

Asymptotic analysis of three-dimensional pressure interference tests: A point source solution

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[1] Well test analyses conducted in steady state and transient flow regimes have complementary strengths and drawbacks. While steady state analyses are easy to interpret, they are useful only on a late-time portion of the data collected during pressure interference tests and do not provide estimates of porosity. Transient type curve and numerical inverse approaches overcome these shortcomings but are harder to analyze, and their reliability can be affected by changing external forcings. We develop a new approach to estimate permeability and porosity from well tests, which is based on an asymptotic analysis of pressure transients during three-dimensional pressure interference tests. Our approach results in a straight line data fitting, rendering the data interpretation straightforward. It also allows one to use intermediate-to-late time pressure data. To illustrate the advantages of the proposed technique, we use it to analyze data from several cross-hole pneumatic injection tests conducted in unsaturated fractured tuff. We demonstrate that the equivalent permeabilities and porosities obtained from our analysis compare well with their counterparts inferred from more complicated approaches, such as type curve and numerical inverse analyses as well as from a steady state analysis.

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1. Introduction

[2] Traditional interpretation of well test data relies on either steady state or transient methods. Steady state approaches use only a fraction of the pressure transient data collected during a pressure interference test, which corresponds to the steady state flow regime. To achieve steady state, pressure interference tests may have to be run for an exceedingly long time. In fact, it is common for the pressure transients to never reach a steady state. In addition, steady state analyses of pressure interference tests allow one to infer only permeability but not porosity. Transient (e.g., type curve approaches and the semilog analysis of *Cooper and Jacob* [1946]) and inverse methods overcome these shortcomings by enabling one to analyze the transient portion of the data.

[3] Various type curve models developed for different hydrogeologic conditions allowed for the transient analysis of the time drawdown data. For the technique to be applicable and the parameter estimates derived from the technique to be meaningful, the time drawdown data must fit the type curves developed for the situation under

consideration. In many cases these requirements are difficult to meet under field conditions due to factors that complicate the analysis. External factors such as recharge and barometric pressure fluctuations can corrupt the pressure transients making well test interpretation by means of traditional techniques difficult. These complications limit the use of analytical type curve approaches to simple situations.

[4] Numerical inverse approaches can overcome many of these difficulties by incorporating the effects of external forcings and heterogeneities, among other things, but these models can be complex and time consuming to develop. Therefore there is a need for alternative yet complementary interpretive approaches for the analysis of pressure interference tests to yield reliable estimates of flow parameters.

[5] The main objective of this study is to present a new approach for estimating equivalent permeability and porosity from three-dimensional pressure interference tests by analyzing the intermediate to late data. The approach allows for a simple graphical interpretation of data and is based on an asymptotic analysis of the point source solution described in section 3. In section 4, we use the technique to infer the equivalent permeability and porosity of unsaturated fractured tuff from the cross-hole pneumatic injection tests conducted by *Illman et al.* [1998; see also *Illman*, 1999]. Section 5 presents the comparison of these estimates of equivalent permeability and porosity with those derived from the same data set by means of type curve [*Illman*

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and Neuman, 2001] and numerical inverse [Vesselinov *et al.*, 2001a, 2001b] analyses, as well as with the permeability estimates obtained from steady state analysis of Illman and Neuman [2003].

2. Problem Formulation

[6] Pneumatic injection tests in partially saturated porous media or fractured rocks induce two-phase flow described by

$$-\nabla \cdot (\rho_f \mathbf{q}_f) = \phi \frac{\partial (S_f \rho_f)}{\partial t}, \quad f = w, a, \quad (1)$$

where the subscripts w and a refer to water and air, respectively; ρ is mass density; \mathbf{q} is flux density; ϕ is porosity; and S is fluid saturation. This formulation assumes isothermal conditions, a rigid porous medium, and the absence of mass transfer between water and air.

