Type curve interpretation of late-time pumping test data in randomly heterogeneous aquifers

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[1] The properties of heterogeneous media vary spatially in a manner that can seldom be described with certainty. It may, however, be possible to describe the spatial variability of these properties in terms of geostatistical parameters such as mean, integral (spatial correlation) scale, and variance. Neuman et al. (2004) proposed a graphical method to estimate the geostatistical parameters of (natural) log transmissivity on the basis of quasi–steady state head data when a randomly heterogeneous confined aquifer is pumped at a constant rate from a fully penetrating well. They conjectured that a quasi–steady state, during which heads vary in space-time while gradients vary only in space, develops in a statistically homogeneous and horizontally isotropic aquifer as it does in a uniform aquifer. We confirm their conjecture numerically for Gaussian log transmissivities, show that time-drawdown data from randomly heterogeneous aquifers are difficult to interpret graphically, and demonstrate that quasi–steady state distance-drawdown data are amenable to such interpretation by the type curve method of Neuman et al. The method yields acceptable estimates of statistical log transmissivity parameters for fields having either an exponential or a Gaussian spatial correlation function. These estimates are more robust than those obtained using the graphical time-drawdown method of Copty and Findikakis (2003, 2004a). We apply the method of Neuman et al. (2004) simultaneously to data from a sequence of pumping tests conducted in four wells in an aquifer near Tübingen, Germany, and compare our transmissivity estimate with estimates obtained from 312 flowmeter measurements of hydraulic conductivity in these and eight additional wells at the site. We find that (1) four wells are enough to provide reasonable estimates of lead log transmissivity statistics for the Tübingen site using this method, and (2) the time-drawdown method of Cooper and Jacob (1946) underestimates the geometric mean transmissivity at the site by 30–40%.

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1. Introduction

[2] The interpretation of pumping tests has traditionally been based on analytical solutions of groundwater flow equations in relatively simple domains, consisting of one or at most a few units assumed to have uniform hydraulic properties. A method to evaluate aquifer characteristics in the presence of a radial discontinuity around a pumping well was described by *Sternberg* [1969]. *Chu and Grader* [1991, 1999] developed a generalized analytical solution for transient pressure interference tests in a composite aquifer which allows considering up to three uniform, isotropic regions of finite or infinite extent having varied geometries; placing active and observation wells at diverse locations within the composite system; prescribing constant flow rate, pressure or slug injection/withdrawal at active wells having zero or finite radius, the latter including storage and skin; and simulating faults or boundaries between fluid banks using "boundary skins" between regions.

[3] *Meier et al.* [1998] investigated theoretically the meaning of results obtained when using the *Cooper and Jacob* [1946] semilogarithmic straight line method to determine aquifer properties graphically from constant rate pumping tests in heterogeneous aquifers. Their analysis supported a number of field studies suggesting that the method yields a relatively narrow range of transmissivity estimates [*Schad and Teutsch*, 1994; *Sánchez-Vila et al.*, 1999]. It led them to conclude that using the Cooper-Jacob method to analyze late drawdown data from various observation wells in a given test yields a narrow range of transmissivity estimates representing an effective value and diverse storativity estimates providing qualitative information about how well the pumping and each observation well are interconnected hydraulically.

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[4] The properties of heterogeneous media vary spatially in a manner that can seldom be described with certainty. It may however be possible to describe the spatial variability of these properties in terms of geostatistical parameters such as mean, (integral) spatial correlation scale and variance. A recent development has been the use of geostatistical inversion to assess the spatial variability of medium properties on the basis of multiple cross-hole pressure interference tests. The approach yields detailed "tomographic" estimates of how these properties vary in three-dimensional space as well as measures of corresponding estimation uncertainty. The idea, originally proposed by *Neuman* [1987], has been used by *Vesselinov et al.* [2001a, 2001b] to obtain high-resolution three-dimensional tomographic images of air permeability and air-filled porosity in unsaturated fractured tuffs at a field site in Arizona, and to quantify the corresponding estimation uncertainties, on the basis of transient cross-hole pneumatic interference tests. Though Vesselinov et al. used a geostatistical method to parameterize medium properties, their flow analysis was deterministic. More recent efforts concerning hydraulic tomography have been discussed by *Zhu and Yeh* [2006].

[5] Numerical inversion is computationally intensive and requires considerable sophistication. It has been suggested by *Yortsos* [2000] that, in principle, one should be able to estimate the variogram parameters of a heterogeneous aquifer from the analysis of pressure transients in multiple wells using a more direct approach. *Copty and Findikakis* [2003, 2004a] used two-dimensional numerical Monte Carlo simulations to investigate the manner in which transient drawdowns due to pumping at a constant rate from a randomly heterogeneous, statistically homogeneous confined aquifer differ from those predicted by the *Theis* [1935] equation. On the basis of these results *Copty and Findikakis* [2003, 2004a] proposed estimating the mean of log transmissivity (related to the geometric mean transmissivity T_G) using methods based on the Theis solution (particularly the *Cooper and Jacob* [1946] semilogarithmic straight line analysis of late time data advocated also by *Meier et al.* [1998]), the integral (spatial correlation) scale from the time needed for drawdown time rate to approach that in a corresponding uniform aquifer, and the variance through a numerical least squares fit of drawdown time rate versus log time with type curves of mean drawdown time rate versus log normalized time, provided for drawdowns measured in the pumping well. The authors found their approach to yield reasonable estimates of geometric mean log transmissivity, acceptable estimates of integral scale but less satisfactory estimates of variance. *Copty and Findikakis* [2004b] proposed estimating the probability density function (pdf) of log transmissivity statistics numerically using Bayesian inversion of time-drawdown data from the pumping well (without considering wellbore storage or skin effects). Their posterior pdfs of the integral scale and the variance improved and sharpened as the number of pumping tests increased from 1 to 10.

[6] *Neuman et al.* [2004] proposed a simple graphical approach to estimate the mean, integral scale and variance of (natural) log transmissivity on the basis of quasi–steady state head data when a randomly heterogeneous confined aquifer is pumped at a constant rate from a fully penetrating well. They consider log transmissivity to vary randomly over horizontal distances that are small in comparison to a characteristic spacing between pumping and observation wells during a test. Experimental evidence and hydrogeologic scaling theory suggest that the corresponding integral scale would be considerably smaller than the maximum well spacing [e.g., *Neuman and Di Federico*, 2003, section 3]. This is in contrast to equivalent transmissivities derived from pumping tests by treating the aquifer as being locally uniform (on the scale of each test), which tend to exhibit regional-scale spatial correlations [e.g., *Anderson*, 1997, Table 1; *Neuman and Di Federico*, 2003, Figure 17]. *Neuman et al.* [2004] showed that whereas the mean and integral scale of local log transmissivity can be estimated on the basis of theoretical ensemble mean variations of head and drawdown with radial distance from a pumping well, estimating the log transmissivity variance requires considering the manner in which the theoretical standard deviation of circumferentially averaged drawdown (about its mean) varies radially.

[7] *Neuman et al.* [2004] based their type curve approach on the conjecture that a quasi–steady state flow regime, during which hydraulic heads vary in space and in time while hydraulic gradients vary only in space, develops in a randomly heterogeneous aquifer as it does in a uniform aquifer. In this paper we confirm their conjecture numerically for the special case where log transmissivity is a statistically homogeneous Gaussian field. We show that whereas random time-drawdown data are difficult to interpret graphically in a statistically meaningful manner, distance-drawdown data representing quasi–steady state are amenable to such interpretation by the type curve method of Neuman et al. Given sufficient data the method yields acceptable estimates of statistical log transmissivity parameters for fields having either an exponential or a Gaussian spatial correlation function, significantly outperforming the time-drawdown method proposed for this purpose by *Copty and Findikakis* [2003, 2004a]. Our paper ends with an application to field data from a sequence of pumping tests conducted in an aquifer near Tübingen, Germany.

2. Computational Analysis

2.1. Problem Definition

[8] When water is withdrawn at a constant volumetric rate Q from a well of negligible radius fully penetrating a uniform confined aquifer of infinite lateral extent, the vertically averaged drawdown s evolves according to the well-known *Theis* [1935] equation. We are interested to know how s would evolve if the transmissivity T of the aquifer was an autocorrelated random field. To address this we consider s to be governed by a two-dimensional stochastic transient flow equation

$$\frac{\partial}{\partial x}\left[T(x,y)\frac{\partial s}{\partial x}\right]+\frac{\partial}{\partial y}\left[T(x,y)\frac{\partial s}{\partial y}\right]-\delta(x-x_0,y-y_0)Q=S\frac{\partial s}{\partial t} \tag{1}$$

in which x and y are horizontal Cartesian coordinates, t is time since pumping starts, $\delta(x - x_0, y - y_0)$ is the Dirac delta function, Q is the rate of pumping from a well located at (x_0, y_0) and S is storativity. As spatial variations in S have lesser impact on flow than do spatial variations in T [e.g.,

Table 1. Number of Monte Carlo Runs Conducted for Various Combinations of $\sigma_Y{}^2$ and λ_Y

Parameters	Number of Runs
$\sigma_Y^2 = 0.5$, $\lambda_Y = 1$	1500
$\sigma_Y^2 = 1.0$, $\lambda_Y = 1$	2600
$\sigma_Y^2 = 1.5$, $\lambda_Y = 1$	2800
$\sigma_Y^2 = 2.0$, $\lambda_Y = 1$	3000
$\sigma_Y^2 = 0.5$, $\lambda_Y = 2$	1500
$\sigma_Y^2 = 1.0$, $\lambda_Y = 2$	2800
$\sigma_Y^2 = 1.5$, $\lambda_Y = 2$	3000
$\sigma_Y^2 = 2.0$, $\lambda_Y = 2$	3200
$\sigma_Y^2 = 0.5$, $\lambda_Y = 3$	1500
$\sigma_Y^2 = 1.0$, $\lambda_Y = 3$	2900
$\sigma_Y^2 = 1.5$, $\lambda_Y = 3$	3100
$\sigma_Y^2 = 2.0$, $\lambda_Y = 3$	3300
$\sigma_Y^2 = 0.5$, $\lambda_Y = 4$	1900
$\sigma_Y^2 = 1.0$, $\lambda_Y = 4$	3000
$\sigma_Y^2 = 1.5$, $\lambda_Y = 4$	3200
$\sigma_Y^2 = 2.0$, $\lambda_Y = 4$	3400

