, p. 366),

$$Q = \frac{K(H_i^2 - H_e^2)}{2L} \quad h^2(x) = -\frac{2Q}{K}x + H_i^2 \quad V = \frac{Q}{nh}. \quad (8)$$

Recalling that $\varphi = -Kh$ and substituting the expression for V into Eq. (5) yields

$$\lambda_L \frac{Q}{nK^2} \frac{\partial}{\partial h} \left(\frac{1}{h} \frac{\partial C}{\partial h} \right) + \lambda_T \frac{Q}{nh} \frac{\partial^2 C}{\partial \psi^2} + \frac{1}{K} \frac{\partial C}{\partial h} = 0 \quad (h, \psi) \in G_w. \quad (9)$$

Due to the last two boundary conditions in Eq. (6), Eq. (9) becomes

$$\lambda_L \frac{Q}{nK} \frac{\partial}{\partial h} \left(\frac{1}{h} \frac{\partial C}{\partial h} \right) + \frac{\partial C}{\partial h} = 0 \quad H_e < h < H_i. \quad (10)$$

Using the transformation $z = h^2$ yields

$$\frac{d^2 C^*(z)}{dz^2} + a \frac{dC^*(z)}{dz} = 0 \quad H_e^2 < z < H_i^2 \quad (11)$$

subject to

$$\begin{aligned} \frac{dC^*(z)}{dz} + aC^*(z) &= 0 \quad z = H_i^2 \\ C^*(z) &= 1 \quad z = H_e^2. \end{aligned} \quad (12)$$

Here $C^* = C/C_e$ and $a^{-1} = 2 Q\lambda_L/nK$. The solution of Eqs. (11) and (12) is

$$C^*(h) = e^{a(H_e^2 - h^2)} = \exp\left(-\frac{n}{D^*} \frac{h^{*2} - 1}{H^{*2} - 1}\right) \tag{13}$$

where $h^* = h/H_e$, $H^* = H_i/H_e$, and $D^* = D_L/VL$ is the dimensionless mechanical dispersion coefficient.

Under different field scenarios, similar solutions were derived by Nikolaevskij (1990, p. 438) and Tartakovsky and Di Federico (1997). The latter authors demonstrated that the applicability of the Dupuit approximation to the contaminant transport is restricted to the ratios $\lambda_T/\lambda_L < 0.1$.

4. Comparison with the constant coefficients solution

We now compare C^* in Eq. (13) with the concentration profile, C_{av} , obtained by the velocity averaging. For each streamline, Dillon (1989) defined the averaged velocity as the distance, L , between the injection and extraction wells, divided by the experimentally determined time of travel (residence time). In a similar spirit, we define our averaged velocity from the Darcy’s law, $q_{av} = K(H_i - H_e)/L$ and $V_{av} = q_{av}/n$. Whereas Dillon (1989) assumed dispersion coefficients D_L and D_T to be constant and independent from V , we define our averaged dispersion coefficients in accordance with Eq. (7), $D_{i_{av}} = \lambda_i V_{av}$ where $i = L$ or T .

By virtue of the argument leading to Eq. (10), substituting V_{av} , $D_{L_{av}}$, and $D_{T_{av}}$ into Eqs. (5) and (6) yields

$$\frac{d^2 C_{av}^*}{d\varphi^2} - b \frac{dC_{av}^*}{d\varphi} = 0 \quad -KH_i < \varphi < KH_e \tag{14}$$

subject to

$$\begin{aligned} \frac{dC_{av}^*(\varphi)}{d\varphi} - bC_{av}^*(\varphi) &= 0 \quad \varphi = -KH_i \\ C_{av}^*(\varphi) &= 1 \quad \varphi = -KH_e \end{aligned} \tag{15}$$

where $C_{av}^* = C_{av}/C_e$ and $b^{-1} = \lambda_L K(H_i - H_e)/nL$. Since $\varphi = -Kh$,

$$C^*(h) = \exp\left(-\frac{n}{D^*} \frac{h^{*2} - 1}{H^{*2} - 1}\right). \tag{16}$$