[7] Furthermore, we assume that both fluxes \mathbf{q}_f ($f = w, a$) are Darcian, i.e., that Darcy's law applies to both water and air. While in general gas flow might become non-Darcian due to inertial effects, Knudsen diffusion, and/or slip flow (the so-called *Klinkenberg* [1941] effect), laboratory [Alzaydi *et al.*, 1978] and field-scale [Guzman *et al.*, 1996] experiments have shown that their effects on test interpretations are negligible. For anisotropic porous media, Darcy's law gives

$$\mathbf{q}_w = -\frac{\rho_w g}{\mu_w} k_w(S_w) \mathbf{T} \nabla h \quad \mathbf{q}_a = -\frac{k_a(S_a)}{\mu_a} \mathbf{T} \nabla p_a, \quad (2)$$

where g is the gravitational constant, μ_w and μ_a are the dynamic viscosities of water and air, k_w and k_a are the relative permeabilities of a porous medium for both water and air, \mathbf{T} is the dimensionless tensor responsible for the directional anisotropy of the medium, and p_a is air pressure. Hydraulic head h is defined by

$$h = \frac{p_w}{\rho_w g} + z, \quad (3)$$

where p_w is water pressure and z is elevation about an arbitrary datum.

[8] The equations for water and air phases are coupled by the following relationships:

$$S_w + S_a = 1 \quad p_c = p_a - p_w, \quad (4)$$

where p_c is capillary pressure. The model formulation is completed by specifying appropriate equations of state, i.e., the functional dependencies of fluid properties on pressure, and relative permeabilities and capillary pressure on saturation.

[9] To analyze airflow around wells during pneumatic injection tests, it is common [e.g., Illman and Neuman,

2000, 2001] to treat the water phase as immobile. Then (1)–(4) reduce to

$$\nabla \cdot \left(\rho \frac{k}{\mu} \mathbf{T} \nabla p \right) = \phi \frac{\partial \rho}{\partial t}, \quad (5)$$

where k is intrinsic permeability, and the subscript a has been omitted. Following the linearization procedure used by Illman and Neuman [2000, 2001] to describe air flow in isotropic media, we approximate (5) by

$$\nabla \cdot (\mathbf{k} \nabla p) = \frac{\phi \mu}{p_{ave}} \frac{\partial p}{\partial t}, \quad \mathbf{k} = k \mathbf{T}, \quad (6)$$

where p_{ave} is average pressure. This, of course, is the standard diffusion equation used to describe flow in saturated anisotropic porous media.

3. Methodology

[10] The full solution for a point source in an infinite three-dimensional anisotropic homogeneous medium is given by [Hsieh *et al.*, 1985, equations (7)–(9)]

$$p_d(t_d) = \operatorname{erfc} \left(\frac{1}{\sqrt{4t_d}} \right), \quad (7)$$

where the dimensionless pressure drop in the monitoring interval and time are given by

$$p_d = \frac{4\pi r p}{q \mu} \sqrt{\frac{D}{k_d}} \quad t_d = \frac{k_d t p_{ave}}{\phi \mu r^2}, \quad (8)$$

respectively. Here r is the distance between the centroids of the injection and monitoring intervals, q is flow rate, and D and k_d are the determinant and the canonical ellipsoid of \mathbf{k} , respectively.

[11] Expanding (7) into an asymptotic series [Abramowitz and Stegun, 1972, equation (7.1.5)],

$$p_d(t_d) = 1 - \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(2n+1)} (4t_d)^{-n-1/2}, \quad (9)$$

gives, for large t_d ,

$$p_d(t_d) \approx 1 - \frac{1}{\sqrt{\pi t_d}}. \quad (10)$$

Hence at large times the change in pressure inside the monitoring interval can be approximated by

$$p = \frac{q \mu}{4\pi r} \sqrt{\frac{k_d}{D}} - \frac{q \mu^{3/2} \phi^{1/2}}{4\pi^{3/2} D^{1/2} p_{ave}^{1/2}} t^{-1/2}, \quad (11)$$

i.e., p varies linearly with $t^{-1/2}$. This leads to the following data interpretation procedure.