Dagan, 1982; *Oliver*, 1993] we treat the former as a deterministic constant, assigning to it an arbitrary (computationally convenient) value of 0.01. On the other hand we take the (natural) log transmissivity $Y = \ln T$ to be a statistically homogeneous multivariate Gaussian random field with variance σ_Y^2 and an isotropic exponential variogram

$$\gamma(r_s) = \sigma_Y^2(1 - \exp(-r_s/\lambda_Y)) \tag{2}$$

where r_s is separation distance and λ_Y the integral (spatial correlation) scale. Ideally, we would like to solve (1) subject to $s = 0$ at initial time and at infinite distance from the pumping well. In reality, we solve the problem by numerical Monte Carlo simulation on a finite difference grid of 501 × 501 square cells measuring 0.2 × 0.2 arbitrary consistent length units. A well pumping at a constant rate of Q = 100 consistent units of volume per time is placed at the central grid coordinate $x = 50.1$, $y = 50.1$. The grid length is chosen so as to place the boundaries far enough to minimize their impact on computed drawdowns around the pumping well, and the cell lengths small enough so as not to exceed one fifth of any integral scale among those considered. Setting $s = 0$ (we actually solve for heads by setting their initial and boundary values equal to 100 length units) along the square grid boundaries would thus necessitate filtering out their effect in a manner we describe later.

2.2. Monte Carlo Simulation of Log Transmissivities and Hydraulic Heads

[9] We start by generating unconditional random realizations of $Y = \ln T$ with zero mean (geometric mean transmissivity $T_G = 1$), variances $0.5 \le \sigma_Y^2 \le 2$ in increments of 0.5 and integral scales $1 \le \lambda_Y \le 4$ in increments of 1, using the public domain code FIELDGEN based on the sequential Gaussian simulator SGSIM [*Deutsch and Journel*, 1998]. We then solve the flow problem corresponding to each of between 1500 and 3400 realizations (depending on σ_Y^2 and λ_Y) using the finite difference code MODFLOW 2000 [*Harbaugh et al.*, 2000] over a time period of 100 consistent units. This time is long enough to insure that computed drawdowns within a radius of 25 units from the pumping well (half the distance to the boundary) in a uniform aquifer match the *Theis* [1935] solution over at

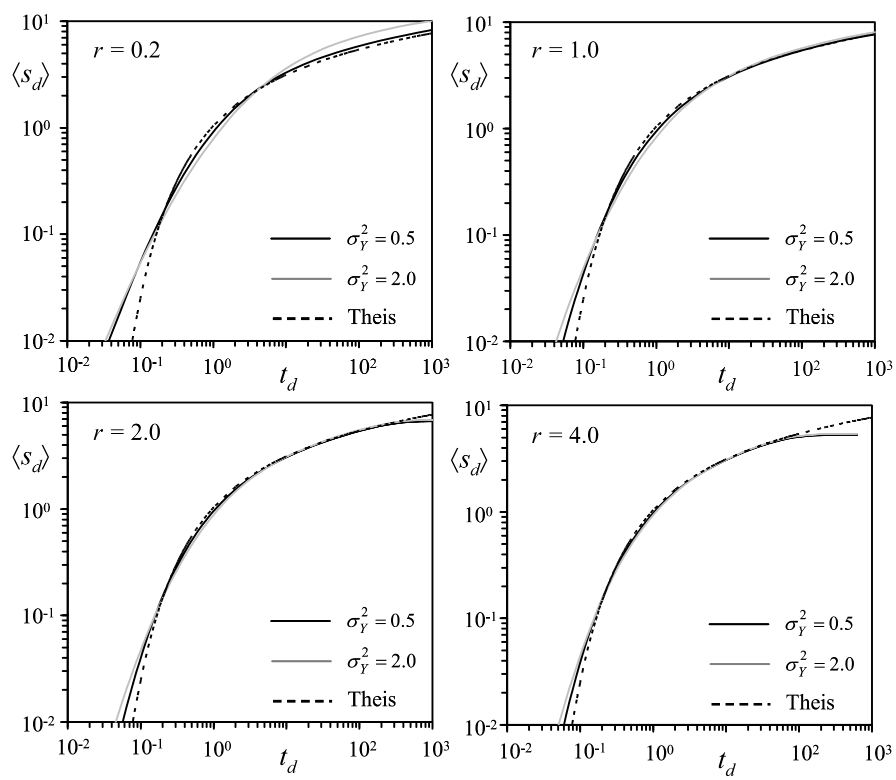

Figure 1. Mean dimensionless drawdown versus dimensionless time on log-log scale for $\lambda_Y = 1$ and $\sigma_Y^2 = 0.5$, 2 superimposed on the *Theis* [1935] curve at radial distances $r = 0.2$, 1, 2, 4.

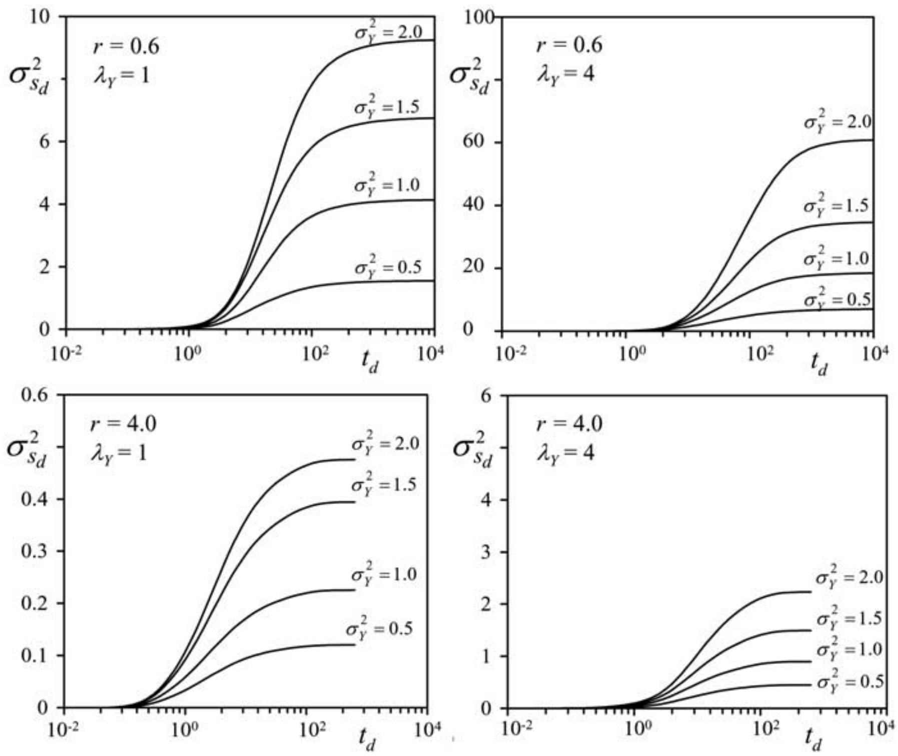

Figure 2. Dimensionless variance of drawdown versus log dimensionless time at radial distances $r = 0.6$, 4 for $\sigma_Y^2 = 0.5$, 1, 1.5, 2 and $\lambda_Y = 1$, 4.

least three logarithmic time cycles. Plots of sample mean and variance of computed head versus the number of Monte Carlo runs can be found in work by *Blattstein* [2006]. We terminate the simulations when fluctuations in sample mean and variance over the last 3 simulations at $t/t_b = 1$ (where t_b is the earliest dimensionless time at which boundary effects become discernible, quantified in the following section), at one node located a unit distance (five cells) from the center node (pumping well), are within 0.1% and 1.0% of each other, respectively. The number of Monte Carlo runs conducted on the basis of these criteria for each choice of log transmissivity statistics is listed in Table 1. Each run took about 3 min on a 2.00 GHz Pentium IV processor with 1.0 GB RAM.

2.3. Temporal Variations in Drawdown

[10] Figure 1 shows how ensemble mean dimensionless drawdown $\langle s_d \rangle = 4\pi T_G \langle s \rangle / Q$ varies with dimensionless time $t_d = tT_G/Sr^2$ at radial distances $r = 0.2$, 1, 2 and 4 when $\lambda_Y = 1$ and $\sigma_Y^2 = 0.5$, 2. The *Theis* [1935] curve corresponding to a uniform aquifer having transmissivity T_G and storativity S is included for reference. The mean drawdown curves show little sensitivity to variations in σ_Y^2 or, as we show elsewhere [*Blattstein*, 2006], to λ_Y. At early dimensionless time the mean drawdown curves at each r lie above the Theis curve but approach the latter as radial distance increases. Considering that the same happens when we set $\sigma_Y^2 = 0$ [*Blattstein*, 2006] suggests that this early deviation from the Theis curve is at least in part a computational artifact caused by insufficient numerical resolution of our finite difference grid close to the pumping well. Other than at $r = 0.2$ (near this well) the mean drawdown curves correspond closely to the Theis curve except at relatively late dimensionless time where they show an increasing tendency to flatten, at earlier and earlier t_d values, as r increases; the same is true at other values of λ_Y [*Blattstein*, 2006]. The flattening is caused by the constant head we impose at the lateral boundaries of the flow domain. One way to detect the onset of this boundary effect is to plot $d\langle s_d \rangle / d\ln(t_d)$ or $ds_d/d\ln(t_d)$ versus dimensionless time $t_d = tT_G/Sr^2$ on log-log scale where $s_d = 4\pi T_G s/Q$ is random dimensionless drawdown corresponding to a single realization. Blattstein demonstrates that during the initial transient flow the derivative increases, then flattens because of the establishment of a quasi–steady state flow regime during which both mean and random drawdowns increase linearly with the logarithm of time and would continue doing so if the aquifer was laterally infinite. The presence of a deterministic constant head boundary causes the rate of increase in drawdown to diminish, bringing about a sharp decline in the derivative. Blattstein's derivative curves suggest that the boundary effect sets in at real time $t_b = 6.0$ at $r = 2$, $t_b = 7.2$ at $r = 12$ and $t_b = 8.6$ at $r = 24$. On the other hand her plots of mean and random drawdown versus time on semilogarithmic scale suggest that the boundary effect sets in at time $t_b = 10$–15 at $r = 2$ and $t_b = 15$–20 at $r \geq 24$. On the basis of these findings we set, for purposes of our discussion, $t_b = 13.62$ which is the closest value to 15 corresponding to a computational time step in our numerical Monte Carlo analyses. None of the results we present below for $t/t_b \leq 1$ are therefore affected to any appreciable degree by lateral boundary effects.