Fig. 3 compares the dimensionless concentration distributions, C^* , (bold lines) with C_{av}^* for $n/D^* = 4.0$ ($n = 0.4$ and $D^* = 0.1$, typical values of porosity and dimensionless mechanical dispersion coefficient, respectively (Domenico and Schwartz, 1990)) and several values of the ratio H_i/H_e . For any pumping regime (any H_i/H_e), C_{av} underestimates the actual concentration distribution C . The accuracy of the averaged velocity solution, C_{av} , deteriorates as the ratio between hydraulic heads, H_i and H_e , as the injection and extraction wells increases. It is to be expected, however, since high hydraulic gradients make the validity of the Dupuit approximation questionable.

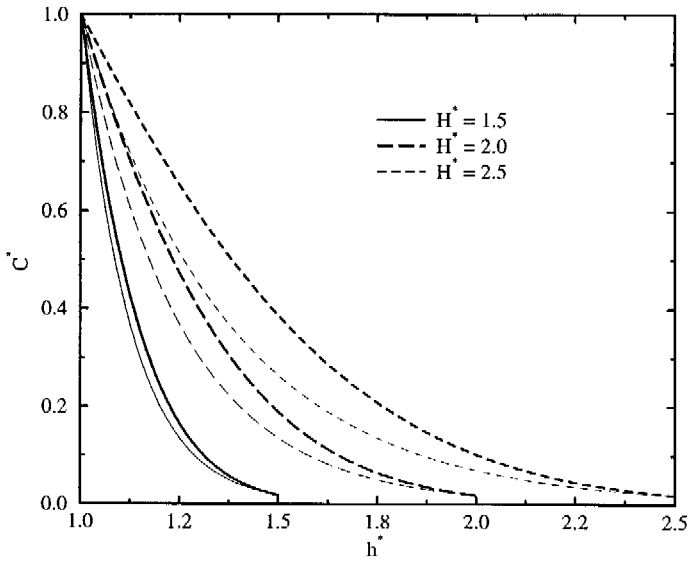


Fig. 3. Normalized concentration distributions for the variable (bold lines) and averaged velocity and dispersivities. Here $n = 0.4$, $D^* = 0.1$, and $H^* = H_i/H_c$.

Fig. 4 shows the relative error, $\mathcal{E} = (C - C_{av})/C$, introduced by the averaging of velocity and dispersion coefficients. Here the ratio between hydraulic heads at the

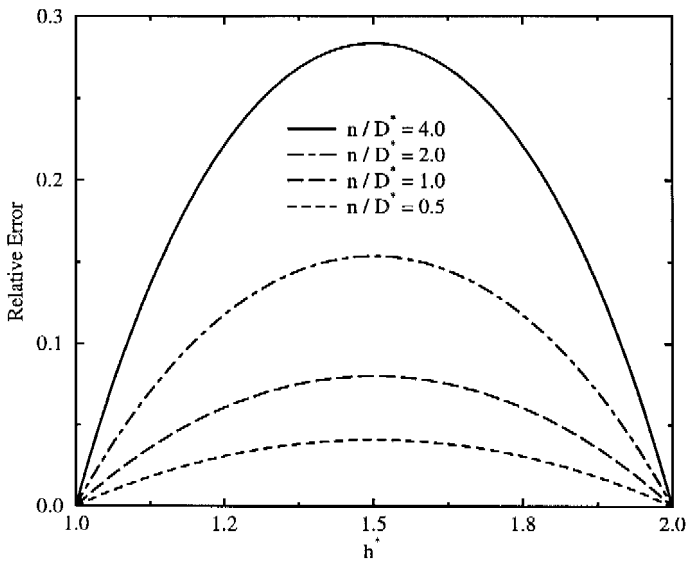


Fig. 4. Relative error, $\mathcal{E} = (C - C_{av})/C$, introduced by averaging velocity and dispersivities for $H^* = 2.0$ and several values of the ratio n/D^* .