[12] First, the data on the change in pressure p at a given monitoring interval are plotted against the reciprocal of the square root of time $t^{-1/2}$. A linear regime should develop for a portion of the data, to which a straight line is fitted. The

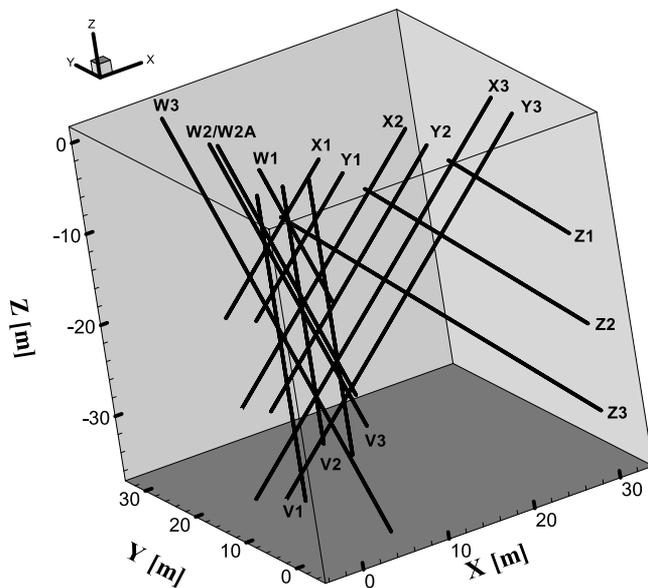


Figure 1. Vertical and inclined boreholes at the Apache Leap Research Site (ALRS) near Superior, Arizona, USA.

intersection of this straight line with the time axis $t^{-1/2} = 0$ is denoted by p^* . Then (11) yields

$$\sqrt{\frac{D}{k_d}} = \frac{q\mu}{4\pi r p^*}, \quad (12)$$

from which the principal components of the permeability tensor and the corresponding canonical ellipse k_d are determined following the procedure outlined by Hsieh *et al.* [1985]. Let t^* denote the time at which the straight line crosses the horizontal coordinate, i.e., the time at which $p = 0$. Then porosity ϕ can be found from (11) as

$$\phi = \frac{\pi k_d p_{ave} t^*}{\mu r^2}. \quad (13)$$

[13] For isotropic media, the dimensionless pressure and time in (8) become $p_d = 4\pi k r p / (q\mu)$ and $t_d = k t p_{ave} / (\phi \mu r^2)$, respectively. Then (12)–(13) simplify to give the following expressions for permeability and porosity:

$$k = \frac{q\mu}{4\pi r p^*} \quad \phi = \frac{q p_{ave} t^*}{4r^3 p^*}. \quad (14)$$

4. Application to Three-Dimensional Pressure Interference Tests

[14] We apply our technique to determine the pneumatic parameters of an unsaturated fractured tuff from a set of three-dimensional pressure interference tests conducted at the Apache Leap Research Site (ALRS).

4.1. Site and Test Description

[15] The ALRS is located near Superior, Arizona, at an elevation of 1200 m above sea level. The test site contains 22 vertical and inclined (at 45°) boreholes that have been completed to a maximum depth of 30 m within a geolog-

ically distinct unit of partially welded unsaturated tuff. The upper 1.8 m of each borehole is cased. Core samples were taken from 9 of the 22 boreholes and a variety of tests were performed by Rasmussen *et al.* [1990, 1993] to determine the interstitial properties of the tuff matrix. Single-hole pneumatic and hydraulic injection tests were initially conducted by Rasmussen *et al.* [1990, 1993] with an injection interval length of 3 m to determine estimates of permeabilities of the fractured tuff. Guzman *et al.* [1996] then conducted over 270 single-hole pneumatic injection tests in 6 of the 22 boreholes with various injection interval lengths. Additional details to these tests and the site are provided by Rasmussen *et al.* [1990, 1993], Guzman *et al.* [1996], and Illman *et al.* [1998].