[11] Figure 2 conveys a sense of the extent to which random time-drawdown curves may deviate from their

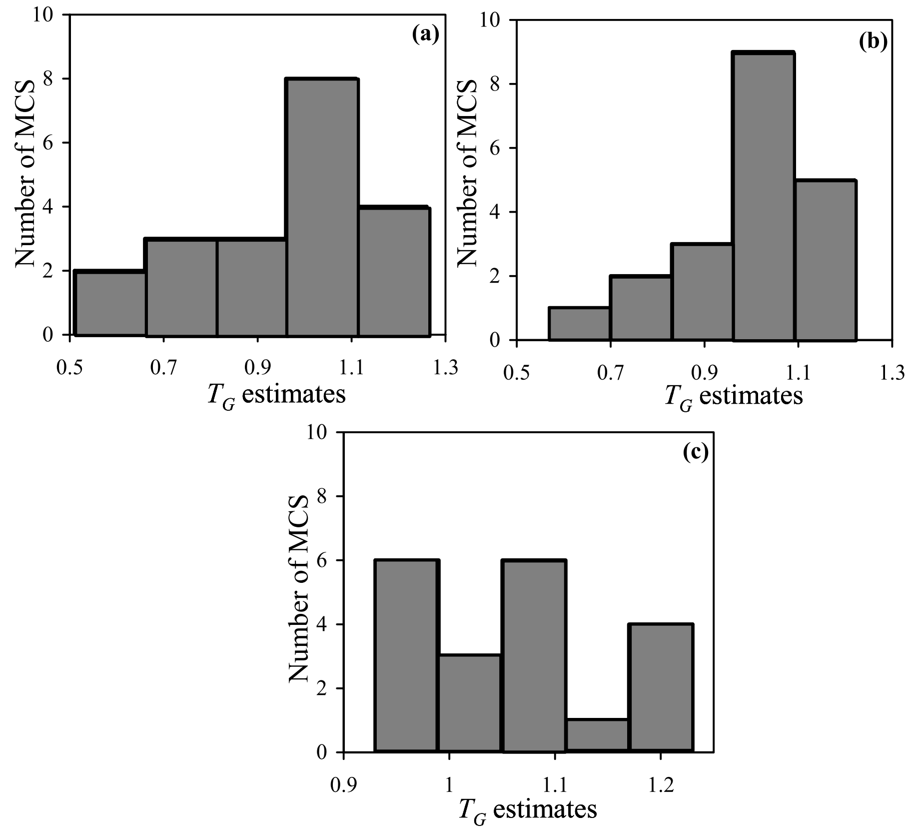

Figure 3. Histograms of T_G estimates obtained upon applying the Cooper-Jacob method to random drawdowns from 20 Monte Carlo realizations corresponding to $\sigma_Y^2 = 2$ and $\lambda_Y = 1$ at (a) $r = 1$, (b), $r = 2$, and (c) $r = 4$.

mean counterparts. Figure 2 depicts dimensionless variance of drawdown, $\sigma_{s_d}^2 = \sigma_s^2/[Q^2/16\pi^2 T_G^2]$ where σ_s^2 is the variance of actual drawdown, versus log dimensionless time at radial distances $r = 0.6$ and 4 for $\sigma_Y^2 = 0.5$, 1, 1.5, 2 and $\lambda_Y = 1$, 4. As expected, the dimensionless variance of drawdown increases systematically with σ_Y^2 and with proximity to the pumping well. It also increases sharply with log transmissivity integral scale, by slightly less than an order of magnitude, as the latter increases from 1 to 4. At late time the dimensionless variance stabilizes because of the influence of the external boundary.

2.4. Parameter Estimation Based on Time-Drawdown Data

[12] It should be evident from Figure 2 (as well as Figure 1) that traditional methods of analyzing time-drawdown data from randomly heterogeneous aquifers, based on the *Theis* [1935] solution for a uniform aquifer, could lead to sizable errors in the estimation of aquifer parameters. We illustrate these errors by plotting in Figure 3 histograms of T_G estimates we obtain by applying the *Cooper and Jacob* [1946] semilogarithmic straight line method to random drawdowns from 20 Monte Carlo realizations corresponding to $\sigma_Y^2 = 2$ and $\lambda_Y = 1$ at $r = 1$, 2 and 4. Following standard practice we apply the method to late time drawdown data that appeared to fall on a straight line when plotted against the logarithm of time. The estimation errors are in our view significant enough to suggest that transmissivities obtained from late time-drawdown data by means of the Cooper-Jacob

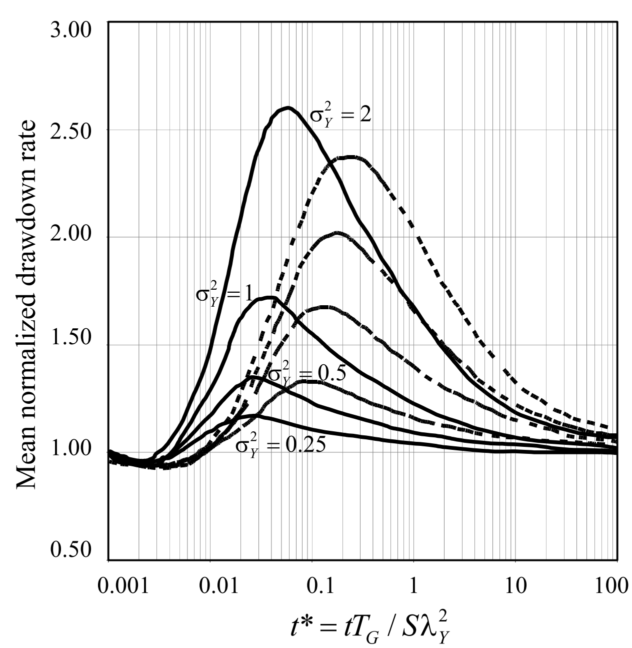

Figure 4. Mean normalized drawdown rate versus normalized time t^* at $r/\lambda_Y = 0$ for $\lambda_Y = 1$ and $\sigma_Y^2 = 0.25$, 0.5, 1, 2 as computed by us (dashed curves) and by *Copty and Findikakis* [2003] (solid curves).

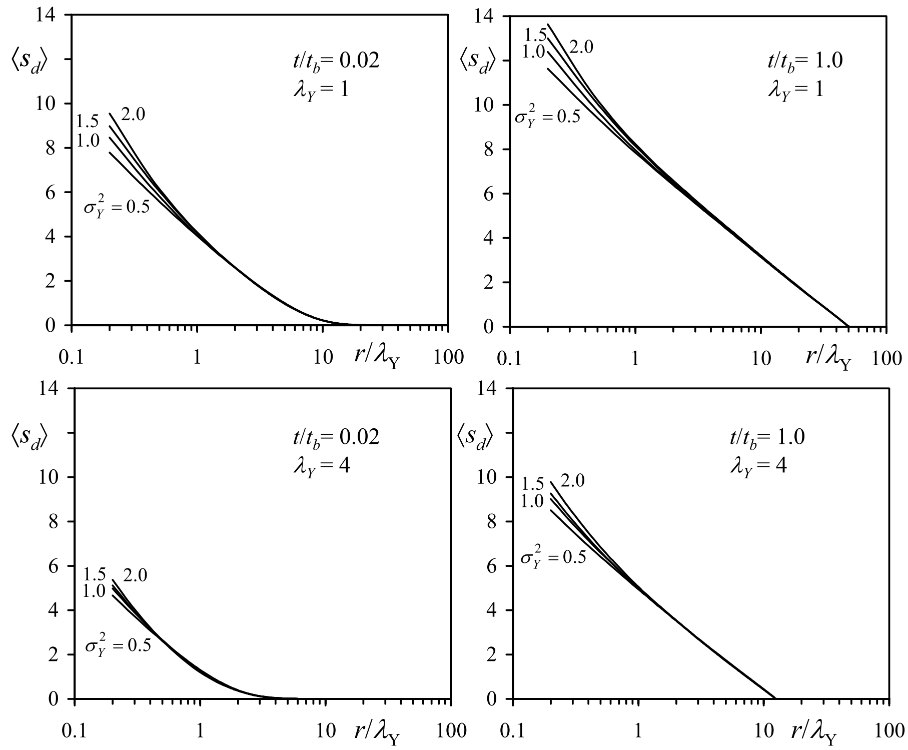

Figure 5. Mean dimensionless drawdown versus dimensionless distance at $t/t_b = 0.02$, 1 for $\sigma_Y^2 = 0.5$, 1, 1.5, 2 and $\lambda_Y = 1$, 4.

method (as proposed by *Meier et al.* [1998], *Sánchez-Vila et al.* [1999], and *Copty and Findikakis* [2003, 2004a]) provide relatively poor estimates of geometric mean transmissivity in all but mildly heterogeneous aquifers.