injection and extraction wells is kept constant, $H_i/H_e = 2.0$, while n/D^* varies. It follows from Figs. 3 and 4 that the smaller the ratio n/D^* , the higher the hydraulic gradients are allowed for C_{av} to remain a reasonable approximation of C . This analysis shows that for any given aquifer (characterized by hydraulic conductivity, K , porosity, n , and dispersivity coefficients, λ_L and λ_T) it is possible to choose a priori a remediation strategy (i.e., the spacing between the injection and extraction wells, L , and the pumping regime, H_i and H_e) such that C_{av} is a good approximation of C .

5. Application to in situ bioremediation

We now apply our results to demonstrate the applicability of the 1-D analytical solutions by Fry et al. (1993) and Fry and Istok (1994) for modeling in situ bioremediation of a portion of the aquifer depicted on Fig. 1. Consider 2-D contaminant transport with rate-limited desorption and decay in the aqueous phase (Fry et al., 1993),

$$\frac{\partial C}{\partial t} + \frac{\rho_b}{\theta} \frac{\partial S}{\partial t} = \nabla \cdot (D \nabla C) - V \cdot \nabla C - \mu C$$

$$\frac{\partial S}{\partial t} = \alpha (K_d C - S) \quad \mathbf{x} \in ABCD \tag{17}$$

subject to the initial,

$$C(\mathbf{x},0) = C_0 \quad S(\mathbf{x},0) = S_0 \quad S_0 = K_d C_0 \quad \mathbf{x} \in ABCD, \tag{18}$$

and boundary,

$$-D_{11} \frac{\partial C(\mathbf{x},t)}{\partial x} + uC(\mathbf{x},t) = 0 \quad \mathbf{x} \in AB$$

$$\frac{\partial C(\mathbf{x},t)}{\partial x} = 0 \quad \mathbf{x} \in CD$$

$$\frac{\partial C(\mathbf{x},t)}{\partial y} = 0 \quad \mathbf{x} \in BC$$

$$\mathbf{n} \cdot \nabla C(\mathbf{x},t) = 0 \quad \mathbf{x} \in AD, \tag{19}$$

conditions. Here t is time (T); $C(\mathbf{x}, t)$ the aqueous concentration ($M L^{-3}$); C_0 the aqueous concentration at $t = 0$; $S(\mathbf{x}, t)$ the sorbed concentration ($M^3 M^{-3}$); S_0 the sorbed concentration at $t = 0$; ρ_b the bulk density ($M L^{-3}$), θ the volumetric water content ($L^3 L^{-3}$); μ the first-order decay rate coefficient (T^{-1}); α the first-order desorption rate coefficient (T^{-1}); and K_d the equilibrium distribution coefficient ($L^3 M^{-1}$).

To obtain their analytical solutions, Fry et al. (1993) and Fry and Istok (1994) made two simplifying assumptions. They (i) ignored spatial variations in the vertical (y) direction, and (ii) used velocity and dispersion coefficients averaged in the horizontal

(x) direction. As should become clear from the preceding discussion, none of these assumptions can be easily justified. Instead, we rewrite Eqs. (17)–(19) for the complex potential domain,

$$\frac{\partial C}{\partial t} + \frac{\rho_b}{\theta} \frac{\partial S}{\partial t} = V^2 \left[\frac{\partial}{\partial \varphi} \left(D_L \frac{\partial C}{\partial \varphi} \right) + \frac{\partial}{\partial \psi} \left(D_T \frac{\partial C}{\partial \psi} \right) - \frac{\partial C}{\partial \varphi} \right] - \mu C$$

$$\frac{\partial S}{\partial t} = \alpha (K_d C - S) \quad (\varphi, \psi) \in G_w \quad (20)$$

subject to

$$C(\mathbf{x}, 0) = C_0 \quad S(\mathbf{x}, 0) = S_0 \quad S_0 = K_d C_0 \quad (\varphi, \psi) \in G_w$$