[16] Core and single-hole pneumatic injection tests provide information only about a small volume of rock in the close vicinity of the injection interval. Fractured rock properties measured on such small scales tend to vary erratically in space so as to render the rock strongly heterogeneous. To determine the properties of the rock on larger scales, Illman *et al.* [1998; see also Illman, 1999] conducted 44 cross-hole pneumatic injection tests between 16 boreholes (one of which included all 22 boreholes), 11 of which have been previously subjected to single-hole testing (Figure 1). The tests consisted of injecting air into an isolated interval within one borehole, while monitoring pressure responses in isolated intervals within this and all other boreholes. The purpose of these tests was to determine the bulk pneumatic properties of larger rock volumes between boreholes at the site, and the degree to which fractures are pneumatically interconnected.

4.2. Results

[17] The data collected from one such test (PP4) was analyzed by means of type curves [Illman and Neuman, 2001], five tests (PP4–PP8) by means of a three-dimensional numerical inverse model [Vesselinov *et al.*, 2001a, 2001b], and eleven tests [PL3, PL4, PL8, PL9, PL10, PL15, and PP4–PP8] by a steady state approach [Illman and Neuman, 2003]. We use these data to demonstrate our asymptotic approach and to show its consistency with alternative (and more complicated) data interpretation techniques. To employ the methodology developed in the previous section, we consider the pressure data collected from tests in which both the injection and monitoring intervals are short enough to be regarded, for purposes of analysis, as points.

[18] Figure 2 shows how the straight line is fitted to pressure records collected from monitoring intervals during cross-hole test PP4. One can see that after an early time, during which the pressure behavior may be dominated by the effects of borehole storage, skin, and heterogeneity, a straight line develops. The data deviate from a straight line at a later time, during which the test is affected by barometric pressure. Figure 2a shows a pressure record, in which a signal-to-noise ratio is relatively large, making the identification of the straight line portion of the pressure transients relatively easy. In contrast, the data plotted in Figure 2b have a low signal-to-noise ratio, rendering the identification of the straight line more difficult.

[19] \log_{10} -transformed permeability values from the asymptotic analysis range from -14.43 ($3.74 \times 10^{-15} \text{ m}^2$) to -12.43 ($3.68 \times 10^{-13} \text{ m}^2$) with a mean of -13.65 ($2.25 \times 10^{-14} \text{ m}^2$), variance of 0.20, and a coefficient of variation of

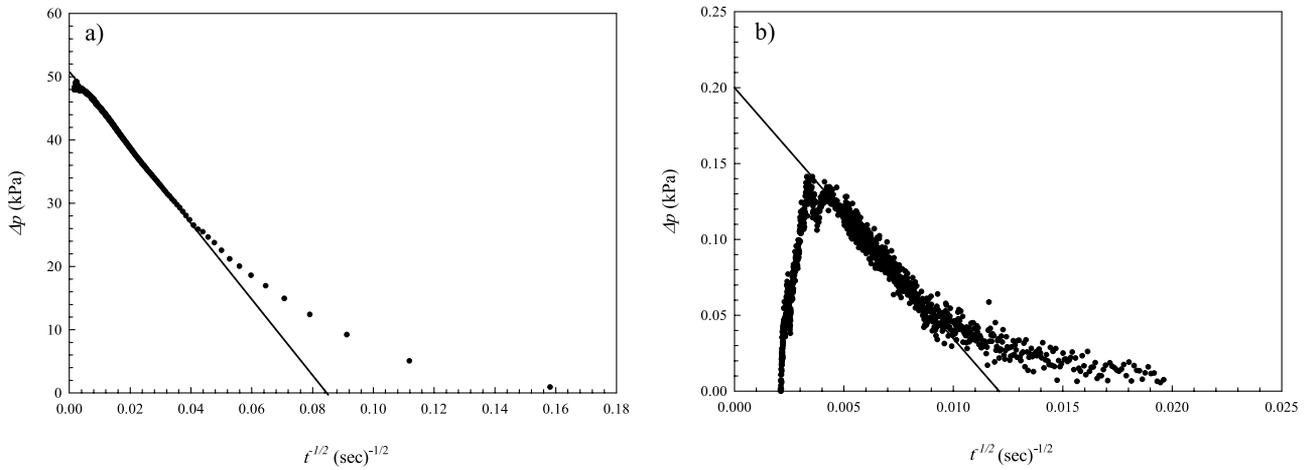


Figure 2. Asymptotic analysis of pressure transients in monitoring intervals (a) W1 and (b) W2AM during cross-hole pneumatic injection test PP4.