[13] *Copty and Findikakis* [2003, 2004a] recommend estimating the log transmissivity integral scale and variance on the basis of mean normalized drawdown rate (MNDDR), defined by them as temporal mean drawdown rate in a heterogeneous aquifer normalized by that in an equivalent homogeneous aquifer having T_G and S values estimated via the Cooper-Jacob method. The authors provide a formula for λ_Y based on the premise that MNDDR becomes insensitive to integral scale and approaches unity at $t^* = tT_G/S\lambda_Y^2 \approx 15$. Copty and Findikakis derived their formula on the basis of drawdown computed at the pumping well. *Blattstein* [2006] found that convergence of MNDDR to unity at or away from the pumping well takes generally much longer, more so as log transmissivity variance increases.

[14] To estimate the variance σ_Y^2 *Copty and Findikakis* [2003] recommend matching observed drawdown rate versus log time in the pumping well, normalized by that in an equivalent homogeneous aquifer having T_G and S values estimated via the Cooper-Jacob method, to type curves of MNDDR versus t^* corresponding to $r/\lambda_Y = 0$, $\lambda_Y = 1$ and $\sigma_Y^2 = 0.5$, 1, 1.5, 2 (solid curves in Figure 4). Upon employing a much larger and finer numerical grid than they do we obtain corresponding type curves (dashed curves in Figure 4) that differ markedly from theirs, the same being true for $r/\lambda_Y = 0.5$, 1 and 2 [*Blattstein*, 2006]. In other words, the curves depend strongly on grid resolution (which in turn impacts the effective radius of the pumping well) and are therefore not suitable for the analysis of real data from the pumping well. Blattstein demonstrates that the curves vary significantly with λ_Y and with r/λ_Y, casting doubt about the possibility of estimating σ_Y^2 by comparing real drawdown with mean drawdown behavior. Only by considering the scatter of random values about their mean could σ_Y^2 be properly estimated, as we propose below.

2.5. Quasi–Steady State

[15] It is well known that, in a uniform aquifer of infinite lateral extent, a quasi–steady state region extends from the well out to a cylindrical surface whose radius increases as the square root of time. On the expanding surface head is uniform and time invariant. Inside this surface head at any time is described by a steady state solution, implying that (1) head varies logarithmically with radial distance from the pumping well and (2) the cone of depression is declining at a uniform logarithmic time rate while preserving its shape. *Neuman et al.* [2004] conjectured that a quasi–steady state flow regime develops in the mean within a randomly heterogeneous aquifer as well. Figure 5 depicts on semi-logarithmic scale the variation of mean dimensionless drawdown with dimensionless radial distance r/λ_Y from the pumping well at $t/t_b = 0.02$ and 1 for various values of log transmissivity variance and integral scale. At $t/t_b = 0.02$ (early transient regime), curves corresponding to various variance values coalesce at $r/\lambda_Y \geq 1.0$ when $\lambda_Y = 1$ and at $r/\lambda_Y \geq 0.5$ when $\lambda_Y = 4$. At $t/t_b = 1$ the curves coalesce at $r/\lambda_Y \geq 2.0$ for all λ_Y values, delineating a straight line representative of mean quasi–steady state. The slope of this straight line is inversely proportional to T_G; the line splits into curves having increasing slopes as effective transmissivity diminishes from T_G toward the harmonic

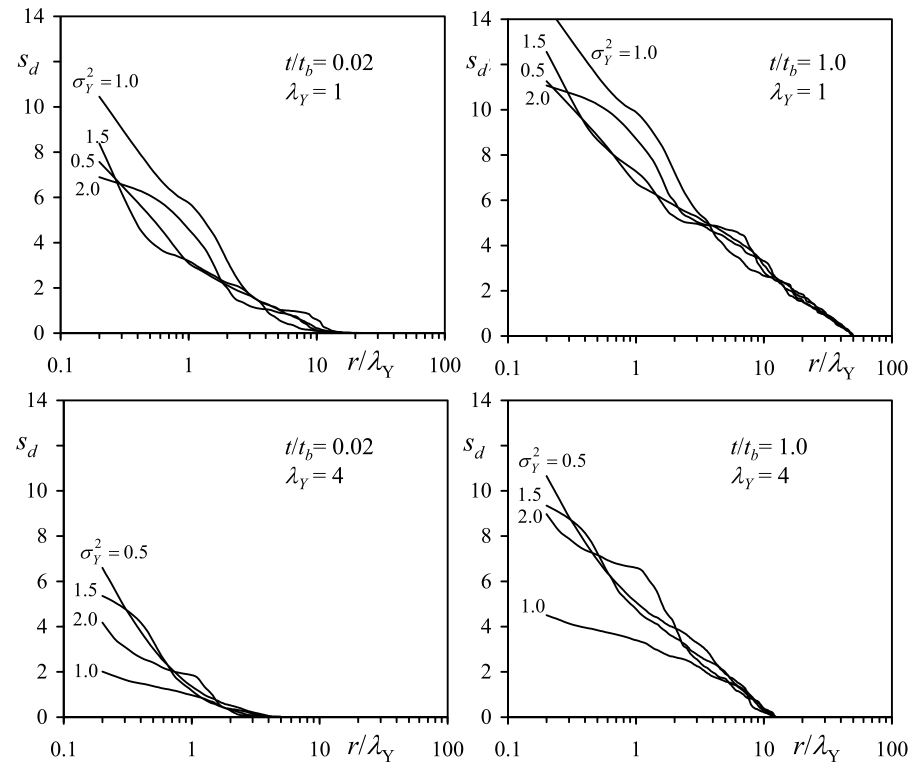

Figure 6. Random dimensionless drawdown versus dimensionless distance at t/t_b = 0.02, 1 for σ_Y^2 = 0.5, 1, 1.5, 2 and λ_Y = 1, 4.

mean T_H with decreasing r/λ_Y, T_H becoming smaller (and the slopes larger) as σ_Y^2 increases.

[16] Figure 6 shows what may happen when one replaces the mean dimensionless drawdown in Figure 5 with random values: these no longer coalesce neatly into a straight line at t/t_b = 1 and $r/\lambda_Y \geq 2.0$.

[17] Figure 7 shows how the normalized variance of drawdown, $\sigma_{s_d}^2 = \sigma_s^2/\sigma_Y^2$, varies with dimensionless distance r/λ_Y at t/t_b = 1 on log-arithmetic scale for various values of log transmissivity variance and integral scale. The behavior is very similar to that obtained by *Riva et al.* [2001] under steady state. In both cases the dimensionless variance of head first decreases sharply with dimensionless distance from the pumping well, then more gradually at a near-constant rate and eventually decreases sharply to zero as one approaches the external Dirichlet boundary.

[18] These findings suggest the possibility of estimating T_G, λ_Y and σ_Y^2 on the basis of late distance-drawdown data, corresponding (at least approximately) to a quasi–steady state flow regime, by using the type curve approach of *Neuman et al.* [2004]. Whereas these authors explored the feasibility of their approach vis à vis steady state data, we explore it below vis à vis synthetic as well as actual late time data.

2.6. Parameter Estimation Based on Quasi–Steady State Distance-Drawdown Data

[19] On the basis of steady state analyses in which λ_Y equals one *Neuman et al.* [2004] developed a set of type curves, and a graphical method of interpreting pumping test data, which they had conjectured would apply to transient data under quasi–steady state flow. To verify this we use our own results to plot in Figure 8 corresponding type curves of sample mean dimensionless drawdown increments ($2\pi T_G \Delta\overline{h}/Q$ where $\Delta\overline{h} = \overline{h}(r/\lambda_Y) - \overline{h}(2)$ and $\overline{h}$ is circumferentially averaged head at any dimensionless radial distance r/λ_Y) versus $r/2\lambda_Y$ on semilogarithm scale at t/t_b = 1 for various values of σ_Y^2 and λ_Y = 1, 4. Figure 8 also shows envelopes of ±2 standard deviations of $2\pi T_G \Delta\overline{h}/Q$ about the mean. Whereas our mean curves correspond almost

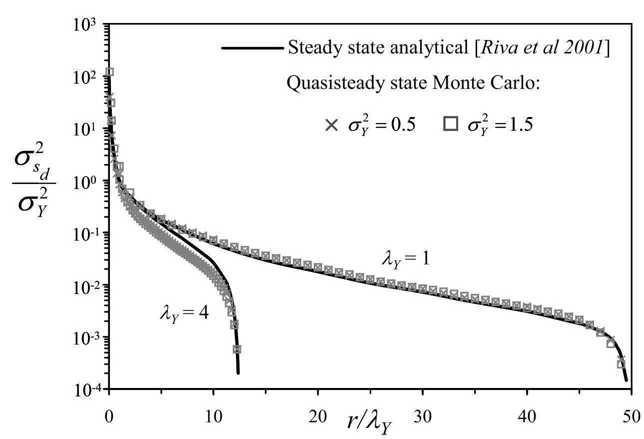

Figure 7. Normalized variance of drawdown versus dimensionless radial distance r/λ_Y at t/t_b = 1 for σ_Y^2 = 0.5, 1.5 and λ_Y = 1, 4 (crosses and squares) compared with second-order steady state analytical solution (solid curve) of *Riva et al.* [2001].