$$-D_L \frac{\partial C(\varphi, \psi, t)}{\partial \varphi} + C(\varphi, \psi, t) = 0 \quad \varphi = -KH_i$$

$$\frac{\partial C(\varphi, \psi, t)}{\partial \varphi} = 0 \quad \varphi = -KH_e$$

$$\frac{\partial C(\varphi, \psi, t)}{\partial \psi} = 0 \quad \psi = 0$$

$$\frac{\partial C(\varphi, \psi, t)}{\partial \psi} = 0 \quad \psi = Q. \quad (21)$$

We now assume that hydraulic heads at the injection and extraction wells, H_i and H_e are such that the conditions derived in the previous section are satisfied. Then it is possible to employ the averaged-along-streamlines velocity and dispersion coefficients. For constant V_{av} , $D_{L_{av}}$ and $D_{T_{av}}$, Eq. (20) and the last two boundary conditions in Eq. (21) suggest that $\partial C / \partial \psi = 0$. Then Eqs. (20) and (21) reduce to

$$\frac{\partial C}{\partial t} + \frac{\rho_b}{\theta} \frac{\partial S}{\partial t} = V_{av}^2 \left[D_{L_{av}} \frac{\partial^2 C}{\partial \varphi^2} - \frac{\partial C}{\partial \varphi} \right] - \mu C$$

$$\frac{\partial S}{\partial t} = \alpha (K_d C - S) \quad -KH_i < \varphi < KH_e \quad (22)$$

subject to

$$C(\varphi, 0) = C_0 \quad S(\varphi, 0) = S_0 \quad S_0 = K_d C_0 \quad -KH_i < \varphi < KH_e$$

$$-D_{L_{av}} \frac{\partial C(\varphi, t)}{\partial \varphi} + C(\varphi, t) = 0 \quad \varphi = -KH_i$$

$$\frac{\partial C(\varphi, t)}{\partial \varphi} = 0 \quad \varphi = -KH_e. \quad (23)$$

Introducing dimensionless variables and parameters, $C^* = C/C_0$, $S^* = S/S_0$, $\gamma^* = \rho_b K_d / \theta$, $\mu^* = \mu L / V_{av}$, $\alpha^* = \alpha L / V_{av}$, $t^* = tL / V$, $\varphi^* = \varphi / (V_{av} L)$, $\varphi_i^* = KH_i / (V_{av} L)$, and $\varphi_e^* = KH_e / (V_{av} L)$, yields

$$\frac{\partial C^*}{\partial t^*} + \gamma^* \frac{\partial S^*}{\partial t^*} = D^* \frac{\partial^2 C^*}{\partial \varphi^{*2}} - \frac{\partial C^*}{\partial \varphi^*} - \mu^* C^*$$

$$\frac{\partial S^*}{\partial t^*} = \alpha^* (C^* - S^*) \quad -\varphi_i^* < \varphi^* < \varphi_e^* \tag{24}$$

subject to

$$C^*(\varphi^*, 0) = 1 \quad S^*(\varphi^*, 0) = 1 \quad -\varphi_i^* < \varphi^* < \varphi_e^*$$

$$-D^* \frac{\partial C^*(\varphi^*, t^*)}{\partial \varphi^*} + C^*(\varphi^*, t^*) = 0 \quad \varphi^* = -\varphi_i^*$$

$$\frac{\partial C^*(\varphi^*, t^*)}{\partial \varphi^*} = 0 \quad \varphi^* = -\varphi_e^* \tag{25}$$

We further notice that Eqs. (24) and (25) are the same as Eqs. (6) and (9) of Fry et al. (1993) with x^* replaced by φ^* . Hence the analytical solution of Eqs. (24) and (25), $C^*(\varphi^*, t)$, is given by their Eqs. (40)–(43). The advantages of using our solution are twofold: (i) all assumptions and limitations necessary for deriving our solution are clearly stated and are easy to verify; and (ii) while being one-dimensional, our $C^*(\varphi^*, t)$ represents the solution of the 2-D ADE. Moreover, evaluation of the total mass of the contaminant removed from the portion of an aquifer, $ABCD$, is now straightforward.