-0.033. Log_{10} -transformed porosity values range from -2.59 (2.59×10^{-3}) to -0.76 (1.74×10^{-1}) with a mean of -1.69 (2.06×10^{-2}), variance of 0.21, and a coefficient of variation of -0.270 .

[20] Following a standard practice, our analysis of pressure transient data assumes that on the scale of the cross-hole test the rock is pneumatically uniform and isotropic. Thus the values of permeabilities and porosities can be viewed as bulk directional properties of the rock associated with given injection and monitoring intervals. In addition, data from different monitoring intervals during a given

three-dimensional cross-hole test are seen to yield different values of equivalent permeabilities and porosities, thereby providing information about their spatial and directional dependence.

5. Discussion

5.1. Comparison With Results From Type Curve Analysis

[21] Permeability estimates obtained from the asymptotic analyses are compared with those derived from the type

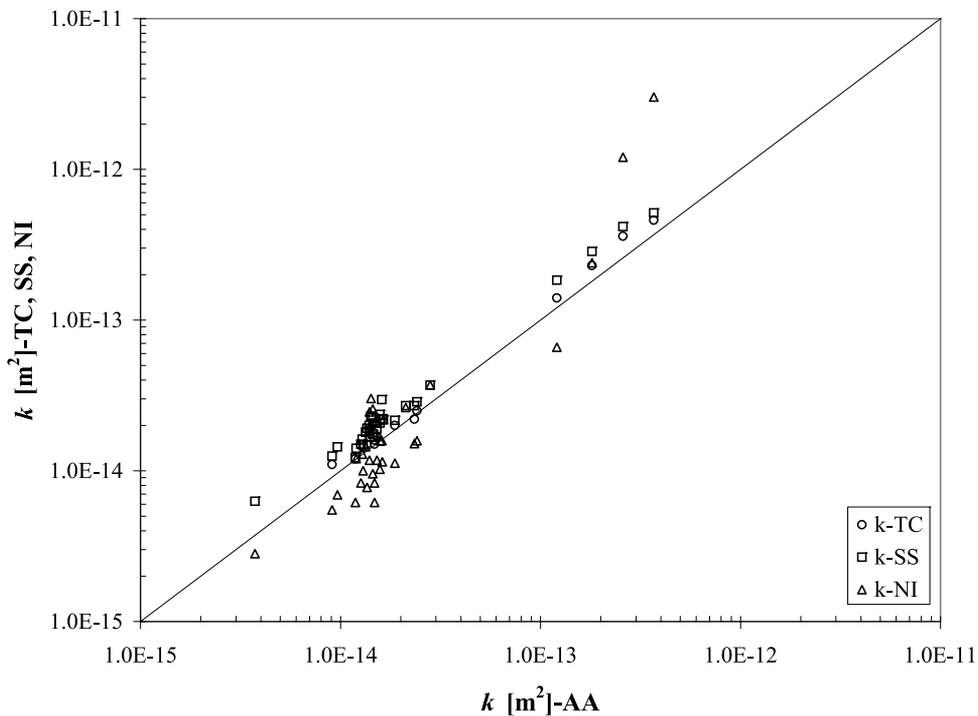


Figure 3. Comparison of permeabilities determined from the asymptotic (AA), transient type curve (TC), steady state (SS), and numerical inverse (NI) analyses of the data from test PP4 with $\beta_1 > 5$ (point source case).