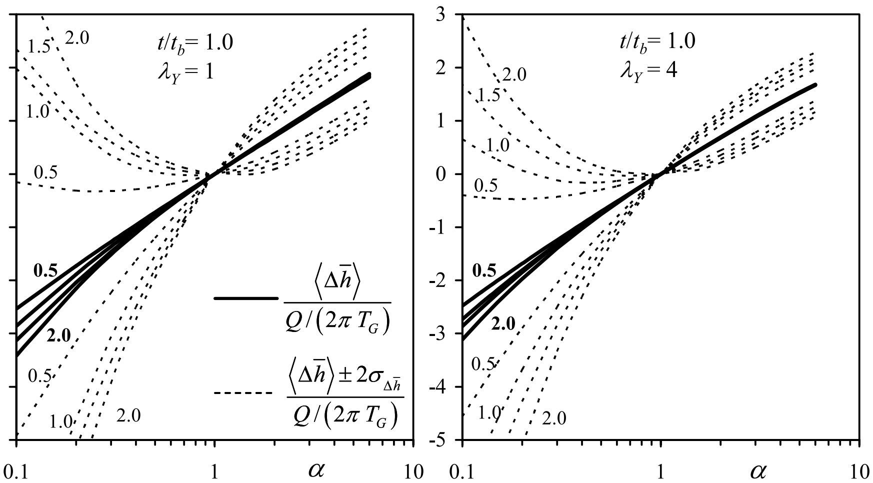

Figure 8. Type curves of sample mean dimensionless drawdown increments (solid curves) versus $\alpha = r/(2\lambda_Y)$ at $t/t_b = 1$ for various σ_Y^2 (0.5, 1.0, 1.5, 2.0) and (left) $\lambda_Y = 1$ and (right) $\lambda_Y = 4$. Dashed curves represent ±2 sample standard deviations about the mean.

exactly to those of Neuman et al. in both cases, our envelopes of ±2 standard deviations are somewhat narrower than theirs when $\sigma_Y^2 > 1$ and $\lambda_Y > 1$. The latter difference may be due in part to the fact that whereas Neuman et al. ran 2000 Monte Carlo simulations using a Gaussian variogram with Galerkin finite elements and bilinear shape functions on a numerical grid of 101 × 101 nodes, we ran 1500–3400 Monte Carlo simulations using an exponential variogram with finite differences on a grid of 501 × 501 nodes. Another reason for the difference may be related to slight differences noted between steady state and quasi–steady state results in Figure 7 when $\lambda_Y = 4$. The discrepancy is small enough to constitute a verification of Neuman et al.'s conjecture about the applicability of their methodology to transient data at quasi–steady state. Like these authors we too fail to confirm a hypothesis that our generated heads are normally distributed at a significance level of 5% [*Blattstein*, 2006]. Therefore our envelopes (likes theirs) may not be strictly proportional to 95% confidence intervals.

[20] We test the methodology of *Neuman et al.* [2004] by using it to estimate T_G, λ_Y and σ_Y^2 on the basis of transient head data extracted from one random realization corresponding to $\sigma_Y^2 = 2$ and $\lambda_Y = 4$ at $t/t_b = 1$. Though Neuman et al. described two ways of analyzing random distance-drawdown data we [*Blattstein*, 2006] use below the following approach.

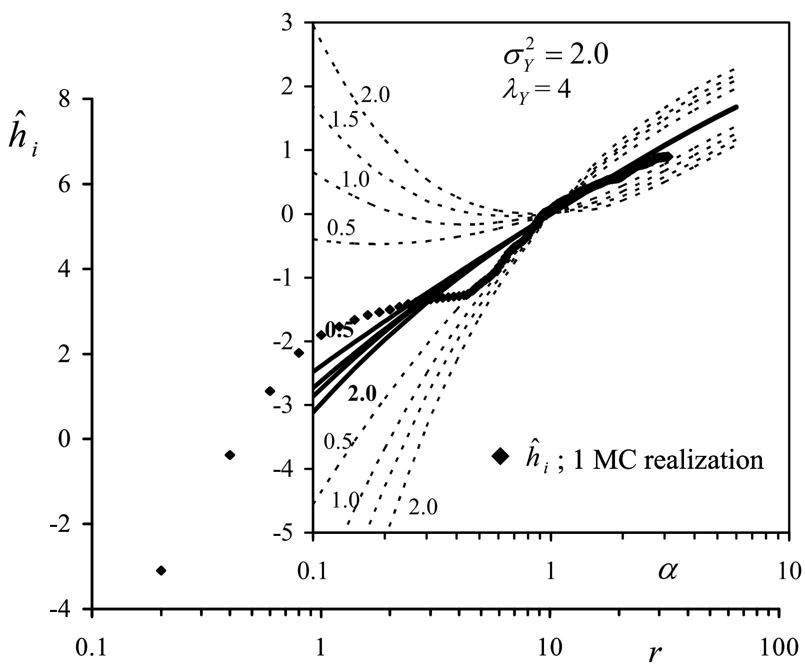

Figure 9. Random dimensionless heads (diamonds) versus r corresponding to $\sigma_Y^2 = 2.0$, $\lambda_Y = 4$ superimposed on transient type curves.

Table 2. Estimates of T_G and Corresponding Estimation Errors Obtained Using Random Heads at $t/t_b = 1$ Along a Single Radius and Along Four Orthogonal Radii From Single Realizations Corresponding to Various Combinations of σ_Y^2 and λ_Y

	Single Radius			Four Radii		
Parameters	T_G Estimate	Estimation Error	%Error	T_G Estimate	Estimation Error	%Error
$\sigma_Y^2 = 0.5,\ \lambda_Y = 1$	0.95	−0.05	−5	1.03	0.03	3
$\sigma_Y^2 = 0.5,\ \lambda_Y = 2$	1.01	0.01	1	0.98	−0.02	−2
$\sigma_Y^2 = 0.5,\ \lambda_Y = 3$	1.25	0.25	25	1.12	0.12	12
$\sigma_Y^2 = 0.5,\ \lambda_Y = 4$	0.89	−0.11	−11	1.01	0.01	1
$\sigma_Y^2 = 1.0,\ \lambda_Y = 1$	1.07	0.07	7	1.03	0.03	3
$\sigma_Y^2 = 1.0,\ \lambda_Y = 2$	1.04	0.04	4	1.02	0.02	2
$\sigma_Y^2 = 1.0,\ \lambda_Y = 3$	1.26	0.26	26	1.07	0.07	7
$\sigma_Y^2 = 1.0,\ \lambda_Y = 4$	0.92	−0.08	−8	1.02	0.02	2
$\sigma_Y^2 = 1.5,\ \lambda_Y = 1$	1.08	0.08	8	1.06	0.06	6
$\sigma_Y^2 = 1.5,\ \lambda_Y = 2$	1.24	0.24	24	1.17	0.17	17
$\sigma_Y^2 = 1.5,\ \lambda_Y = 3$	1.10	0.10	10	1.20	0.20	20
$\sigma_Y^2 = 1.5,\ \lambda_Y = 4$	1.18	0.18	18	1.02	0.02	2
$\sigma_Y^2 = 2.0,\ \lambda_Y = 1$	0.90	−0.10	−10	1.02	0.02	2
$\sigma_Y^2 = 2.0,\ \lambda_Y = 2$	1.03	0.03	3	0.98	−0.02	−2
$\sigma_Y^2 = 2.0,\ \lambda_Y = 3$	0.90	−0.10	−10	0.93	−0.07	−7
$\sigma_Y^2 = 2.0,\ \lambda_Y = 4$	0.86	−0.14	−14	0.94	−0.06	−6

[21] Let quasi−steady state head values measured at discrete radial and angular locations (r_i, θ_i), where i indicates well number/location, be denoted by $h_i = h(r_i, \theta_i)$. One starts by plotting h_i versus r_i on semilogarithmic scale, fitting a straight line to data corresponding to large r_i values and obtaining $T_G = 2.303Q/(2\pi m)$ from the slope m of this line. One then estimates $2\lambda_Y$ by equating it to the radial distance at which the data start deviating from the straight line. This allows computing and plotting dimensionless heads $\tilde{h}_i = 2\pi T_G h_i/(2.303Q)$ at all observation wells versus r_i on semilogarithmic scale and superimposing them on type curves of mean dimensionless drawdown increments as illustrated in Figure 9. The match should yield $\lambda_Y = r/(2\alpha)$ where $\alpha = r/(2\lambda_Y)$ is the horizontal coordinate in Figure 8; if it does not, one modifies the previous estimate of λ_Y iteratively till it does. Finally, one estimates the variance σ_Y^2 on the basis of envelopes of ±2 standard deviations within which about 95% of the data lie, excluding about 5% of the data in the vicinity of $r = 2\lambda_Y$ where all dashed type curves coalesce.

[22] Table 2 lists estimates of T_G and corresponding estimation errors obtained using random heads at $t/t_b = 1$ along a single radius and along four orthogonal radii from single realizations corresponding to various combinations of σ_Y^2 and λ_Y. As estimates along four radii are based on four times as many data as those along a single radius, they generally have smaller estimation errors though the average estimation error of the latter (3%) is smaller than that of the former (3.8%). Both sets of estimates fluctuate with much smaller amplitude about the true value $T_G = 1$ than do those obtained by applying the Cooper-Jacob method to individual time-drawdown records [*Blattstein*, 2006] of the kind illustrated in Figure 3.

[23] Table 3 lists estimates of λ_Y, and corresponding estimation errors, obtained using random heads at $t/t_b = 1$ along a single radius from single realizations corresponding to various combinations of σ_Y^2 and λ_Y. Whereas the estimates in Table 3 were obtained using the steady state type curves of *Neuman et al.* [2004], estimates obtained using type curves we developed on the basis of late time transient data (Figure 8) are of comparable quality [*Blattstein*, 2006]. The estimation errors range from small to considerable with a tendency for small λ_Y values (1 and 2) to be overestimated and large values (3 and 4) to be underestimated. Relying on four times as many data along four orthogonal radii has reduced the average estimation error from 41.8% to 25.3%.