6. Summary

We considered contaminant transport between the injection and extraction wells during aquifer remediation. The problem was analyzed in the vertical cross-section passing through the wells. Employing (i) the Dupuit assumption to evaluate velocity distribution and (ii) conformal mapping onto the complex potential domain, we derived the analytical solution, C , for the two-dimensional advection–dispersion equation (ADE) with space-variable velocity, V , and velocity-dependent dispersion coefficients, D_L and D_T .

Our analytical solution was then used to ascertain the validity of the commonly used approach, wherein constant (averaged) velocity, V_{av} , and dispersion coefficients, $D_{L_{av}}$ and $D_{T_{av}}$, are used to simplify mathematical analysis of various transport phenomena. We demonstrated that the solution, C_{av} , of ADE with the averaged coefficients, V_{av} , $D_{L_{av}}$ and $D_{T_{av}}$, underestimates the “true” solution, C . Given aquifer parameters (hydraulic conductivity, K , porosity, n , and longitudinal and transverse dispersivities, λ_L and λ_T) and remediation strategy (spacing between the injection and extraction wells, L , and hydraulic heads at the wells, H_i and H_e), the accuracy of the approximate

solution, C_{av} , can be assessed a priori. Alternatively, for any given aquifer, one can choose a remediation strategy which can be modeled by simple one-dimensional analytical solutions.

Acknowledgements

The author thanks the two reviewers, whose helpful comments and suggestions greatly improved the paper. This work was supported by Los Alamos National Laboratory under LDRD 98604.

References

- Bear, J., 1972. Dynamics of Fluids in Porous Media. Elsevier, New York.
- Dagan, G., Zeitoun, D.G., 1998. Free-surface flow toward a well and interface upconing in stratified aquifers of random conductivity. *Water Resour. Res.* 34, 3191–3196.
- Dillon, P.J., 1989. An analytical model of contaminant transport from diffuse sources in saturated porous media. *Water Resour. Res.* 25, 1208–1218.
- Domenico, P.A., Schwartz, F.W., 1990. Physical and Chemical Hydrogeology. Wiley, New York.
- Fry, V.A., Istok, J.D., Guenther, R.B., 1993. An analytical solution to the solute transport equation with rate-limited desorption and decay. *Water Resour. Res.* 29, 3201–3208.
- Fry, V.A., Istok, J.D., 1994. Effects of rate-limited desorption on the feasibility of in situ bioremediation. *Water Resour. Res.* 30, 2413–2422.
- Indelman, P., Fiori, A., Dagan, G., 1996. Steady flow toward wells in heterogeneous formations: mean head and equivalent conductivity. *Water Resour. Res.* 32, 1975–1984.
- Nikolaevskij, V.N., 1990. Mechanics of Porous and Fractured Media. Taeneck, N.J. Singapore.
- Polubarinova-Kochina, P.Ya., 1962. Theory of Ground Water Movement. Princeton Univ. Press, Princeton.
- Tartakovsky, D.M., Di Federico, V., 1997. An analytical solution for contaminant transport in nonuniform flow. *Transp. Porous Media* 27, 85–97.
- van Genuchten, M.Th., Wagenet, R.J., 1989. Two-cite/two-region models for pesticide transport and degradation: Theoretical development and analytical solutions. *Soil Sci. Soc. Am. J.* 53, 1303–1310.
- van Genuchten, M.Th., Alves, W.J., 1982. Analytical solutions of the one-dimensional convective-dispersive solute transport equation. *Tech. Bull.*, 1661, US Department of Agriculture, Riverside, CA.
- Youngs, E.G., 1965. Horizontal seepage through unconfined aquifers with hydraulic conductivity varying with depth. *J. Hydrol.* 3, 283–296.