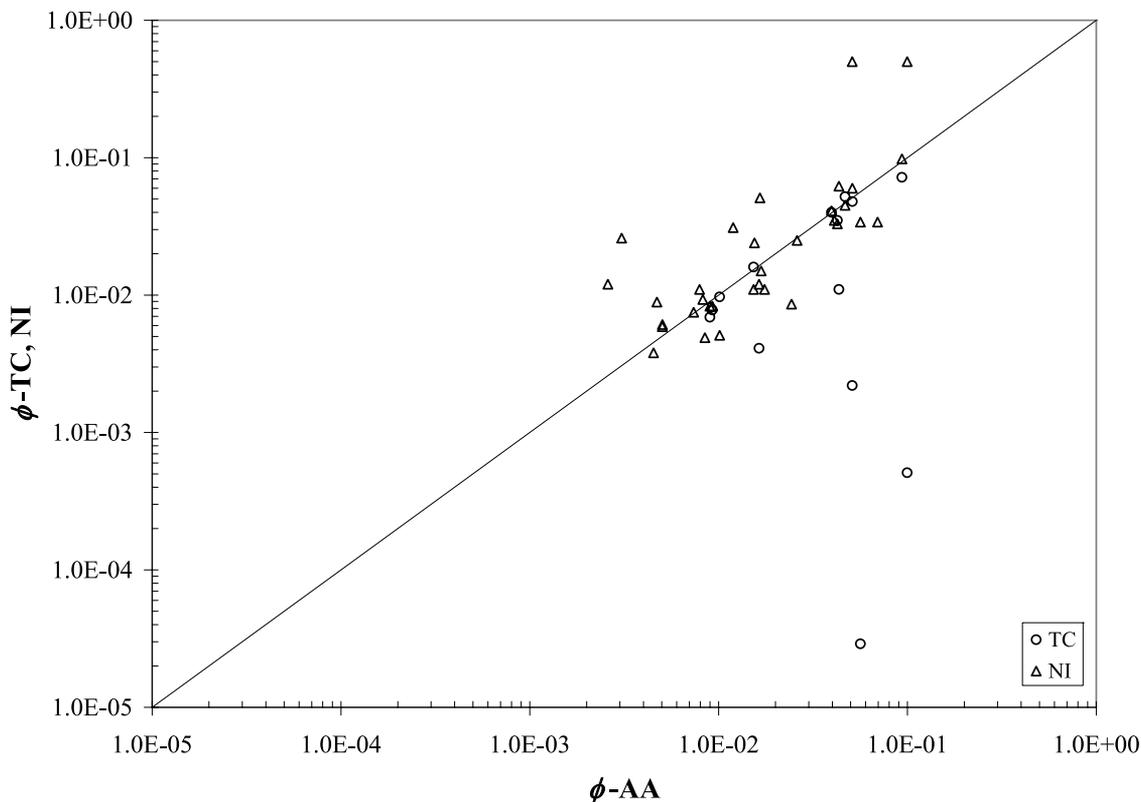


Figure 4. Comparison of porosities determined from the asymptotic (AA), type curve (TC), and numerical inverse (NI) analyses of the data from test PP4 with $\beta_1 > 5$ (point source case).

curve analysis of *Illman and Neuman* [2001]. Figure 3 shows that the agreement between the two estimates is excellent.

[22] Figure 4 provides a similar comparison of porosity estimates. While most estimates are in good agreement, several type curve estimates are heavily biased toward lower porosity values. This discrepancy stems from the data collected in monitoring intervals of boreholes Y3, Z2 and Z3. *Illman and Neuman* [2001] interpreted these data [see, e.g., *Illman and Neuman*, 2001, Figure 10j] to have a very high well bore storage, which significantly alters the analysis of the early time data and, in our opinion, causes the porosity estimates to be artificially low. Since our asymptotic analysis uses primarily the intermediate data, it is not affected by the well bore storage effect and, hence, yields porosity estimates that are more consistent with those obtained from the numerical inverse interpretation discussed in section 4.2.

[23] Another factor that can corrupt estimates of porosity is a poor match between type curves and early time data. For instance, the majority of the early time data from test PP5 failed to match the type curves of *Illman and Neuman* [2001]. One of these data sets is shown in Figure 5. One can see that pressure transients arrive later than the theoretical curves, which implies the presence of a low-permeability region between the injection and monitoring intervals. This causes the horizontal match to be nonunique, even when pressure derivatives and recovery techniques [*Illman and Neuman*, 2001] are used, and renders the porosity estimates derived from such matches highly unreliable. Our asymp-

totic analysis eliminates this ambiguity in porosity estimates by focusing on the fit to the straight line portion of the data.