[24] Table 4 lists estimates of log transmissivity variance σ_Y^2 and corresponding estimation errors obtained using the type curves of *Neuman et al.* [2004] with random heads at $t/t_b = 1$ along a single radius and along four orthogonal radii from single realizations corresponding to various combinations of σ_Y^2 and λ_Y. The estimation errors range from zero to 200% with an average of 13.4% for data along a single radius and 44.8% for data along four radii.

3. Field Application and Verification

[25] We demonstrate the applicability of the quasi−steady state graphical distance-drawdown method of *Neuman et al.* [2004] to late time pumping test data from a heterogeneous

Table 3. Estimates of λ_Y, and Corresponding Estimation Errors, Obtained Using Random Heads at $t/t_b = 1$ Along a Single Radius From Single Realizations Corresponding to Various Combinations of σ_Y^2 and λ_Y

Parameters	λ_Y Estimate	Estimation Error	%Error
$\sigma_Y^2 = 0.5,\ \lambda_Y = 1$	2.27	1.27	127
$\sigma_Y^2 = 0.5,\ \lambda_Y = 2$	2.50	0.50	25
$\sigma_Y^2 = 0.5,\ \lambda_Y = 3$	2.63	−0.37	−12.3
$\sigma_Y^2 = 0.5,\ \lambda_Y = 4$	2.94	−1.06	−26.5
$\sigma_Y^2 = 1.0,\ \lambda_Y = 1$	2.08	1.08	108
$\sigma_Y^2 = 1.0,\ \lambda_Y = 2$	2.17	0.17	8.5
$\sigma_Y^2 = 1.0,\ \lambda_Y = 3$	2.63	−0.37	−12.3
$\sigma_Y^2 = 1.0,\ \lambda_Y = 4$	2.50	−1.50	−37.5
$\sigma_Y^2 = 1.5,\ \lambda_Y = 1$	2.78	1.78	178
$\sigma_Y^2 = 1.5,\ \lambda_Y = 2$	1.92	−0.08	−4
$\sigma_Y^2 = 1.5,\ \lambda_Y = 3$	2.50	−0.50	−16.7
$\sigma_Y^2 = 1.5,\ \lambda_Y = 4$	2.78	−1.22	−30.5
$\sigma_Y^2 = 2.0,\ \lambda_Y = 1$	2.50	1.50	150
$\sigma_Y^2 = 2.0,\ \lambda_Y = 2$	1.43	−0.57	−28.5
$\sigma_Y^2 = 2.0,\ \lambda_Y = 3$	2.78	−0.22	−7.3
$\sigma_Y^2 = 2.0,\ \lambda_Y = 4$	3.33	−0.67	−16.8

Table 4. Estimates of σ_Y^2 and Corresponding Estimation Errors Obtained Using Type Curves With Random Heads at t/t_b = 1 Along a Single Radius and Along Four Radii From Single Realizations Corresponding to Various Combinations of σ_Y^2 and λ_Y[a]

	Single Radius			Four Radii		
Parameters	σ_Y^2 Estimate	Estimation Error	%Error	σ_Y^2 Estimate	Estimation Error	%Error
σ_Y^2 = 0.5, λ_Y = 1	1	0.50	100	1	0.50	100
σ_Y^2 = 0.5, λ_Y = 2	1.5	1.00	200	2	1.50	300
σ_Y^2 = 0.5, λ_Y = 3	0.5	0.00	0	0.5	0.00	0
σ_Y^2 = 0.5, λ_Y = 4	0.5	0.00	0	1	0.50	100
σ_Y^2 = 1.0, λ_Y = 1	2	1.00	100	2	1.00	100
σ_Y^2 = 1.0, λ_Y = 2	1	0.00	0	1	0.00	0
σ_Y^2 = 1.0, λ_Y = 3	0.5	−0.50	−50	1.5	0.50	50
σ_Y^2 = 1.0, λ_Y = 4	2	1.00	100	2	1.00	100
σ_Y^2 = 1.5, λ_Y = 1	1	−0.50	−33.3	2	0.50	33.3
σ_Y^2 = 1.5, λ_Y = 2	1.5	0.00	0	1.5	0.00	0
σ_Y^2 = 1.5, λ_Y = 3	1	−0.50	−33.3	2	0.50	33.3
σ_Y^2 = 1.5, λ_Y = 4	0.1	−1.40	−93.3	1.5	0.00	0
σ_Y^2 = 2.0, λ_Y = 1	2	0.00	0	1.5	−0.50	−25
σ_Y^2 = 2.0, λ_Y = 2	1.5	−0.50	−25	1	−1.00	−50
σ_Y^2 = 2.0, λ_Y = 3	1.5	−0.50	−25	1.5	−0.50	−25
σ_Y^2 = 2.0, λ_Y = 4	0.5	−1.50	−75	2	0.00	0

[a]Type curves are from *Neuman et al.* [2004].

fluvial aquifer at the Lauswiesen site in the Neckar river valley near Tübingen, Germany (Figure 10a). The aquifer consists of sandy gravel overlain by stiff silty clay and underlain by hard silty clay (Figure 11). We analyze simultaneously quasi–steady state drawdown data from five consecutive pumping tests conducted in wells B1–B5 (Figures 10b and 11) by pumping each of them at a constant rate while treating the other four as observation wells. The wells fully penetrate the aquifer which, during these tests, included a water table beneath the upper silty clay. The pumping well and rate, start date, duration, average initial saturated thickness b and largest as well as smallest initial saturated thickness during each test are listed in Table 5. We treat flow during each test as being horizontal and correct the drawdown s for variations in saturated thickness according to [*Jacob*, 1944]

$$s_c = s - \frac{s^2}{2b} \qquad (3)$$

where s_c is corrected drawdown (under the assumption of horizontal flow this correction applies equally to uniform and nonuniform media). In all wells, during all five pumping tests, late values of s_c varied logarithmically with time. Upon analyzing these late time-drawdown data from each observation well (except those associated with the pumping of well B1, for reasons explained later) by the method of *Cooper and Jacob* [1946] we obtain a narrow range of transmissivity having an arithmetic average of 1.71×10^{-2} m^2/s with a standard deviation of 1.56×10^{-3} m^2/s (coefficient of variation equal to 0.09), a geometric average of 1.70×10^{-2} m^2/s and natural log transmissivity variance equal to 7.72×10^{-3}. Doing the same for late time data from the pumping wells yields systematically lower values equal to 1.65×10^{-2} m^2/s for B2, 4.08×10^{-3} m^2/s for B3, 1.02×10^{-2} m^2/s for B4 and 1.54×10^{-2} m^2/s for B5. This is consistent in principle with stochastic theory [e.g., *Neuman and Orr*, 1993; *Neuman et al.*, 2004] according to which the apparent transmissivity of a randomly heterogeneous, statistically homogeneous aquifer decreases from the geometric mean at some distance from the pumping well to the harmonic mean at the pumping well.

[26] To estimate T_G, λ and σ_Y^2 by using the quasi–steady state graphical distance-drawdown method of *Neuman et al.* [2004] we analyze simultaneously all corrected drawdowns at a relatively late time of 60 min (at which all s_c values vary logarithmically with time). As pumping rates Q vary from test to test (Table 5), we plot in Figure 12 the negative normalized corrected drawdowns $-s_c/Q$ (in m × s/m^3) versus radial distance r (in m) in observation wells (left) and in all wells (right). Values of $-s_c/Q$ obtained during the pumping of well B1 are seen to lie well below all other values while exhibiting a more or less similar slope, suggesting the possibility that the recorded pumping rate is in error. Being unsure about the reason, we exclude data associated with this test from our analysis. The remaining values from observation wells can be represented by a regression line $-s_c/Q = 7.305\ln r - 33.652$ with a relatively high coefficient of determination, R^2 = 0.863. From the slope of this regression line we estimate T_G to be 2.18×10^{-2} m^2/s, which exceeds the geometric average of the Cooper-Jacob estimates (1.70×10^{-2} m^2/s) by nearly 30%. All values of $-s_c/Q$ from the pumping wells are seen to lie well below this regression line, suggesting that the line starts curving downward at distances smaller than r = 5 m. According to *Neuman et al.* [2004] this yields an estimate of λ_Y equal to about 2.5 m. It is of interest to note that the latter is one tenth the largest radial distance of 25 m spanned by all B wells; this is consistent with an observation [*Gelhar*, 1993; *Neuman*, 1994] that the apparent spatial correlation scales of natural log hydraulic conductivities and transmissivities worldwide, obtained upon treating these quantities as samples from statistically homogeneous random fields, tend to be 1/10 of the characteristic length of their sampling window as the latter ranges between 1 m and 450 km. This relationship is in turn consistent with a view of log hydraulic conductivity and transmissivity as a trun-