5.2. Comparison With Results From Steady State Analysis

[24] Next, we compare our permeability estimates with those derived from the steady state analysis of *Illman and Neuman* [2003]. Figure 3 reveals that this comparison is quite good, with a slight bias toward the steady state estimates of permeability. This may be due to the fact that the steady state estimates are associated with a larger volume of the rock, since these estimates are based on late time data. Such a time dependence of permeability was observed by *Schulze-Makuch and Cherkauer* [1998] during their analysis of pumping test data in fractured carbonates.

5.3. Comparison With Results From Numerical Inverse Analysis

[25] Finally, we compare our estimates of permeability and porosity with those obtained by *Vesselinov et al.* [2001a, 2001b] from a three-dimensional numerical inverse interpretation of the same data.

[26] *Vesselinov et al.* [2001a, 2001b] analyzed data one pressure record at a time, which rendered their approach similar in spirit to the analytical interpretive techniques described here. Each numerical inversion required ≈ 80 forward simulations, so that the complete analysis took about four hours on the University of Arizona SGI Origin multiprocessor supercomputer. To interpret the cross-hole tests with the inverse model, *Vesselinov et al.* [2001a, 2001b]

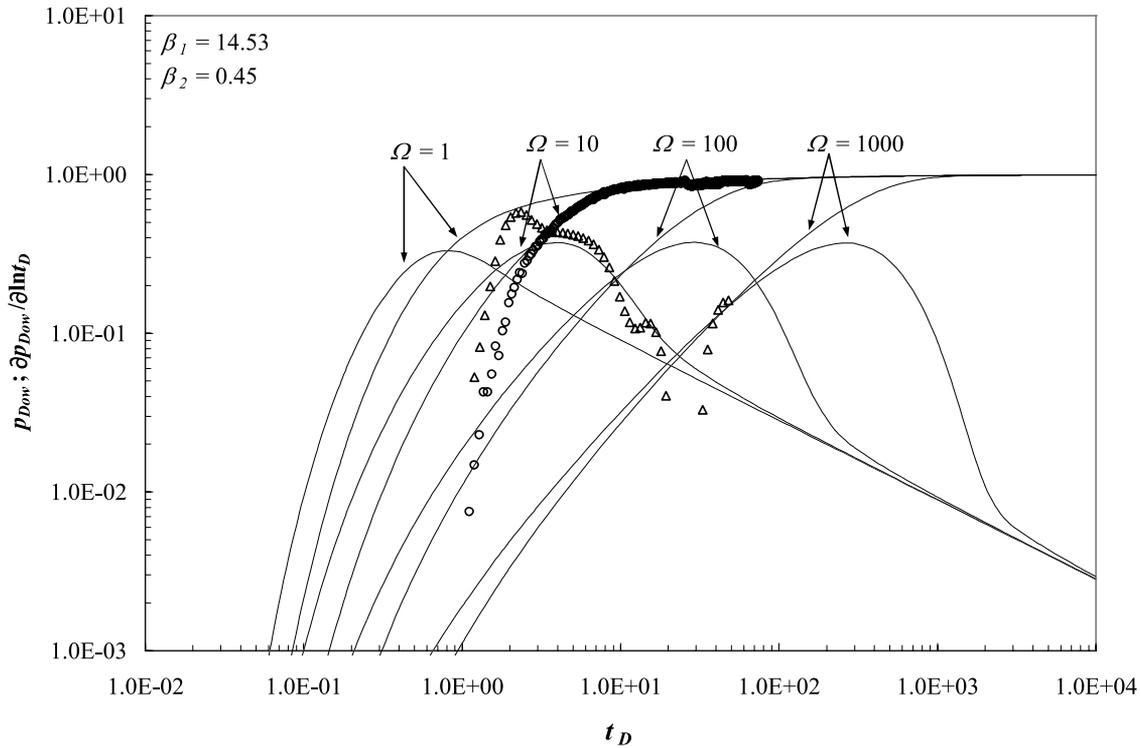


Figure 5. Pressure (circles) and its derivative (triangles) from monitoring interval V3M during cross-hole test PP5 matched against cross-hole type curves of varying geometrical parameters (β_1 , β_2) and dimensionless well response time (Ω). The solid curve represents dimensionless pressure (p_{Dow}), while the dashed curve represents the dimensionless pressure derivative ($\partial p_{Dow}/\partial \ln t_d$). See *Illman and Neuman* [2001] for additional details on the definition of parameters.

selected pressure records in which pressure transients were caused primarily by air injection, thus reducing the large set of recorded pressures to a manageable number without the significant loss of information. They did so by ignoring the portions of a pressure record they deemed to be strongly influenced by barometric pressure fluctuations and/or other extraneous phenomena and by representing the remaining portions via a relatively small number of “match points.” The match points were distributed more or less uniformly along the log-transformed time axis, so as to capture with equal fidelity both rapid pressure transients at early time and more gradual pressure variations at later time. Matching was done with equal weighting using the match points with the numerical inverse interpretation.

[27] Figure 3 demonstrates that our estimates of permeability compare well with those based on the inverse model that treats the rock as uniform. The scatter between the two estimates is greater than the scatter between our estimate and the estimates based on the type curve and steady state approaches.

[28] A similar comparison of porosity estimates is shown in Figure 4. It reveals a much larger scatter, which is not surprising since the porosity estimates are more uncertain than the estimates of permeability. This uncertainty manifests itself through wider confidence intervals for the estimates of porosity [*Vesselinov et al.*, 2001a, 2001b].

6. Conclusions

[29] We developed a new approach to estimate permeability and porosity from well tests, which relies on the

asymptotic analysis of pressure transients resulting from three-dimensional pressure interference tests. This approach was then used to analyze data from the cross-hole pneumatic injection tests conducted by *Illman et al.* [1998; see also *Illman*, 1999]. The estimates of permeability and porosity obtained from this analysis were compared with the estimates derived from the type curve [*Illman and Neuman*, 2001], numerical inverse [*Vesselinov et al.*, 2001a, 2001b], and steady state [*Illman and Neuman*, 2003] analyses. This study leads to the following major conclusions.

[30] 1. At intermediate-to-late time t , pressure transients vary linearly with $t^{-1/2}$, which paves the way for a straight line data interpretation. A major advantage of our approach over currently used type curve and numerical inverse models lies in its simplicity.

[31] 2. The estimates of permeability obtained from the asymptotic analysis are in excellent agreement with those obtained from the type curve analysis of *Illman and Neuman* [2001]. They also compare well with the estimates of permeability obtained from the steady state approach of *Illman and Neuman* [2003], although the latter are slightly biased toward higher values. Finally, estimates of permeability obtained from the asymptotic analysis are in good agreement with those obtained from the numerical inverse approach of *Vesselinov et al.* [2001a, 2001b], but there is more scatter in the data.

[32] 3. The agreement between the asymptotic and type curve estimates of porosities is not as good as that for permeabilities. However, it should be noted that the porosity estimates obtained from type curve analyses are often less

reliable, since early time data are usually influenced by borehole storage, skin, and subsurface heterogeneity. The comparison of the estimates of porosity obtained from the asymptotic analysis and numerical inverse modeling shows increased scatter, suggesting the higher uncertainty in the parameter.

[33] 4. The asymptotic analysis is much easier to conduct than either the transient type curve [Illman and Neuman, 2001] or numerical inverse [Vesselinov *et al.*, 2001a, 2001b] analyses. In particular, the computational demands of the numerical inverse approach are so heavy, that it could only be applied to a relatively few single- and cross-hole tests.

[34] 5. Our asymptotic approach works well even for pressure records, whose signal-to-noise ratio is too low to allow meaningful transient analysis. This includes cases where pressure transients are heavily influenced by borehole storage, external forcings, and heterogeneities, all of which cause data to depart from type curve and numerical inverse methods that ignore these and other phenomena.

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