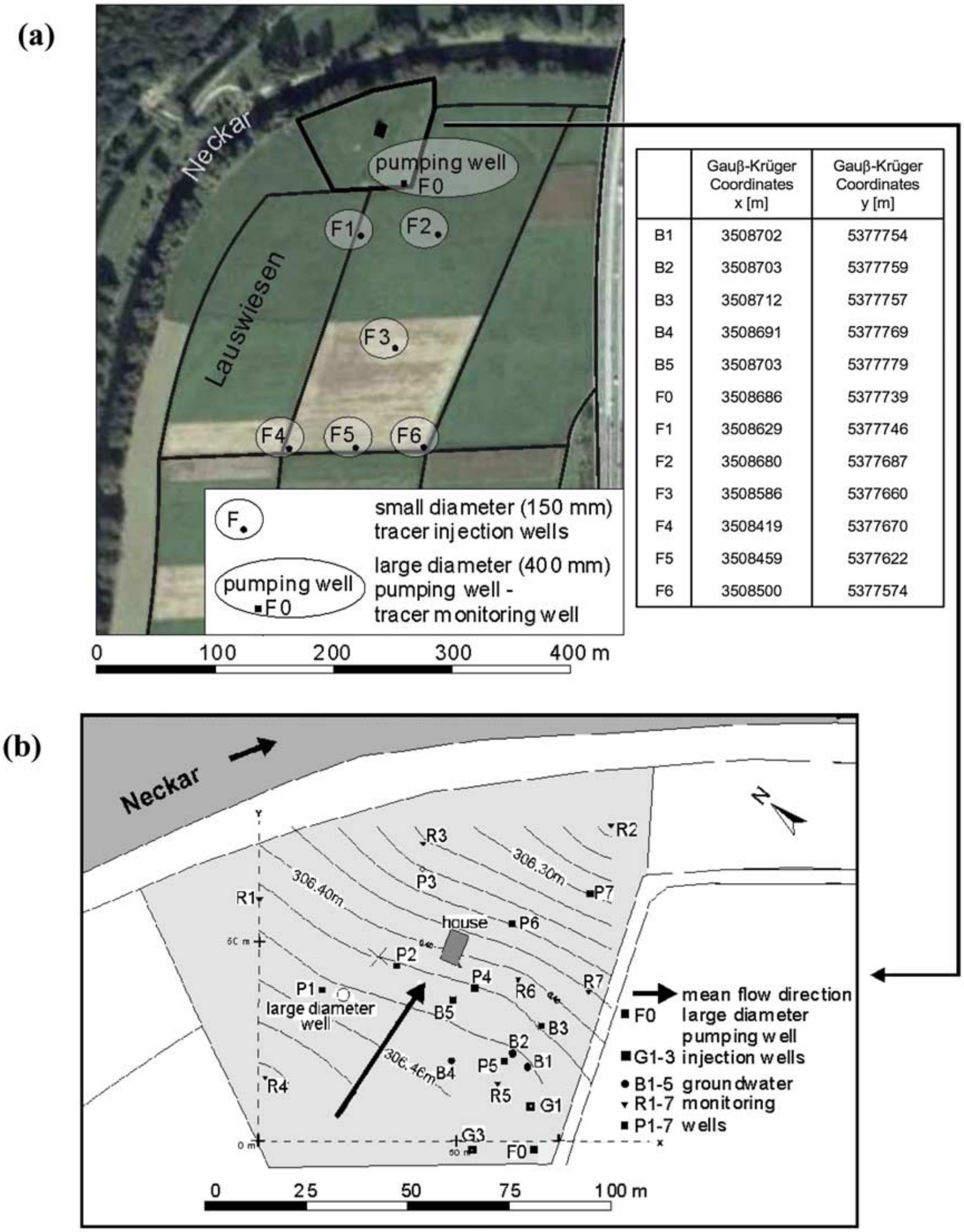

Figure 10. Layout of the Lauswiesen site near Tübingen, Germany. The test discussed here was conducted in boreholes B1–B5 [after *Riva et al.*, 2006].

cated random fractal [*Di Federico and Neuman*, 1997; *Di Federico et al.*, 1999; *Neuman and Di Federico*, 2003]. Such a fractal is characterized by a truncated power variogram which, as we show elsewhere (S. P. Neuman et al., On the geostatistical characterization of hierarchical media, submitted to *Water Resources Research*, 2007), is often difficult to differentiate from traditional exponential or Gaussian variograms of the kind utilized in developing the type curves of *Neuman et al.* [2004].

[27] Figure 13 is a plot of corrected dimensionless drawdowns $2\pi T_G[\langle s_c(r = 2\lambda_Y)\rangle - s_c]/Q$ versus $\alpha = r/(2\lambda_Y)$ in observation wells (left) and in all wells (right) superimposed on the type curves of *Neuman et al.* [2004]. Whereas ignoring drawdowns in pumping wells yields an estimate of σ_Y^2 equal to 0.5, taking such drawdowns into consideration yields a higher estimate of 1.5. This is so because values of $-s_c/Q$ from the pumping wells are the only data in our possession exhibiting a significant scatter, most other

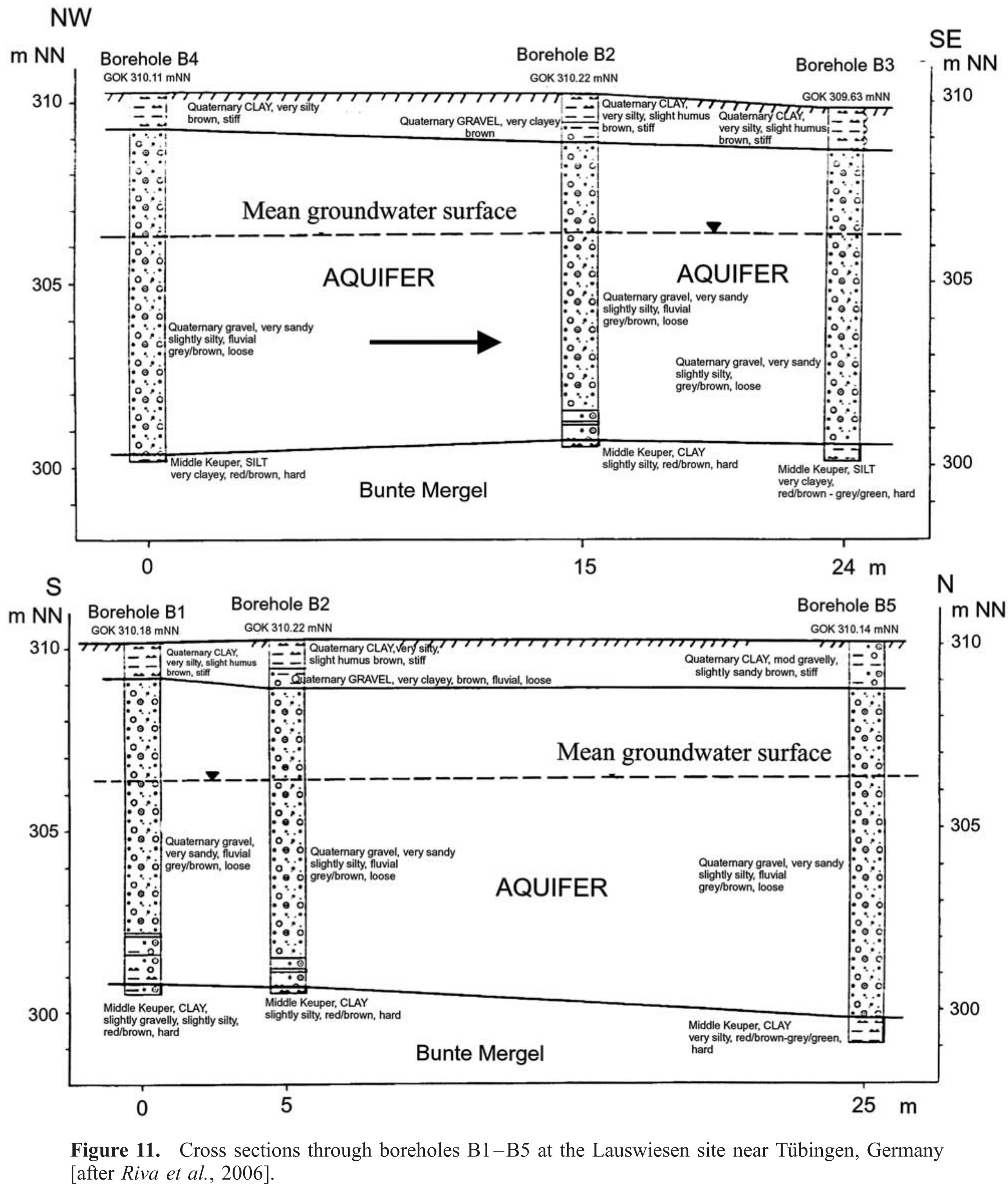

Figure 11. Cross sections through boreholes B1–B5 at the Lauswiesen site near Tübingen, Germany [after *Riva et al.*, 2006].

data being too close to the regression line to provide a satisfactory estimate of the variance.

[28] To verify the above estimates we compare them with those obtained by us independently on the basis of 312 hydraulic conductivity values obtained using a flowmeter in all B and F wells across the site (Figure 10). The conductivities correspond to vertical intervals ranging in length from 3 to 40 cm, varying over five orders of magnitude between 4.60×10^{-6} and 1.91×10^{-1} m/s. The natural log conductivity values have mean −6.17, median −6.12, mode −6.65, variance 2.38, kurtosis 1.35 and skewness −0.33. Setting the transmissivity of each well equal to the product of its weighted arithmetic average conductivity and the average saturated thickness (5.14 m) in the B wells (no information is available about contemporary water table elevations in the F wells) yields a T_G estimate of 2.38×10^{-2} m^2/s, very close to the pumping test estimate of 2.18×10^{-2} m^2/s and higher by 40% than the Cooper-Jacob

Table 5. Pumping Rate Q, Initial Saturated Thickness, Date, and Duration of Each Test in Wells B1–B5 at the Lauswiesen Site Near Tübingen, Germany

Well Being Pumped	Pumping Rate Q, L/s	Start Date	Duration, h	Mean Initial Saturated Thickness, m	Largest Initial Saturated Thickness	Smallest Initial Saturated Thickness
B1	1.03	3 May 1996	2.67	4.71	5.19	4.37
B2	5.27	2 May 1996	3.46	4.81	5.28	4.33
B3	3.00	30 Apr 1996	3.34	5.04	5.51	4.56
B4	5.48	29 Apr 1996	3.93	5.64	5.80	4.85
B5	5.52	27 Apr 1996	2.33	5.49	5.96	5.01

estimate. The corresponding (natural) log transmissivity variance is 1.4, very close to the pumping test estimate of 1.5. As the flowmeter data yield local estimates of transmissivity at only 12 boreholes, we were not able to estimate an integral scale of Y on the basis of these data corresponding to any sampling window, whether that spanning the B wells or another spanning both the B and F wells.

4. Conclusions

[29] We have conducted numerical Monte Carlo simulations of pumping at a constant rate from a well of zero radius that fully penetrates a confined aquifer having uniform storativity and randomly varying Gaussian log transmissivity with variance and integral (spatial correlation) scale within the ranges $\sigma_Y^2 = 0.5, 1, 1.5, 2$ and $\lambda_Y = 1, 2, 3, 4$, respectively. Our results indicate that:

[30] 1. In the absence of a lateral boundary effect (i.e., in an infinite-acting aquifer) mean dimensionless drawdown varies with dimensionless time in a manner virtually identical to the *Theis* [1935] solution corresponding to a uniform aquifer having transmissivity equal to the geometric mean T_G. Deviations from this behavior in the immediate vicinity of the pumping well appear to be at least in part a numerical artifact due to insufficient numerical resolution of our finite difference grid close to this well.

[31] 2. Random dimensionless drawdown varies with dimensionless time in a manner that may differ substantially from the mean. The dimensionless variance of drawdown about the mean increases systematically with σ_Y^2 and λ_Y, decreasing with normalized radial distance r/λ_Y from the pumping well.

[32] 3. Consequently, traditional methods of analyzing time-drawdown data from randomly heterogeneous aquifers which are based on the *Theis* [1935] solution for a uniform aquifer, may lead to sizable errors in the estimation of aquifer parameters. The estimation errors are in our view significant enough to suggest that transmissivities obtained from late time-drawdown data by means of the Cooper-Jacob method provide relatively poor estimates of geometric mean transmissivity in all but mildly heterogeneous aquifers.

[33] 4. It has been suggested in the literature that log transmissivity integral scale and variance be estimated through comparison of actual and mean temporal drawdown rates in the pumping (and perhaps an observation) well, normalized by corresponding drawdown rates in an equivalent homogeneous aquifer having T_G and storativity values estimated via the Cooper-Jacob method. Our analysis casts doubt about the reliability of this approach.

[34] 5. Our analysis supports a conjecture made by *Neuman et al.* [2004] that mean flow in a randomly heterogeneous aquifer evolves toward a quasi–steady state within a cylindrical domain having an inner radius $r = 2\lambda_Y$ and an outer radius that expands at a rate proportional to the logarithm of time t. Within this domain mean drawdown varies linearly with $\log(t/r^2)$ at a rate that is inversely proportional to T_G. The variance of random head fluctuations about the mean at quasi–steady state varies with dimensionless distance from the pumping well in a manner similar to that obtained by *Riva et al.* [2001] under steady state.

[35] 6. On the basis of steady state analyses in which (natural) log transmissivity was taken to have a Gaussian variogram with unit integral scale λ_Y *Neuman et al.* [2004]

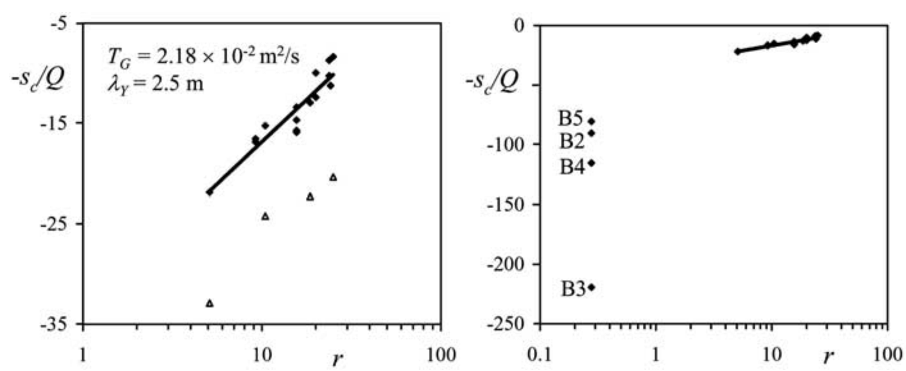

Figure 12. Negative normalized corrected drawdowns $-s_c/Q$ (in m $\times$ s/m^3) versus radial distance r (in m) (left) in observation wells (triangles represent responses to the pumping of B1) and (right) in all wells (excluding responses to the pumping of B1).

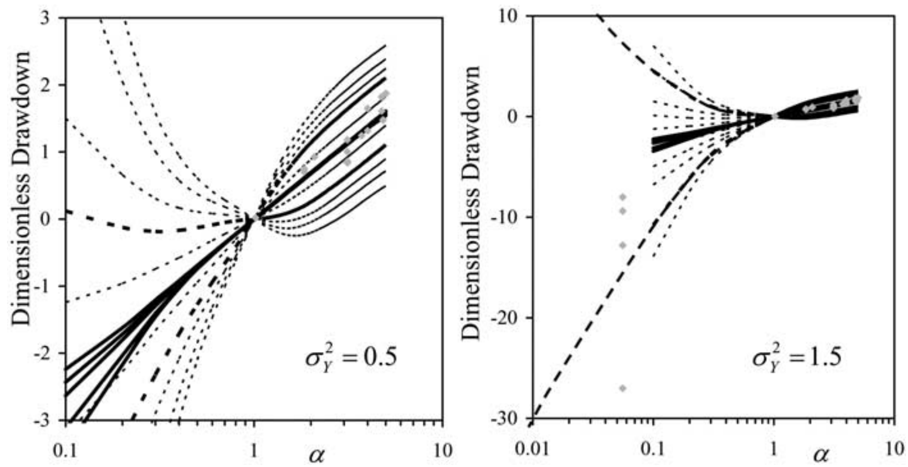

Figure 13. Dimensionless corrected drawdowns $2\pi T_G[\langle s_c(r = 2\lambda_Y)\rangle - s_c]/Q$ versus dimensionless distance $\alpha = r/(2\lambda_Y)$ (left) in observation wells and (right) in all wells, excluding responses to the pumping of B1, superimposed on selected type curves.

developed a set of type curves, and a graphical method of interpreting pumping test data, which they had conjectured would allow estimating T_G, λ_Y and σ_Y^2 on the basis of transient drawdowns under quasi–steady state. We have confirmed their conjecture for log transmissivity fields having an exponential variogram and integral scales at least as large as 4.

[36] 7. We applied the distance-drawdown method of *Neuman et al.* [2004] to synthetic random drawdowns at quasi–steady state corresponding to 16 combinations of $\sigma_Y^2 = 0.5$, 1, 1.5, 2 and $\lambda_Y = 1$, 2, 3, 4. The analysis yielded estimates of T_G that fluctuate with much lesser amplitude about the true value of this parameter than do estimates obtained by applying the Cooper-Jacob method to individual time-drawdown records. It yielded estimates of λ_Y and σ_Y^2 having acceptable average estimation errors over the 16 cases.

[37] We analyzed transient data from a series of pumping tests conducted in four wells within a heterogeneous unconfined aquifer near Tübingen, Germany, using both the *Cooper and Jacob* [1946] time-drawdown method and the quasi–steady state graphical distance-drawdown method of *Neuman et al.* [2004]. In both cases, we treated flow between the fully penetrating wells during each test as being horizontal and corrected the drawdown for variations in saturated thickness.

[38] 8. Applying the Cooper-Jacob method to individual observation wells yielded a narrow range of transmissivities having a geometric mean of 1.70×10^{-2} m^2/s and natural log transmissivity variance equal to 7.72×10^{-3}. Applying the method to pumping wells gave systematically lower values. This is consistent with stochastic theory [e.g., *Neuman and Orr*, 1993] according to which the apparent transmissivity of a randomly heterogeneous, statistically homogeneous aquifer decreases from the geometric mean at some distance from the pumping well to the harmonic mean at the pumping well.

[39] 9. Applying the quasi–steady state graphical distance-drawdown method of *Neuman et al.* [2004] simultaneously to late drawdowns from the four tests gave a geometric mean transmissivity estimate of 2.18×10^{-2} m^2/s. The latter exceeds the Cooper-Jacob estimate of 1.70×10^{-2} m^2/s by nearly 30%.

[40] 10. The method of *Neuman et al.* [2004] yielded an estimate of 2.5 m for the integral scale of natural log transmissivity. This is one tenth the distance of 25 m spanned by the test wells, consistent with an observation [*Gelhar*, 1993; *Neuman*, 1994] that the apparent spatial correlation scales of natural log hydraulic conductivities and transmissivities worldwide, obtained upon treating these quantities as samples from statistically homogeneous random fields, tend to be 1/10 of the characteristic length of their sampling window as the latter ranges between 1 m and 450 km. This relationship is in turn consistent with a view of log hydraulic conductivity and transmissivity as a truncated random fractal [*Di Federico and Neuman*, 1997; *Di Federico et al.*, 1999; *Neuman and Di Federico*, 2003].

[41] 11. The method of *Neuman et al.* [2004] yielded an estimate of 1.5 for the variance of natural log transmissivity.

[42] 12. To verify the above estimates we compared them with those obtained by us independently on the basis of 312 hydraulic conductivity values, varying over five orders of magnitude, obtained using a flowmeter in the four test well and eight additional wells across the site. Setting the transmissivity of each well equal to the product of its weighted arithmetic average conductivity and the average saturated thickness yielded a geometric mean transmissivity estimate of 2.38×10^{-2} m^2/s. The latter is very close to the pumping test estimate of 2.18×10^{-2} m^2/s but exceeds the Cooper-Jacob estimate by 40%. The corresponding (natural) log transmissivity variance is 1.4, very close to the pumping test estimate of 1.5. As the flowmeter data yield local estimates of transmissivity at only 12 boreholes, we were not able to estimate an integral scale of log transmissivity on the basis of these data on any length scale of relevance.

[43] 13. Four wells were enough to estimate lead statistics of log transmissivity at the site using the distance-drawdown method of *Neuman et al.* [2004].

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